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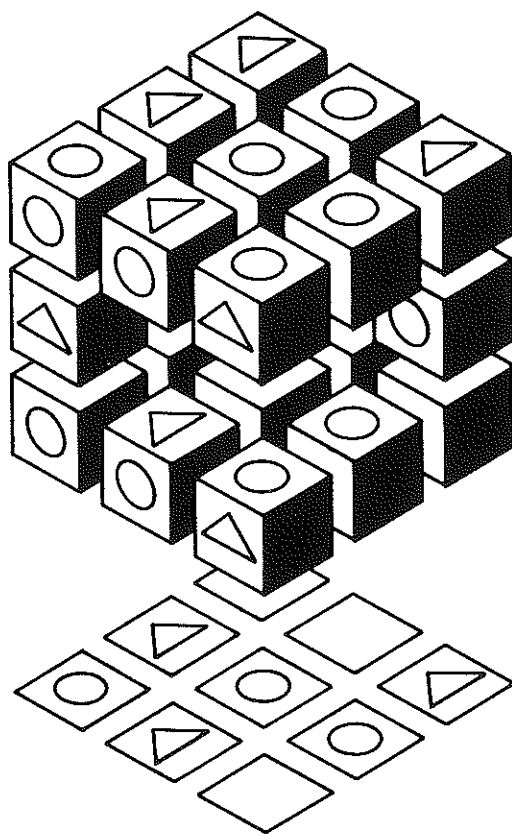
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REPRESENTING
KINSHIP:
SIMPLE MODELS
OF
ELEMENTARY STRUCTURES



F. E. TJON SIE FAT

Note: According to the PhD Regulations at Leiden University, the PhD candidate is required to submit a dissertation and accompany this thesis with a number of propositions (“stellingen”) which must be defensible “with scientific arguments”. As the thesis itself can be very specialized, the propositions allow the candidate to display a more general field of interest. The final proposition may be more playful.

The rather quaint constraints on the propositions are set out in Article 12:

“Article 12. Propositions

1. As soon as possible after the approval as referred to in Article 10, the PhD candidate submits to the supervisor at least four propositions relating to the subject of the dissertation, at least four scientific propositions relating to the field of the subject of the dissertation, and at most four propositions on one or more subjects of the candidate's choice.
2. It must be possible to defend the propositions with scientific arguments.
3. The supervisor informs the PhD candidate whether the propositions in his opinion meet the required standards as referred to in the first and second paragraphs. In the affirmative case, the supervisor sends the texts of the propositions and his assessment of them to the Dean, who may, if he wishes, test them himself against the required standards.”

The seven propositions supplementing my 1990 thesis are included below.

Franklin Tjon Sie Fat

Leiden, April 15, 2014

Franklin E. Tjon Sie Fat (1990). *Representing Kinship: Simple Models of Elementary Structures*. Leiden (thesis).

Proposition 1: Weyl (1982 [1950]:144) has advocated the following guiding principle for research in mathematics: 'Whenever you have to do with a structure-endowed entity Σ , try to determine its group of automorphisms'. This principle is equally important as a prescription for research in structural anthropology.

Weyl, H. (1982 [1950]). *Symmetry*. Princeton: Princeton University Press.

Proposition 2: A major problem with Dumont's (1980:239-243) definition of a hierarchical relation lies in the confusion of element/set or class inclusion rules with various types of mereological (i.e., part-to-whole) relationships.

Dumont, L. (1980). *Homo Hierarchicus: The Caste System and Its Implications*. Chicago: University of Chicago Press (revised English edition).

Proposition 3: Dumont's hierarchical opposition (i.e., 'the encompassing of the contrary'), defined in terms of set theory (1980:239-243), is only a 'logical scandal' (1980:242) if the standard ZF (Zermelo-Fraenkel) axiom system is assumed. Under the elegant new alternative developed by Aczel (1988; cf. Barwise and Etchemendy 1989), circular phenomena (including self-membership and self-reference) may be modeled in a straightforward manner.

Aczel, P. (1988). *Non-Well-Founded Sets*. Stanford: CSLI Lecture Notes Number 14.

Barwise, J. and J. Etchemendy (1989). *The Liar: An Essay on Truth and Circularity*. Oxford: Oxford University Press.

Dumont, L. (1980). *Homo Hierarchicus: The Caste System and Its Implications*. Chicago: University of Chicago Press (revised English edition).

Proposition 4: According to Howard (1988), an ethnographic account is best treated as an exercise in multisequential, multimedia programming, i.e. as an extension of *hypertext*, not *text* — thus eliminating many of the constraints imposed by the linear sequential mode of the standard written presentation. Howard's suggestions merit serious consideration by anthropologists.

Howard, A. (1988). Hypermedia and the future of ethnography. *Cultural Anthropology* 3(3):304-315.

Proposition 5: Héritier's (1981:19, 39-45) 'missing' equation $[MB = F] \neq FB$ has not (as she implies) been wholly ignored in previous kinship studies. As one of the five logically possible kin terminological partitions on the set of G+I relations $\{F, FB, MB\}$, this equation figures prominently in Greenberg's (1966: 72-87) discussion of the universals of kinship terminology.

Greenberg, J.H. (1966). *Language Universals. With Special Reference to Feature Hierarchies*. The Hague: Mouton.
Héritier, F. (1981). *L'exercice de la parenté*. Paris: Gallimard.

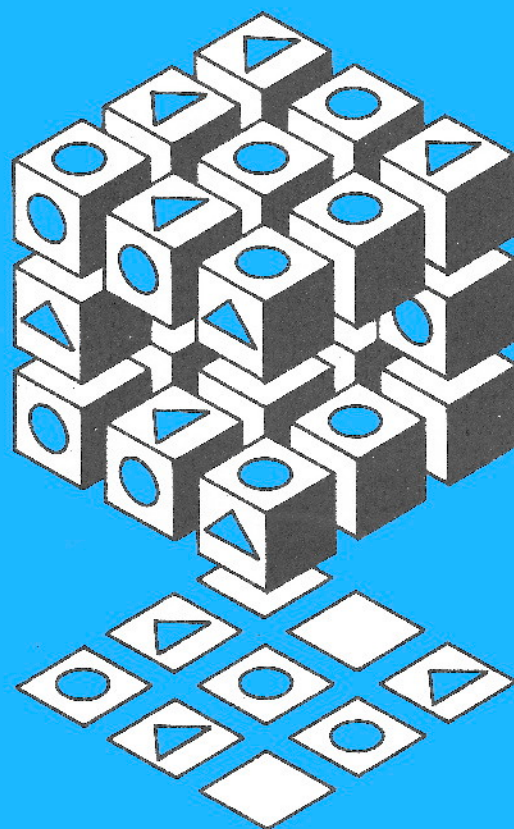
Proposition 6: Proposition 6, not its direct successor, is the equivalent of the traditional 'final proposition'.

Proposition 7: More sophisticated theories of kinship and social organization may emerge if the present framework is extended to include concepts, methods and techniques developed for the modeling and simulation of 'complex systems' (cf. Langton 1989 and Stein 1989).

Langton, C.G. (ed.) (1989). *Artificial Life*. Santa Fe Institute Studies in the Science of Complexity, Volume 6. Redwood City, Ca.: Addison-Wesley.

Stein, D.L. (ed.) (1989). *Lectures in the Sciences of Complexity, Volume 1*. Santa Fe Institute Studies in the Sciences of Complexity. Redwood City, Ca.: Addison-Wesley.

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OF
ELEMENTARY STRUCTURES



F.E. TJON SIE FAT

R E P R E S E N T I N G K I N S H I P :

S I M P L E M O D E L S O F E L E M E N T A R Y S T R U C T U R E S

PROMOTIECOMMISSIE

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R E P R E S E N T I N G K I N S H I P :

S I M P L E M O D E L S O F E L E M E N T A R Y S T R U C T U R E S

PROEFSCHRIFT

ter verkrijging van de graad van Doctor
aan de Rijksuniversiteit te Leiden
op gezag van de Rector Magnificus
Dr. J.J.M. Beenakker, hoogleraar in de
faculteit der Wiskunde en Natuurwetenschappen,
volgens besluit van het college van dekanen
te verdedigen op dinsdag
27 november 1990 te klokke 15.15 uur

door

Franklin Edmund Tjon Sie Fat
geboren te Willemstad, Curaçao in 1947

To the Memory of my Father

To my Mother

To Mildred and Lisa

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*The book is written in the mathematical language,
and the symbols are triangles, circles and
other geometrical figures, without the help of which
it is impossible to conceive a single word of it ...*

Galileo Galilei, *Opera IV*¹

*The simplicities of natural laws arise
through the complexities of the languages
we use for their expression.*

E.P. Wigner²

*For seven and a half million years, Deep Thought
computed and calculated, and in the end announced
that the answer was in fact Forty-two — and so
another, even bigger, computer had to be built
to find out what the actual question was.*

Douglas Adams,
*The Restaurant at the
End of the Universe*³

1 and 2: cited in Mackay (1977:62, 162).
3: volume 2 of Adams (1979-1985).

PREFACE

The book that follows is concerned with the formalization of anthropological theory, in particular, with theories of kinship. The central thesis of my research is that a systematic treatment of theory-structure is a prerequisite to the development of an adequate framework for representing and integrating kinship phenomena, and that a structural reconstruction of major segments of kinship theory is fundamental to a critical assessment of the dynamics of theory-change.

Earlier versions of parts of Chapter 2 and Chapter 3 were published in *Current Anthropology* and *American Ethnologist* respectively. Many sections of the thesis were first presented at the seminar on cognitive and structural anthropology (CASA), a research programme of the Institute of Cultural Anthropology at Leiden University. Sections of Chapter 5 were recently worked up for a lecture at the Department of Anthropology, University College London, in the context of the Erasmus exchange programme. I am grateful to my colleagues and friends, and to the editors and anonymous referees of the journals concerned for their numerous helpful suggestions and perceptive criticisms.

The Netherlands Foundation for the Advancement of Tropical Research (WOTRO) provided financial assistance during 1981 and 1982; for the forbearance in awaiting a thesis that bears but little resemblance to the research originally proposed I am indeed indebted.

My intellectual debts are many. Unfortunately, academic tradition at Leiden University does not allow me to thank the individual members of the staff (past and present) who have influenced my thinking about anthropology and who have provided assistance in the preparation of this thesis. Without their encouragement, this book would never have been written. It is a belated and unequal counter-prestition.

Finally, then, to Mildred, for her unstinting support and patience: many, many thanks.

0. PROLOGUE

I am not trying to argue that we can use mathematics to solve anthropological problems. What I do claim is that the abstraction of mathematical statement has great virtues in itself. By translating anthropological facts into mathematical language, however crude, we can get away from excessive entanglement in empirical facts and value loaded concepts.

E.R. Leach (1959), *Rethinking Anthropology. The First Malinowski Memorial Lecture.*¹

An emerging trend in the current cycle of anthropological theorizing, perhaps motivated by the sense of malaise experienced by the community of anthropologists in the 1970s and 1980s, is the great emphasis placed on relativism and reflexivity as core values. In *Against Method: Outline of an Anarchistic Theory of Knowledge* (1984 [1975]) Paul Feyerabend had already argued that science (only one of the many forms of thought developed by man) is much closer to myth than orthodox scientific philosophy is prepared to admit. One should focus on the study of scientific practice, not on method and the way in which a scientific result is ultimately presented and justified. What is required is an understanding of the particular *Weltanschauung* or perspective which conceptually shapes the way one experiences the world, and which determines the selection of legitimate problems as well as the criteria for their acceptable solution. From this perspective, a critical analysis of the history of ideas and the sociological factors influencing their development, persistence, and transformation is required. Anthropological method is thus accorded a privileged position: 'My argument presupposes, of course, that the anthropological method is the correct method for studying the structure of science (and, for that matter, of any other form of life)' (Feyerabend 1984:252).

Recent critics of anthropology have now taken Feyerabend's programme one step further. Focussing mainly on the study and interpretation of ethnography as texts, and explicitly concerned with the epistemology of textual (de)construction and with the (re)presentation of ethnography as objective discourse, anthropology itself has now become the object of anthropological self-critique. Under the more radical forms of 'postmodern' skepticism, the rejection of science as legitimating value is complete: anthropology, in achieving phenomenological validity, is reduced to a form of literary culture critique, trapped in the infinite regress of meta-self-reflexivity.²

The debate has now commenced, with much posturing and potent rhetoric from both sides of the divide. (See, for example, Sangren's recent (1988) polemic, with comments, reactions, and his reply.) However, if the history of anthropological theory is anything to go by, one should not be overly optimistic about the possibility of cumulative progress arising through discussion and the elimination of past errors.

Thus, Barrett (1984), applying a Kuhnian framework to the history of anthropological theory, argues that theory development within the discipline has failed to be cumulative. Specific theoretical orientations emerge, disappear, and reappear, expressing a sequence of transformations based on a limited number of underlying 'conceptual contradictions' which are themselves never resolved (Barrett 1984:73-99). The character of anthropological theory is repetitive, oscillatory, and cyclic, and the analysis of its history closely resembles the Lévi-Straussian analysis of myth. Thus (Barrett 1984:4): 'Like myth, theories do not become "better" over time, and are "good to think" even if they do not explain'. Kuper has recently developed a similar argument. In *The Invention of Primitive Society. Transformations of an Illusion* (1988) Kuper explains the emergence and persistence of one of the 'central orthodoxies' of social

anthropology in much the same terms. 'Primitive' society, conceived as the mirror image of 'modern' society, generated an entire series of models and specific theories of society, with each new variant often merely a straightforward structural transformation of its predecessor (1988:5-14).

Other examples abound. In the first Malinowski Memorial Lecture of 1959 Edmund Leach set out to 'rethink' anthropology, sketching in broad strokes a programme for achieving genuinely unbiased generalizations. This new programme was formulated in direct opposition to the (then prevailing) tendency of writing impeccably detailed historical ethnographies of particular societies, as well as to the method of social-structure comparison championed by Radcliffe-Brown (a type of analysis scorned by Leach as mere 'butterfly collecting'; 1971:2-6). Leach argued that valid generalizations could only be obtained if anthropologists were willing to consider societies mathematically. His formal paradigm was topology, roughly, the branch of mathematics concerned with describing the properties of geometrical figures that are unaffected by continuous transformations such as stretching.

Formulated in opposition to the orthodox views of functionalist anthropology, Leach's programme of topological generalization was never carried out. Only five years later, in a wholly negative review of Harrison White's *An Anatomy of Kinship: Mathematical Models for Structures of Cumulated Roles* (1964), the use of mathematics in kinship theory is denounced. By 1978 Leach's recantation and self-refutation is complete. In a reaction to the topological approach developed by Hillier et al. (1978) for the comparative generalization of spatial structure (expressed through building forms and settlement patterns) from various societies, his pronouncements are positively Malinowskian.³ Thus (Leach 1978:400):

In kinship studies we learned long ago that, precisely at

the point where model building begins to turn into formal mathematics, the whole exercise becomes analytically worthless. This has happened repeatedly over the past sixty years and comes about because the mathematical model fails to take account of the complexities of the 'real' situation.

The shift in perspective is striking: if, in 1959, the use of mathematics should enable us to 'get away from excessive entanglement in empirical facts and value loaded concepts', by 1978 an identical approach 'fails to take account of the complexities of the "real" situation' (Leach 1971:13; 1978:400). To quote Lévi-Strauss (1970 [1964]:199): 'the armature remains constant, the code is changed, and the message is reversed'.

There are, indeed, sound methodological reasons for rejecting *both* conceptions on the use of mathematics held (in succession) by Leach. In attacking the comparative studies of Radcliffe-Brown and his successors, Leach was inveighing against the application of a naive positivist and empiricist framework, in which the proliferation of typologies and classificatory schemes merely reflected the *a priori* verbal categories of the anthropologist, not the reality of ethnographic fact (Leach 1971:25-27). However, Leach's proposal — substituting the supposedly value-free categories of a mathematical (topological) calculus, and then translating anthropological facts into mathematical language — remains firmly bound to logical positivist conceptions. Thus, under the so-called Received View on Theories, scientific theories are construed as partially interpreted axiomatic systems (formal calculi), i.e., as linguistic entities in which theoretical terms are given a partial observational interpretation by means of correspondence rules. Thinking about society in a mathematical way (Leach 1971:7) is not a sufficient premise for a radical rethinking of anthropology.

Leach's later critique of mathematical modelling is particularly inapt. The map is not the world; the menu is never as nourishing as the food; never equate the model

with reality. If a model fails to take account of the complexities of the 'real' world which are to be modelled, it should be replaced by a more adequate representation. There is no *a priori* reason for rejecting mathematical formulations as being fundamentally deficient (as opposed to say, anthropological description or some other form of non-mathematical representation).

One way of breaking out of the sterile cycle of non-cumulative theory substitution is to provide a detailed historical critique of how a particular tradition or model has persisted, constraining our perceptions and limiting our strategies for research. This is the method adopted by Kuper (1988). My strategy is more conventional (grounded in merely modernist epistemology). First, whatever its limitations, I hold that formal representation provides the primary technique for exploring and clarifying conceptual problems and for making explicit the foundational assumptions of scientific theories (cf. Suppes 1968). Second, the formal representation and reconstruction of theories is a prerequisite to their comparison. A sufficiently rich framework (to include 'pragmatic' and 'socio-historical' concepts; see Balzer et al. 1987:205-246) will enable us to illuminate and to evaluate more precisely the significant features associated with theory-change or non-cumulative theory transformation.

With the Received View now repudiated (see Suppe 1977 and 1989 for a definitive account of its demise), positivistic treatments of theory formalization are no longer tenable. More specifically, the fundamental assumption that theories are linguistic entities must now be rejected. Under the most promising of the alternative views that have come to the fore since the 1970s, theories no longer consist of an axiomatization in mathematical logic, together with an empirical interpretation.

For example, the 'semantic' (as opposed to 'syntactic') conception of theories is summarized as follows (Van Fraassen (1987 [1980]:64):

To present a theory is to specify a family of structures, its *models*; and secondly, to specify certain parts of those models (the *empirical substructures*) as candidates for the direct representation of observable phenomena. The structures which can be described in experimental and measurement reports we can call *appearances*: the theory is empirically adequate if it has some model such that all appearances are isomorphic to empirical substructures of that model. [original emphasis].

Theories are thus specified as classes of models and their substructures, not as the interpreted statements and formulae of a logical-mathematical calculus. Under the new conception of theories, the 'semantics' are provided directly by defining a specific class of models, not by correspondence rules (as in the 'syntactic' view on theories) linking the formal system to the world of phenomena. A theory's models are mathematical structures, and the relationship of model to phenomena is one of isomorphism: the empirical claim is then that the system of relations discerned in some particular empirical domain is isomorphic to certain substructures (parts of the theory's models).

The particular methodological approach that I have adopted as the general framework for my analysis of kinship theory is quite similar. It is the so-called *non-statement* or *structuralist* view of theories developed and advocated by Joseph Sneed, Wolfgang Stegmüller, Wolfgang Balzer and others. The structuralist approach shares most of the crucial characteristics of the semantic approach, except that the latter tends to be applied less formally in the context of actual analyses of scientific theories.⁴

One aspect of the structuralist approach that I find especially interesting is the claim that it may be applied to the reconstruction of Kuhn's conception of theory-change, a point acknowledged by Kuhn himself (see Stegmüller 1976:135-271 and Kuhn 1976). Thus (Kuhn 1976:184): 'To a far greater extent and also far more naturally than any previous mode of formalization, Sneed's lends itself to

the reconstruction of 'theory dynamics, the process by which theories change and grow'. Specifically, the structuralist view on the structure and dynamics of theories provides a sufficiently rich and elaborate framework for investigating the structure of kinship theory. This is the general thesis of the book.

With regard to the plan of the book, in the first chapter I introduce some of the early kinship models developed under the traditional Leiden programme of structural comparison. Next, I discuss the structuralist or non-statement view on theories and apply the general scheme to obtain a formal re-presentation of the classic models of double-descent and circulating connubium as classes of set-theoretic structures. (The paradigmatic kinship model is derived from group theory.) I then derive the complete set of 'latent' or 'reduced' structures implied by the formal model and compare these results with the kinship structures described by other anthropologists.

Having introduced the basic concepts and mathematical tools, I move next to the formalization and generalization of Claude Lévi-Strauss's seminal theory of elementary kinship structures (1970 [1949]). In Chapter 2, I define a family of structures based on generalized exchange and then demonstrate that certain substructures are isomorphic to partial structures and relationships reported in the ethnographic data. The extended family of structures of generalized exchange is obtained by defining successive exchange cycles recursively, as automorphisms of a basic set of exchange relations.⁵

The analogous procedure is carried out in Chapter 4, formalizing and extending Lévi-Strauss's theory of restricted exchange. In Chapter 3 I demonstrate how the basic model of generalized exchange with exclusive matrilateral cross-cousin marriage may be reformulated and made more complex so as to cope with the wide range of age differences generally reported in the ethnographic

record. The strategy I adopt is to superimpose a metrical structure on the underlying group-theoretic structure. Again, after deriving an entire family of age-constrained kinship structures, I establish isomorphisms between the models and ethnographic data from particular societies. The family of helical exchange models thus formulated also has important consequences for the fundamental Lévi-Straussian assumption that structures of alliance are invariably based on the exchange of a *sister* for a *spouse*. I demonstrate that age-constrained helical models are compatible with the formulation of exchange in terms of close female kin other than sisters, and that such formulations are isomorphic to descriptions of empirical societies.

In Chapter 5, I focus on the Lévi-Straussian opposition between 'elementary' and 'complex' kinship structures. I provide a summary account of recent developments in the theory of complexity, and introduce examples of discrete dynamical systems from the theory of cellular automata. Taking up again some of the points that emerged from the analysis in the preceding chapters, I argue that the fundamental opposition between 'elementary' and 'complex' systems must now be reconsidered, in the light of recent ethnographical research as well as the new developments in modelling complex systems. To accommodate these new factors, the standard models for representing kinship structures (including the families of algebraic models introduced in this volume) must be refined. I conclude with a number of concrete suggestions for developing a more elaborate series of structural models for investigating and understanding the structure and development of kinship systems.

Since much of the material in this book requires some familiarity with basic mathematical concepts, I have provided a series of appendices at the end of each chapter to make the technical material in the rest of the book accessible to those with little knowledge in these areas.

NOTES

- 1 Reprinted as Chapter 1 of Leach (1971 [1961]).
- 2 Reflexive and critical anthropology has yet to address the variety of *tu quoque* arguments in which reflexive and critical approaches are applied to the critics themselves. For a stunning exploration of reflexivity (including attempts to go beyond the constraints of *tu quoque* reflexive critique), I recommend Malcolm Ashmore's *The Reflexive Thesis: Wrighting Sociology of Scientific Knowledge* (1989) wholeheartedly!
- 3 There is a splendid irony in choosing the Malinowski Lecture for recasting anthropology in a mathematical mould, given Malinowski's vehement pronouncements on the 'dehumanization of kinship' by 'mock algebra' and 'pseudo-mathematical treatments' (cf. Malinowski 1930). Malinowski studied physics and philosophy in Poland before reading anthropology. Leach had a Cambridge undergraduate education in mathematics and engineering before becoming a student of Malinowski's. Other, even more impassioned critics of the use of mathematics in anthropology must be considered less well-informed. See, in particular, Korn and Needham (1970) for a polemical article riddled with errors and in which the authors ignore some of the more relevant publications. For other pronouncements on the relevance of mathematics in anthropology roughly contemporaneous with Leach's topological programme of 1959, see Radcliffe-Brown (1957; lecture notes 1937) and Lévi-Strauss (1955).
- 4 For comprehensive reviews of the semantic approach, see Suppe (1977:221-230, 709-712; 1989), Van Fraassen (1987 [1980]; 1989), Giere (1988), and Thompson (1989). Key texts on the structuralist (non-statement) view of theories are: Sneed (1979 [1971]), Stegmüller (1976, 1979), Balzer et al. (1987). See also the references provided in the following chapters. There are now literally hundreds of publications on the non-statement view. Diederich et al. (1989) provide a comprehensive bibliography up to 1988. For a recent overview, see Diederich (1989). Surprisingly, although the semantic and structuralist views share a common ancestry and there are obvious common concerns, there has been almost no discussion between adherents of the two approaches. Diederich has now announced his intention of comparing both views in a forthcoming paper (see Diederich 1989:383).
- 5 Thus introducing, in a technical sense, a modicum of 'reflexivity' as a definitive aspect of the theory's structure.

1. LEIDEN, LÉVI-STRAUSS, AND THE MATHEMATICS OF DOUBLE DESCENT

*For last year's words belong
to last year's language
And next year's words await
another voice*

T.S. Eliot
Little Gidding

The *débris* of previous scientific discourse is often much undervalued - so much so that, with the repudiation of a past paradigm, a scientific community will simultaneously renounce, as not relevant for continued professional study, most of the publications in which that paradigm had been embodied. This is the view presented by Thomas Kuhn in the final section of his celebrated essay on *The Structure of Scientific Revolutions*.¹ Thus (Kuhn 1970:167):

Scientific education makes use of no equivalent for the art museum or the library of classics, and the result is a sometimes drastic distortion in the scientist's perception of his discipline's past. More than the practitioners of other creative fields, he comes to see it as leading in a straight line to the discipline's present vantage. In short, he comes to see it as progress.

Anthropologists are, fortunately, less prone than most to ignoring their past. The fledgling student is introduced to the original sources as well as the contemporary literature, and is thus made aware of the classic problems and discussions, together with the bewildering variety of incommensurable proposals offered in solution (cf. Kuhn 1970:165). To paraphrase Lévi-Strauss (1974:22), anthropological thought (like the

pensée sauvage which is the object of its contemplation), in producing results in the form of events, is constantly reordering, elaborating and extending the structures which constitute its hypotheses and theories. Next year's language emerges from the reconstructed *débris* of last year's words.²

TRADITION

Included among the modest collection of maps held in the library of the Institute of Cultural Anthropology at Leiden are certain items of a curious nature: five large charts with figures and diagrams carefully rendered in Indian ink. Relegated to the basement, they have been rescued from oblivion by our librarian and are now listed in the catalogue as 'kinship charts (*stokkaarten verwantschap*) 1, 5, 7, 9 and 10'.³

Charts number 5 and 9 are line drawings of, respectively, the classic 'Kariera' and 'Aranda' systems with Dutch abbreviations denoting the genealogical relations. Chart number 7 is a variant of the 'Murngin' diagram presented as *Chart A* in J.P.B. de Josselin de Jong's (1952) monograph, *Lévi-Strauss's Theory on Kinship and Marriage*. These charts were evidently used by P.E. de Josselin de Jong as exemplary illustrations in his kinship lectures during the late 1950s and the 1960s.⁴

The two remaining charts are arguably older, possibly even dating back to the 1930s. Chart number 1 is identical to a diagram in F.A.E. van Wouden's 1935 thesis where it is used to demonstrate the existence of a double two-phratry system logically entailed by his celebrated model of double descent and circulating connubium. I shall return to this diagram shortly.⁵

Chart number 10 is the real prize. It is reproduced below as figure 1.1. The chart is composed of two

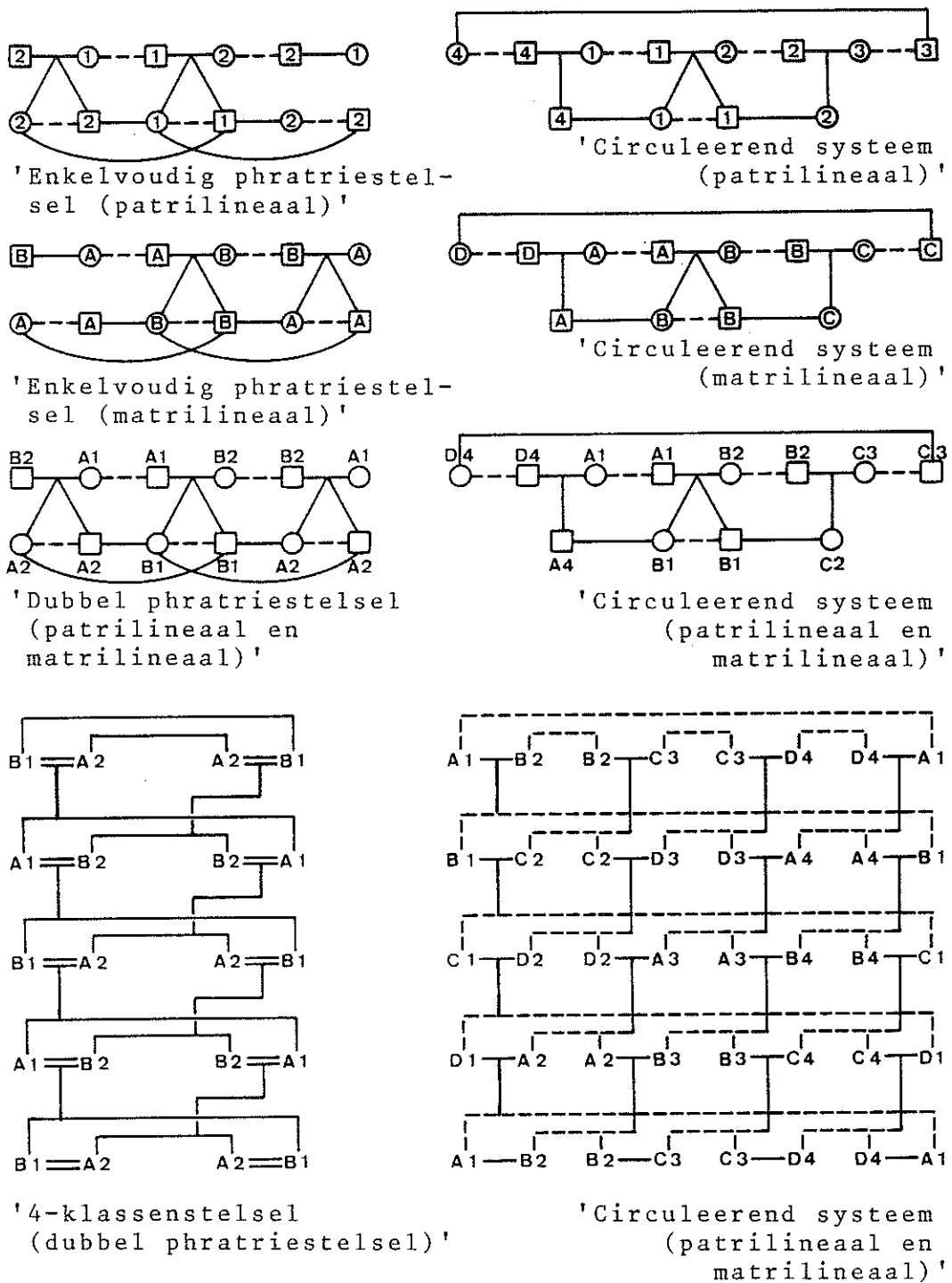


Fig. 1.1. Kinship chart number 10. See the text for detailed information.

parallel series of diagrams set out in four rows. According to the key provided, squares denote males, circles females. Broken lines connect siblings (except in the figure at the bottom left-hand corner), while unbroken lines link spouses. I have translated the Dutch captions to the figures in the first series (starting at the top row, left) as: 'simple phratry system (patrilineal)', 'simple phratry system (matrilineal)', 'double phratry system (patrilineal and matrilineal)', and 'four-class system (double phratry system)'. The captions corresponding to the second series of four diagrams (right) read: 'circulating system (patrilineal)', 'circulating system (matrilineal)', and 'circulating system (patrilineal and matrilineal)', with the third caption repeated for the final diagram.

Chart number 10 is obviously meant to express a theoretical statement of some sophistication. First, the principles of symmetric exchange (*échange restreint*) and asymmetric or generalized exchange (*échange généralisé*) are recognized as fundamentally distinct and contrasted by means of the two series of kinship models. Second, within each series, it is demonstrated that one single structural model of the connubial system can serve for three basic types of descent: patrilineal, matrilineal, or double descent. Finally, the entire family of models comprises a coherent explanatory scheme for the study of structural variation. By focussing on the diverse possibilities implicit in the models the chart serves as a framework for articulating the formal properties of kinship systems, not all of which will necessarily be realised in phenomena from actual societies. Moreover, as I shall demonstrate, the final diagram combining the principles of double descent and asymmetric exchange, actually encompasses *all* of the other models. Under a suitable homomorphism they are recovered as 'latent' or 'reduced' structures, each preserving specific aspects of the more complex, encompassing whole.⁶

In the general sense of the term, a paradigm refers to the entire constellation of shared commitments of a given scientific community. However, in the second, more fundamental sense identified by Kuhn (1970:175, 1977: 294) a paradigm is an exemplary past achievement, a particularly important concrete puzzle-solution which, employed as a model or shared example, can even replace explicit rules as the basis for seeking solutions to the problems identified by a particular scientific group. In this sense of the term, chart number 10 is a unique summary representation of the classic Leiden paradigm for structural research on kinship and marriage as elaborated by J.P.B. de Josselin de Jong and his students.

The emergence of a distinctive Leiden trend in structural anthropology is conventionally dated to 1935, with the presentation of J.P.B. de Josselin de Jong's famous second inaugural lecture, *The Malay Archipelago as a Field of Ethnological Study*, and the exceptional theses prepared by F.A.E. van Wouden and G.J. Held under his supervision and defended in the same year.⁷ The concept of a 'field of ethnological study'⁸ expounded in the inaugural address (with special emphasis on principles of social organization articulating its 'structural core'), together with the clear focus on what would now be termed the analysis of kinship models as a system of transformational variants, would in effect serve as a programme of research for generations of Leiden scholars, particularly those specializing in Indonesia.

The history of the early phases of Leiden structural anthropology (up to the mid-1950s) is now fairly well documented, with previously inaccessible Dutch publications now widely available in English translation.⁹ In recent years many of the early concepts have been re-examined and discussed. A conference was held in 1982 to re-evaluate the central notion of a 'field of

ethnological study'. The participants paid particular attention to certain fundamental questions: to what extent had the concept been revised or transformed since 1935? What should be done to improve its usefulness? The conference papers have since been published (see P.E. de Josselin de Jong 1984), eliciting further debate.¹⁰ As Fox has recently remarked, the original formulation of the programme enunciated in 1935 has seen a significant transformation - although continuities are clearly recognizable. 'What remains is a concern to examine the comparative interrelationship between connubium, descent and dualism within a holistic cultural framework' (Fox 1988:181).

Many of the key elements of the early Leiden paradigm summarized above (cf. figure 1.1) are already set out in publications and isolatable from other sources predating 1935. Thus, in J.Ph. Duyvendak's dissertation (published in 1926; J.P.B. de Josselin de Jong was his supervisor), a structure made up of unilateral marriage relations linking four groups (exclusive matrilinear cross-cousin marriage) is described and shown to be compatible with a phratry division (1926:124-129). Duyvendak was not only familiar with the concept of a model - he applied the term itself: 'The old exogamous tribal division, presumably embodying an elaborate system of classification, was the *model* on which all other oppositions were *modelled*' (1926:135; my emphasis and translation). This use of the term is derived from Durkheim and Mauss's 1903 essay, *De quelques formes primitives de classification: contribution à l'étude des représentations collectives*, by way of F.D.E. van Ossenbruggen and W.H. Rassers.¹¹

The model concept also turns up in J.P.B. de Josselin de Jong's paper on *The Natchez Social System*, presented at the 1928 Congress of Americanists. Here the 'Omaha' type of social structure (encompassing a social dichotomy of society into complementary, exogamous halves, one of

which is held to be superior) serves as a model for reconstructing an earlier phase of Natchez social organization (J.P.B. de Josselin de Jong 1930:555-558). According to Locher (1988:58-59) there are important parallels between the approach sketched in the Natchez paper and the earlier discussion of comparison and 'reconstruction' in Duyvendak's thesis. As Locher recalls, in the late 1920s when the model articulating the important principle of double descent with the connubial system was formulated by J.P.B. de Josselin de Jong and discussed with his students, he not only introduced the concept of structure in his lectures but the word as well (Locher 1968:vi; 1988:60-62).

In the discussion following the presentation of the Natchez paper, J.R. Swanson voiced his doubt: if De Josselin de Jong's position were correct 'it would mean the derivation of a number of different forms of social organization from a single highly specialized type' (J. P.B. de Josselin de Jong 1930:561). That is precisely the point. In interpreting the American Indian data by means of a structural model (i.e., the 'Omaha' type of social structure), J.P.B. de Josselin de Jong is able to demonstrate that diverse possibilities are implicit in the model. These may be realised in actual societies as disparate forms of social organization: as totemic, clan or town moieties, as a matrilineal or patrilineal dual division, etc.

The early Leiden position has been aptly summarized by Locher (1968:x):

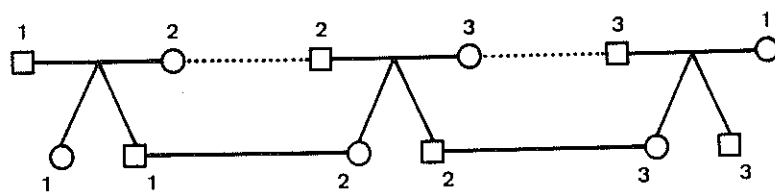
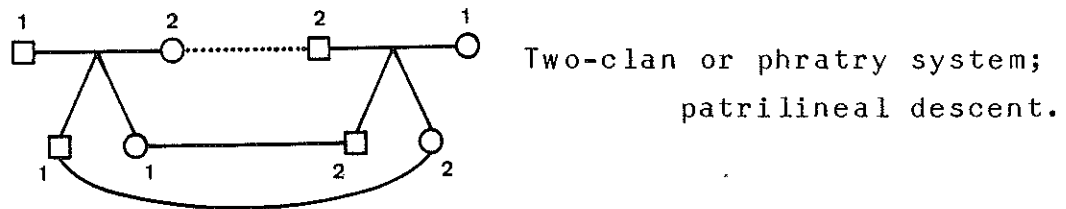
The great advance in understanding effected in the thirties was primarily the idea that accentuated matrilineal grouping, similarly marked patrilineal grouping, and double-unilineal grouping could belong to one and the same structure. For an area ... in which all three forms of organization occurred, this structural insight provided an entirely different perspective from that afforded by starting from three sharply distinguished main types of unilineal kinship systems, studied exclusively in a functionalist way or else

placed in a diffusionist or evolutionary historical sequence, as for the most part had previously been done.

These ideas are clearly represented in a series of four diagrams in H.J. Friedericy's thesis on the Bugis and Makassarese (1933:141-142): a scheme representing a 'two-clan or phratry system with matrilineal descent' is shown, by a simple process of renumbering, to exhibit the identical underlying structure to a 'two-clan or phratry system with patrilineal descent'. The analogous proposition is demonstrated for models of circulating connubium with, respectively, a matrilineal or a patrilineal descent rule. Two of Friedericy's diagrams are reproduced in figure 1.2 (top).¹²

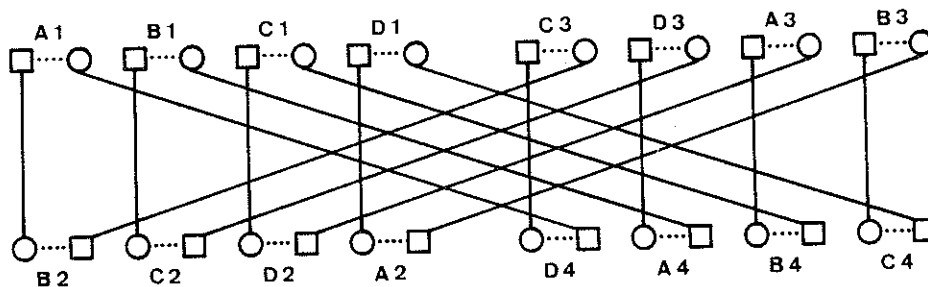
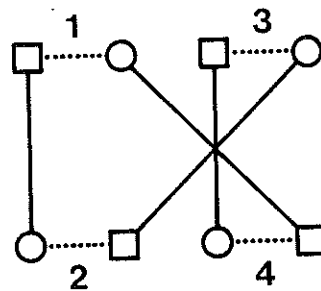
Two years later, in 1935, this now familiar pattern is displayed in even greater detail in Held's thesis, *The Mahābhārata*. Thus, after presenting an 'illustrative plan' of a two-clan system with patrilineal descent, the plan is reproduced in the succeeding diagram, with the patrilineal dichotomy now crossed by a matrilineal division. The structure of double cross-cousin marriage remains identical (Held 1935:54; cf. 56, notes 1, 2, 3). In the same section, three alternative representations are provided for a four-clan patrilineal system with exclusive matrilateral cross-cousin marriage and Held shows that an exogamous phratry division is implied (1935:63; the second diagram is reproduced in figure 1.2 (penultimate diagram)). In a final series of three diagrams, a patrilineal four-clan system with asymmetric exchange is transformed into a system of double descent by superimposing a matrilineal four-clan grouping. The resulting scheme is identical to the final diagram of kinship chart number 10 (see figure 1.1).

Held's work later became known to a wider audience through the critical comments by Claude Lévi-Strauss in *Les Structures élémentaires*.¹³ Lévi-Strauss reproduced Held's scheme articulating double descent



Circulating system; patrilineal descent.

Patrilineal four-clan system
with exclusive matrilineal
cross-cousin marriage;
exogamous phratry division.



System with double descent and exclusive matrilineal
cross-cousin marriage; exogamous phratry division.

Fig. 1.2. Diagrams adapted from Friedericy (top) (1933: 141-142) and Held (bottom) (1935:63, 95).

and circulating connubium in adapted form (1949:501, fig. 77; 1970:405, fig. 76). However, Lévi-Strauss does not render Held's ideas quite correctly, a point made long ago by J.P.B. de Josselin de Jong (1952:54-56). What appears to be at issue is Lévi-Strauss's curious reluctance (at least in *Les Structures*) to admit the possibility of combining double descent with asymmetric connubium in an *explanatory model* (as opposed to the empirical question of whether or not all the features of such a model are ever realized, or recognized in practice by the participants).¹⁴

Lévi-Strauss's emphasis is on the structure of generalized exchange as it appears in its 'simplest' form, i.e., with a unilineal rule of descent, or in a 'harmonic regime' (1970:215-217, 233, 265, 273). Bilineal systems are considered to be secondary elaborations, with double descent a logically redundant feature of the exchange structure. In contrast, under the traditional Leiden paradigm of the 1930s, the double descent formulation of circulating connubium is, at a theoretical level, the most comprehensive, encompassing structure. It is a 'possibilistic' model (a term recently introduced by P.E. de Josselin de Jong and H.F. Vermeulen)¹⁵ generating a well-defined family of 'latent' structures which, in combination, will render intelligible the variation at the level of ethnographic facts. This is the paradigm so admirably summarized in kinship chart number 10 (fig. 1.1).

The original Leiden approach to social organization was never exclusively concerned with *generalized* exchange. From the start, the perspective is much more embracing, with an early emphasis on social-cosmological dualism as a pervasive system of classification, and on the study of kinship systems based on a phratry division.¹⁶ Locher, citing correspondence with J.P.B. de Josselin de Jong regarding his initial Ph.D. research (1988:60-62),¹⁷ emphasizes that the double descent framework was, at the

time, already being applied to the Australian systems embodying *symmetric* exchange - and with some success. 'At a theoretical level we had by then surpassed Radcliffe-Brown ... from our viewpoint, much of the available material could be given a new interpretation' (Locher 1988:61; my translation).

The full ($n \times n$) double descent model, with n patrilineal 'groupings' intersected by n matrilineal 'clans' linked in asymmetric connubium can, for n an even number, be shown to entail a double-phratry or four-class system with symmetric exchange. This can be done in stages. Thus, Held (1935:95), after constructing a 4×4 double descent scheme (cf. figure 1.1, bottom, right), provides a diagram (reproduced here in figure 1.2, bottom) showing how a patrilineal phratry division may be obtained by a regrouping of the original components. By implication, a matrilineal dual division is also possible. A latent four-class system can also be obtained by direct reduction. What would now be termed a homomorphic mapping onto a quotient structure is represented in kinship chart 1, reproduced as figure 1.3 (top) (with minor changes in notation to make it fully compatible with Held's example and with figure 1.1). The reduced structure on four classes P_x , P_y , Q_x and Q_y (dotted lines denote matrilineal descent, broken lines patrilineal descent, and unbroken lines symmetric exchange) is isomorphic to the 'Kariera' structure, one of the Australian systems described by Radcliffe-Brown. In sum: the structure with asymmetric connubium and double descent encompasses the classic structures with symmetric exchange.

Thus, in 1935, when Van Wouden applied the general paradigm to data on the kinship systems of eastern Indonesia, its internal logic had already been discussed in some detail. A generation later, in P.E. de Josselin de Jong's thesis, *Minangkabau and Negri Sembilan* (1951),¹⁸ the basic model would again be applied with success to a

		P		Q	
		A	C	D	B
x	2	IV □ ○	II □ ○	III □ ○	I □ ○
	4	II □ ○	IV □ ○	I □ ○	III □ ○
y	3	III □ ○	I □ ○	II □ ○	IV □ ○
	1	I □ ○	III □ ○	IV □ ○	II □ ○

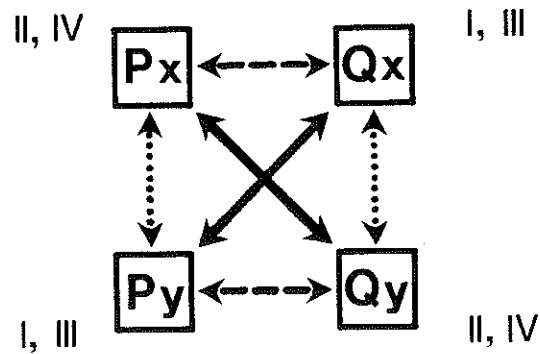


Fig. 1.3. Double two-phratry system logically implied by a scheme with double descent and circulating connubium (top; adapted from kinship chart number 1). I, II, III, IV successive generations, P and Q matrilineal phratries, x and y patrilineal phratries. Reduced model of a four-class system (bottom). Cf. Van Wouden 1935:97.

significant corpus of data. The seminal ideas of the early 1930s, developed and carried forward into the mid-1950s, provided the necessary conceptual focus for maintaining and extending the Leiden perspective during its formative years.

In the remaining sections of this chapter I first introduce the basic methodological apparatus and the simple mathematical tools necessary for the formal reconstruction of kinship theory. The methodology is provided by the 'non-statement' or 'structuralist' (in a related sense of the term) approach to scientific theories arising from the work of Patrick Suppes, and elaborated by Joseph Sneed, Wolfgang Stegmüller and their collaborators. This scheme is fully commensurate with the traditional Leiden structuralist paradigm as set out above. I then apply the mathematical concepts to obtain a formal re-presentation of the classic model of double descent and circulating connubium. Finally, after deriving the complete lattice of 'latent' kinship structures implied by the formal model, I compare my results with the kinship structures described by previous generations of Leiden scholars.

THEORIES, MODELS, AND STRUCTURES

Knowledge, according to Gilles-Gaston Granger, can only become scientific by 'progressing from vulgar error - that is, from unformulated, ambiguous knowledge - to scientific error, that is, to *refutable knowledge*' (1983: 3). Thus, the lived experiences and facts directly perceived as significant must be reconstituted and transformed, by a *découpage* of the phenomena, into objects which satisfy the formal restrictions characteristic of scientific language and which constitute a structure. This is the central thesis of his important work, *Pensée formelle et sciences de*

l'homme, first published in 1960 and recently translated into English.

In this process of reconstruction, *axiomatization*, defined as the substitution of a *simple* idea for a *common-sense* idea, plays a decisive role (1983:135). Thus (Granger 1983:145):

It is axiomatization which reveals the interdependence of hypotheses, and their strategic value in a model. ... it calls forth and assumes this eidetic variation of models that is one of the essential conditions of the construction of effective and coherent concepts.

As Granger argues, in order to overcome the entrenched wisdom encapsulated in ordinary language that has so often led to scientifically sterile *discourse*, the social and behavioural sciences must forgo the ordinary meanings of common-sensical knowledge and formulate *structural* descriptions of the phenomena that they wish to explain.

By 'structural descriptions' Granger is referring to a general methodology applicable to all the sciences. In linguistics it is exemplified by the original and fruitful ideas of Ferdinand de Saussure. As summed up, the basic ideas of Saussurian linguistics are (Granger 1983:xxiii-xxiv):

... language, considered independently of the entire context of concrete activities of expression and their historical evolution, constitutes a legitimately *découpe* object of science, forming a *system* whose intrinsic determinations can be described as such. Once the reduction of the phenomenon to the abstract object that is *language* has been effected, the second Saussurian idea assigns this object its nature; ... each of the elements of the system of language can be defined only in terms of its relations of opposition to all the others ... and assumes value, function and meaning only relative to that from which it is demarcated within the entire system.

Complementing these ideas is a notion of *structure* developed in the early 1930s by the Bourbaki group of

mathematicians.¹⁹ Under the Bourbaki programme, great emphasis is laid on the use of informal set theory and abstract algebra in an attempt to axiomatize the whole of mathematics, seen as an ordered hierarchy of structures, from the general to the particular. According to Granger (1983:xxiv):

The idea that is essential, and at its foundation common to the mathematicians and Saussure, is that the object is perceptible in its depth not so much as the bearer of inner properties - in the image of perceived qualities - but as the system of relations between elements not otherwise marked, whose only envisaged properties derive from these relations themselves. So that the true object of mathematical knowledge is the structure, not the element: what the analyst aims at ... are the formal properties of a *system* of objects ... Each branch of mathematics thus explores a structure, or a complex of structures ...

Granger finds a third, independent source of structuralist methodology in certain attempts in the 1950s to analyse a philosophical work.²⁰ Thus (Granger 1983:xxv):

The structuralist idea in the history of philosophy consists in considering a work in itself, as a relatively closed and autonomous system which the analyst wants to understand as such. Thus the Saussurian idea of language is rediscovered and applied to a phenomenon of culture at once less extensive and more complex ... the second principle ... consists in positing that the elements of the system are 'significations', not 'meanings'.²¹

It is from the superposition of these three modes of objectification that Granger explicates his more specific theme: the axiomatization of the social sciences.

Having failed in many cases to cope with the fundamental problem of the objectification of experience, the social and behavioural sciences have ensnared themselves in the scientifically sterile web of ordinary language descriptions, or in the equally sterile manipulation of metrically quantifiable variables.

Granger is most emphatic: real progress will only be effectuated by a structural treatment of the social phenomena. While formal axiomatization is characteristic of 'mature' theories in the natural sciences, in the social and behavioural sciences an axiomatic presentation is a necessary *initial* step on the path to objectification. For the sciences of man, attempts at axiomatization, however awkward or partial, offer the sole means of escape from the confused implications of concrete reality and ideology (Granger 1983:136-137, 145-149).

After more than a quarter century, Granger's critical reflections on formal thought and the innovative role of axiomatization continue to be eminently relevant for contemporary social sciences. In the context of the early 1960s it should be seen as an original attempt to provide an alternative methodological perspective to logical empiricism, the then predominant approach to scientific knowledge.

The main features of logical positivist methodology are well-known. Thus, under the 'standard' or *Received View*²², scientific theories are treated as classes of statements, some of which can only be established as true or false by empirical means. The logical skeleton of the explanatory system is obtained by specifying a suitable axiomatic calculus or formal language L such that a relevant subset B of L 's statements may be logically derived from a finite proper subset A of B . (L is usually first-order predicate logic.) The non-logical vocabulary of L is divided into observational terms and theoretical terms, with the latter given a partial observational interpretation by means of correspondence rules. These rules in effect assign an empirical content to the abstract calculus by relating it to the concrete material of observation and experiment.

Later versions of the Received View stress the central role of models: independent semantic

interpretations of the axioms such that the theorems derived for the theory are true. Moreover, according to Nagel (1961) and Hesse (1965, 1966, 1980), a theory's models are simultaneously mathematical models (i.e., true interpretations) and iconic models (systems structurally isomorphic or similar to what is modelled). As Nagel argues, a model 'supplies some flesh for the skeletal structure in terms of more or less familiar conceptual or visualizable materials' (1961:90). In effect, in providing an independent semantic interpretation for the theoretical terms and statements one maintains an isomorphism between the structure of the model and the non-observable (and partially interpreted) portion of the empirical domain under consideration.²³

Much has happened since the 1960s. The logical empiricist treatment of science has come under heavy attack, with the Received View now repudiated or found wanting by most philosophers of science. In the meantime the more extreme of the alternative *Weltanschauungen* views advocated by Toulmin, Feyerabend, Hanson, and Kuhn, after a brief decade of relative popularity, have themselves gone into eclipse.²⁴ Perhaps in critical reaction, the most promising recent developments are now committed to a new scientific realism or to a programme of rational reconstruction for the structure and dynamics of scientific knowledge.²⁵ I shall introduce one of the important new conceptual frameworks: the 'non-statement', 'set-theoretic', 'structuralist', or 'Sneedian' approach to scientific theories. These labels refer to a research programme arising from the work of Patrick Suppes and later refined and elaborated by Joseph Sneed, Wolfgang Stegmüller, and their collaborators.²⁶ As I understand it, the non-statement view is consonant with Granger's general position on axiomatization. It also offers a rich conceptual framework for the reconstruction of structural theories

in anthropology. Throughout the rest of this book the non-statement approach will provide the basic framework for the explication and elucidation of kinship theory.

THE NON-STATEMENT PROGRAMME

Under the Received View, scientific theories are treated as classes of statements, with any given theory axiomatized within a precisely described formal language. The Suppes-Sneed-Stegmüller conception of theories is altogether different. Suppes has characterized the approach by the slogan: 'to axiomatize a theory is to define a predicate in terms of notions of set theory' (1957:249). Here theories are defined directly as set-theoretic structures by means of their models (structures satisfying the predicate), together with a collection of distinct intended applications (structures representing the actual empirical systems, processes, configurations, etc. from the domains to which the theory is intended to apply). No logical concepts or formal linguistic statements are involved - hence the characterization as 'non-statement' or 'structuralist'.

The paradigmatic illustration advanced by Suppes in his early work (1957:246-305) is the example of group theory. As he explains, many mathematically significant theories (e.g., group theory, lattice theory and so forth) can be axiomatized by employing the first order predicate calculus. Unfortunately, even in mathematics, the formalization of more complicated theories assuming more than first-order logic is largely impractical. According to Suppes, the standard method of axiomatization becomes even more unrealistic in the case of real empirical theories. Hence the challenge is to develop a viable approach in analogy to an alternative method for presenting a mathematical theory: directly,

by specifying a suitable set-theoretic predicate. Thus, in the example of group theory, an adequate axiomatization of the predicate 'is a group' is the following:

GROUP: x is a group if and only if there exists a G and $*$ such that

- (1) $x = (G, *)$;
- (2) G is a non-empty set;
- (3) $*$ is a function whose domain is $G \times G$ and whose range is a subset of G ;
- (4) for all a, b and c in G ,
 $a * (b * c) = (a * b) * c$;
- (5) for all a and b in G , there is an e in G such
 that $a * e = a$;
- (6) for all a in G , there is an a^{-1} in G such that
 $a * a^{-1} = e$.

This is of course the familiar group definition in terms of an ordered pair consisting of a non-empty set together with an associative binary operation, with the group identity and inverse elements well-defined.²⁷ The predicate 'is a group' specifies a class of set-theoretic structures and is satisfied by all and only those objects that one wishes to designate by the term 'group'.²⁸

The Suppes approach received fresh impetus with the publication of Joseph Sneed's *The Logical Structure of Mathematical Physics* in 1971. Building on Suppes's ideas for characterizing the internal structure of scientific theories, Sneed introduced further elaborations. The relation between set-theoretic structures and the real world situations to which the theory is intended to apply, he claimed, requires the consideration of three classes of models. The first is the class M_p of so-called *potential models*, in other words, the class of entities with some minimal common 'type' of structure. Roughly speaking, the shared structure determines the formal properties of the theory's conceptual apparatus. (In

technical terms the class M_p is defined as a *species of structure* in the sense of Bourbaki. See Balzer 1983 and Balzer et al. 1987.)

The second class M of *actual* or *proper models* is a subset of M_p and picks out of the class of all potential models exactly those which satisfy the laws of the theory.²⁹ Thus, while potential models are just those kinds of structure that one might intelligibly claim to be models for a particular theory (cf. Sneed 1984:97), the subset of proper models represents those structures which, in addition, exhibit the theory's fundamental theoretical content.

The third class of models is M_{pp} , a theory's *partial potential models*. The introduction of M_{pp} hinges on the crucial distinction between theoretical and non-theoretical terms. According to Sneed, the usual division (under the Received View) of a theory's non-logical vocabulary into 'theoretical' and 'non-theoretical' terms is untenable. Thus, the traditional dual-level conception of theories imposes an arbitrary or subjective dichotomy, with 'theoretical' negatively characterized as 'non-observational'. Furthermore, one is supposed to link a theory's theoretical concepts to the independent 'reality' of observation and experiment by devising suitable correspondence rules or 'coordinative definitions'.³⁰ In the final analysis, the goal is to explicate and define all theoretical concepts by reference to the 'observable': one attempts to transfer 'meaning' from the basic empiricist language to the purely theoretical superstructure.

According to Sneed (1979), the 'theoretical - non-theoretical' and 'non-observational - observational' dichotomies do not coincide. In addition, and in contrast to the common view, he argues that theoreticity must always be explicated in relation to the existing expositions of a particular physical theory, not absolutely, by reference to sublanguages of a general

language of science. Sneed's solution (as summarized by Stegmüller (1976:41-42)) is roughly this:

Exactly those quantities or functions whose values cannot be calculated without recourse to a theory T (more specifically, to the successfully applied theory T) are theoretical in relation to this theory T .

Sneed's criterion is therefore for a relativized concept 'T-theoretical', not an absolute definition of theoreticity. In the original formulation (Sneed 1979) the class M_{pp} of a theory's partial potential models is then defined by removing from each potential model of the class M_p all components that are T-theoretical. M_{pp} thus contains all structures which can be described in the non-theoretical vocabulary of the theory, i.e., the elements of M_{pp} represent the potential applications (real systems described as structures) of the theory T .

Sneed's original definition of the 'T-theoretical' criterion referred to quantities occurring in developed theories of mathematical physics (e.g., functions in classical particle mathematics). In such fully developed theories the basic functions are metrical concepts. For qualitative theories (e.g., theories with binary relations instead of functions, or where the function value is just a truth value) the concept of T-dependent measurement would be replaced, in analogy, by the concept of say, T-dependent determination of truth value (cf. Stegmüller 1976:53 and Balzer 1982:22). Hence the criterion can in principle be adapted to non-metrical concepts in anthropological theories.

Balzer has recently (1982, 1983) proposed a more general definition of M_{pp} , introducing partial potential models as arbitrary substructures of a theory's potential models, independent of the criterion of T-theoreticity. Under this modification a substructure is obtained from a potential model by restricting its functions and relations to a smaller subset of the potential model's

basic sets of components. Balzer thus admits as a partial potential model every structure which is 'inbetween' a potential model and the trivial or zero structure consisting of empty components only. It can be easily demonstrated that, under this definition, the class M_{pp} (together with the substructure relation) is a complete lattice with a least element.³¹ The criterion of T -theoreticity is important - a point fully acknowledged by Balzer (1982, 1983). However, even if one accepts the Sneedian distinction, it is in practice often quite difficult to determine which terms of a particular theory are actually T -theoretical and which are T -non-theoretical. By defining M_{pp} in a manner independent of the criterion of T -theoreticity, Balzer's approach allows for the formal reconstruction of a theory to be undertaken prior to the fundamental investigation of theoreticity. This plan of action is particularly appropriate to the reconstruction of theories in anthropology, where it is often difficult to apply the criterion of T -theoreticity in a straightforward manner. Thus, in the present work I find it convenient to follow Balzer and to define M_{pp} as an ordered hierarchy of substructures of M_p .

There are two other components of a theory. The class C of *constraints* express 'cross connections' between several potential models. They guarantee that certain features remain constant or invariant across particular combinations of potential models. For example, the values assumed by functions in fully mathematical theories in one model must be compatible with those assumed in other applications connected by the same constraint. (Similar restrictions may be formulated for theories of a qualitative nature.) Finally, I is the class of structures representing a theory's *intended applications*. Roughly, I contains those real systems the theory is intended to deal with. I is a subset of M_{pp} ; hence, as defined by Balzer (1982, 1983)

intended applications are substructures. Any theory will in principle have countless intended applications. Hence I is largely an open set (cf. Stegmüller 1979:11-12), containing in most cases a subset I_0 of the theory's historic applications or paradigm examples as it develops over time.

A theory-element T is then an ordered quintuple:

$$T = \langle M_p, M, M_{pp}, C, I \rangle,$$

where M_p , M , and M_{pp} are, respectively, classes of potential models, proper models, and partial potential models, C is a class of constraints, and I the class of intended applications. $\langle M_p, M, M_{pp}, C \rangle$ denotes the mathematical formalism or core K of the theory-element. Hence $T = \langle K, I \rangle$, with the theory-element encompassing both the theoretical frame and the theory's domain of intended applications.

With the structuralist conception of theories summarized above, one can now formulate the *empirical claim* of theory-element T . Let I be the set of intended applications, i.e., real systems perceived in terms of the theory and partially exhibiting its concepts. Represented as set-theoretic substructures, intended applications describe or take into account only those parts of real systems that are actually identified, observed, or in some way capture the known data about some system. The empirical claim then simply says that the substructures are in fact parts of the theory's proper models and can be augmented or extended to complete structures. Thus, the empirical claim can be stated as follows: *There exists a set X of extensions of I so that X is a set of proper models which also satisfies the constraints C of the theory* (see Balzer 1982 and 1983, and Balzer et al. 1987). Thus formulated, empirical claims are assertions with testable consequences: they are either true or false.³²

ELEMENTARY KINSHIP STRUCTURES AND DOUBLE DESCENT

In the kinship models developed by the prewar school of Leiden anthropology, a positive marriage rule regulates the choice of a spouse, either by prescribing marriage with a certain type of relative (e.g., with male ego's matrilineal cross-cousin), or by defining the spouse category in terms of a particular wife-giving line or double descent 'marriage class'.³³ Similar models would later play a crucial role in Claude Lévi-Strauss's *Les Structures élémentaires de la parenté*, intended as the introduction to a general theory of kinship. Thus, in the preface to the first (1949) edition (see Lévi-Strauss 1970:xxiii), *elementary kinship structures* are defined as:

... those systems in which the nomenclature permits the immediate determination of the circle of kin and that of affines, that is, those systems which prescribe marriage with a certain type of relative, or, alternatively, those which, while defining all members of the society as relatives, divide them into two categories, viz., possible spouses and prohibited spouses.

The correspondences between the Leiden approach and Lévi-Strauss's more ambitious scheme are evident. Nevertheless, one should not allow the obvious similarities to obscure fundamental issues on which the two approaches disagree. Significant differences were noted and commented upon by J.P.B. de Josselin de Jong in his essay-length review of *Les Structures*, Lévi-Strauss's *Theory on Kinship and Marriage* which appeared in 1952.

One crucial feature of Lévi-Strauss's theory, stressed repeatedly, is his belief that the significance of exchange (resulting in a limited set of elementary structures) does not, in the final analysis, depend on the nature of the gifts exchanged. In marriage, the archetype of exchange, women are the supreme gift. However, the integrating effect and the solidarity which

bind groups together, uniting the gift and the counter gift, and one marriage with other marriages, are ultimately independent of the value of the signs communicated (cf. Lévi-Strauss 1970:116, 483, 495-496).³⁴

In opposition, J.P.B. de Josselin de Jong introduces data from Indonesian societies where categories of 'male' and 'female' goods are distinguished, with both kinds of goods playing an essential role in the transaction of matrilineal cross-cousin marriage, circulating in opposite directions through the whole community. Thus (J.P.B. de Josselin de Jong 1952:58):

It seems indisputable to us that in this category of 'échange général' (all-embracing exchange) the intrinsic nature of the exchanged values is not by any means irrelevant. The functional value of the exchange in such cases results quite as much from the nature of the goods as from the act itself and the positions of the exchange partners in the whole system. It would seem therefore that we have to distinguish between two types of exchange which are probably co-existent in all communities with 'elementary structures of exchange', viz. one in which the effect is felt to reside exclusively in the act itself and one in which it is conceived as resulting from specific goods being exchanged by definite parties.

From this perspective there are clearly no *a priori* grounds for classifying women exclusively as 'neutral' exchange objects, to be relegated to the first-mentioned type of exchange.³⁵

The second fundamental difference stressed by J.P.B. de Josselin de Jong concerns the status of double descent. Lévi-Strauss appears firmly convinced that asymmetric marriage always presupposes a unilineal mode of descent. Double descent, and more generally, all bilineal modes of reckoning of descent are seen as secondary elaborations as compared with the system of exchange. Lévi-Strauss's argument is largely formal: asymmetric systems need not become bilineal in order to become all-embracing, whereas a symmetric unilineal system cannot become all-embracing unless it becomes

bilineal or, transforming its structure of exchange, asymmetric. As a consequence, Lévi-Strauss is strongly inclined 'not to consider the possibility of double descent as a *structural factor* unless it is so evident that it could not be ignored' (J.P.B. de Josselin de Jong 1952:36, 51; my emphasis).³⁶

As I have already stressed in the introductory section to this chapter, the Leiden argument is very different. Thus, in the work of Van Wouden, Held, and others, a diagram articulating a principle of double descent with a principle of exchange emphasizing exclusive cross-cousin marriage and asymmetric connubium is intended as a structural model, i.e., a consistent formal representation of the relationship of all relevant concepts to each other, not as a factual description of any single kinship system. Indeed, whether or not the structural possibilities explicated in such a model are actually realized in data from empirical societies is a question for further research. From this perspective (and in opposition to the ideas expressed in *Les Structures*), the principle of double descent is a necessary component of the structural model. The early Leiden views on the crucial relationship between models of double descent and asymmetric exchange as complete structures, and the partial or selective manifestations of these principles in actual data are clearly congruous with the themes developed more fully in the non-statement programme.³⁷

A third important point of difference emphasized by J.P.B. de Josselin de Jong in his 1952 essay relates to the classification of exchange systems. Under the Lévi-Straussian paradigm, a sister is exchanged for a spouse and the type of marriage rule is the significant criterion: a system is symmetric if men exchange sisters directly, asymmetric if this is not the case. As an exchange formula, a marriage rule corresponds to an intra-generational series linking a man to his wife's

brother, etc.

De Josselin de Jong (1952:46-49) argues for a more extensive set of distinguishing features: knowledge of the marriage rule alone may not provide sufficient information for a truly systematic characterization of structures. Hence one should also determine the structural type of other important series of 'exchange' relations, within generations (e.g., the brother-in-law series: a man, his sister's husband, etc.) as well as across generations (e.g., the father-in-law/son-in-law series: a man, his daughter's husband, etc.). Each series may then be classified as symmetric or asymmetric, and the length of its cycle determined. Indeed, cross-generational exchange cycles may well be decisive in systems where the exchange of women is formulated as the bestowal of a daughter as a spouse, or as the exchange of close female kin other than sisters or daughters. Furthermore, the direction in which women circulate in successive generations or exchange cycles (and which may vary from generation to generation) is itself an important structural feature.

To illustrate his scheme, J.P.B. de Josselin de Jong compares the models representing five Australian systems (the Kariara and Aranda, both with direct exchange of sisters, and the Murngin, Karadjeri and Mara, all embodying an asymmetric marriage rule). His analysis leads to a correction of Lévi-Strauss's conclusions. Thus (1952:49): 'Our comparison of the Aranda and Murngin systems has led us to the conclusion that there is no fundamental difference of structure between them - as Lévi-Strauss has it - but only a difference of degree ...'.

At the level of the model, the Aranda structure combines long cycles of exchange (asymmetric from generation to generation, but in opposite directions in the implicit matrilineal moieties) with the same-generation symmetric exchange of sisters. In the Mara

structure with patrilateral cross cousin marriage, all-embracing long cycles (asymmetric in the same generation, but in opposite directions in two successive generations) are articulated with a symmetric father-in-law/son-in-law relation (symmetric from generation to generation). Consequently, the fundamental pairs of oppositions postulated in *Les Structures* must be modified.³⁸

For example, if one wishes to maintain the opposition *matrilateral/patrilateral*, it should be based on the contrast *unidirectional continuous exchange cycle/bidirectional interrupted exchange cycle*, not, as stated by Lévi-Strauss (1970:452, 465) on the opposition *long exchange cycle/short exchange cycle* (J.P.B. de Josselin de Jong 1952:56-57).³⁹

The theoretical standpoint of the early Leiden structuralists has seldom been explicitly formulated. J.P.B. de Josselin de Jong's essay, the result of an intensive seminar on *Les Structures élémentaires de la parenté* in which fifteen graduate students participated during the academic year 1950-1951, is a rare example of certain fundamental principles and concepts being discussed and brought more fully into relief in contrast to what was to become the new paradigm in post-war French anthropology. As the points of divergence noted above prove, the Leiden view, though inspired by many of the same sources on which Lévi-Strauss was to draw, does not merely anticipate or converge on selected features of Lévi-Strauss's theory of kinship.⁴⁰

Intertheory translation is central to the process of rational theory comparison, a point argued forcefully by Pearce (1987). Here I adopt a more limited strategy. Following Sneed and Stegmüller, by characterising kinship theories as structures of a certain type, I shall develop a framework for the comparison of the Leiden and Paris research traditions in kinship studies.

MODELLING ELEMENTARY KINSHIP STRUCTURES

Under the non-statement programme sketched above, the class of proper models for a simple mathematical theory of elementary kinship structures is introduced by means of a set-theoretic predicate.

X is an *elementary kinship structure* (EKS) if and only if there exists an S , h , m and f such that

- (1) $X = (S, h, m, f)$;
- (2) S is a non-empty set;
- (3) h , m and f are permutations of S ;
- (4) under the usual composition of permutations,
 $(x)f = ((x)h)m$ for all elements x of S .

Since a permutation of S is a one-to-one mapping of S onto itself, the inverse permutations h^{-1} , m^{-1} and f^{-1} are well-defined. Let e denote the identity permutation, i.e., $(x)e = x$ for all x in S . Then, under the usual composition of permutations, (S, G) is the group with identity e and elements g_i generated by the products $\alpha_1 \alpha_2 \dots \alpha_n = g_i$, where the α_i are equal to h , m , f or to their inverses. Hence (S, G) is the permutation group generated by the elementary kinship structure (S, h, m, f) . Where no confusion can arise, the elementary kinship structure will be denoted by (h, m, f) and the associated permutation group by G .

The kinship structure (S, h, m, f) is called *regular* if and only if the associated permutation group (S, G) is regular, that is, if and only if

- (1) for all elements g of (S, G) such that $g \neq e$,
 $(x)g \neq x$ for all x in S ;
- (2) for every pair of elements x and y in S there is an element g of (S, G) (i.e., a permutation of S) such that $(x)g = y$.

Unless otherwise stated, only regular structures will be considered as proper kinship models.

The origin of the class of kinship models (S, h, m, f) can be traced back to the formal appendix prepared by the Bourbaki mathematician André Weil at the request of Lévi-Strauss for the first edition of *Les Structures élémentaires* in 1949. Extensively modified and amended by mathematicians and anthropologists, the original algebraic treatment has generated numerous characterizations of families of special kinship models.⁴¹ Under the Courrège-Lorrain formulation employed in this chapter (cf. Courrège 1965, 1974, Lorrain 1975), the formal models are linked to genealogically defined aspects of kinship systems in the 'real' world. Thus, under the standard interpretation, elements of the set S may be taken to represent either descent lines or marriage 'classes'. Moreover, h , m and f represent, respectively, the conjugal or marriage permutation (linking the line or class of a man to that of his wife), the matrilineal permutation (linking a woman and her children), and the patrilineal permutation (linking a man and his children). Descent and marriage are articulated by the basic equation $(x)f = ((x)h)m$ representing the general rule that the children of a man of descent line or class x who is himself married to a woman of $(x)h$, will belong to descent line or class $((x)h)m$.

More specifically, Lorrain (1974:98; 1975:127-128) interprets the basic model as representing any genealogical network reduced according to the following two principles: (a) the unity of the sibling group (i.e., same-sex siblings and parallel cousins are considered structurally equivalent and are therefore identified with each other), and (b) marriage prescription (i.e., if two persons of the same sex are structurally equivalent, then their spouses are also considered structurally equivalent). Hence actual kinship systems are assumed to be partitioned into a set S of non-overlapping equivalence classes, i.e., reduced sibling

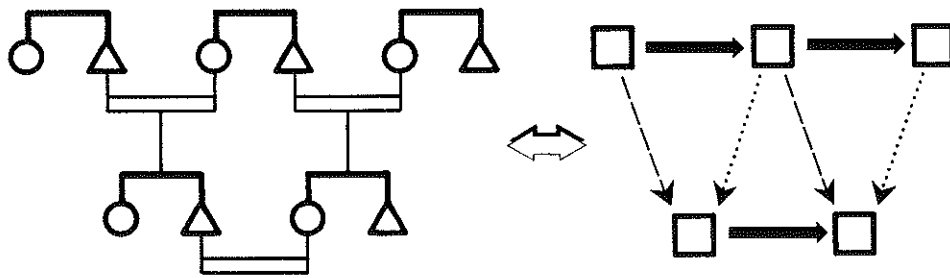
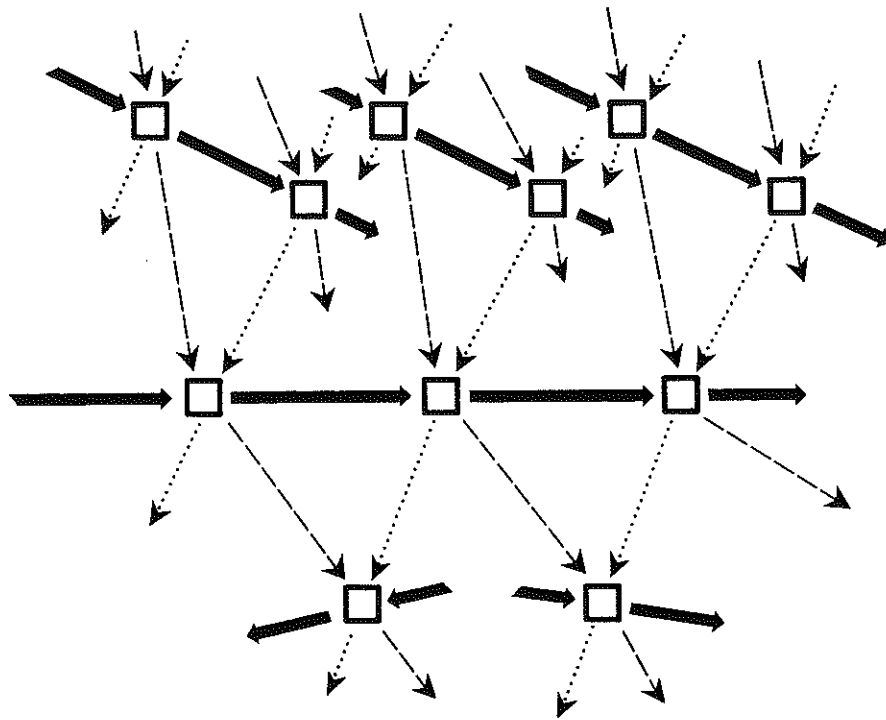


Fig. 1.4. A kinship network reduced according to principles (a) and (b) (top). (Adapted from Lorrain 1975:127, fig. 13.) A structure with exclusive matrilineal cross-cousin marriage (bottom). Structurally equivalent positions (not differentiated as to sex) are denoted by squares. The nodes in the kinship networks are linked by the three basic mappings h (solid arrows), m (dotted arrows), and f (broken arrows).

groups. (Lorrain's interpretation of the model's intended applications is obviously phrased in terms of Radcliffe-Brown's classic theories of kinship structures; cf. Radcliffe-Brown 1930-31.) A kinship network reduced according to these principles is graphically represented in figure 1.4 (top).

For the elementary kinship structure (S, h, m, f) , let S be the set of nodes of a genealogical network reduced according to Lorrain's principles (a) and (b). Assume the mappings h, m, f and their inverses to be suitably defined. (The inverse mappings h^{-1}, m^{-1} and f^{-1} are obtained by reversing the direction of the arrows in figure 1.4.) Composite mappings can now be defined on S by taking different combinations of integral powers of the generating permutations h, m and f . Such composite mappings may be brought into correspondence with the standard kintype notation for genealogical relations. Correspondences between a basic set of kintypes (defined for some male ego in node x of S) and kinship mappings (i.e., elements of the group (S, G)) are listed under the first two columns of table 1.1. Distinct kintype relations are mapped onto any one kinship mapping. For example, $FBC = f^{-1}f = e$, $MZC = m^{-1}m = e$, $FFBSC = f^{-1}f^{-1}ff = e$, $Sb = e$. Conversely, any particular kinship mapping will represent a number of structurally equivalent kintypes. The basic list of table 1.1 may of course be expanded so as to include any other well-formed kintype.

Marriage rules

Thus far the structural properties attributed to the kinship models (S, h, m, f) and the associated permutation group (S, G) are of a very general nature. In order to obtain classes of proper models whose properties reflect more specific features of the structure of real kinship systems (in particular, types of marriage exchange and the classification of spouses) further constraints must be imposed on the generator permutations h, m and f .

Table 1.1. Kintypes and kinship mappings (male ego).

Kintype	Kinship mapping	Kinship mapping (commutative)	Corresponding node of the 4 x 4 model with double descent (ego at CI; see figs. 1.1 and 1.5)
FFZDC	$f^{-2}m^2$	$f^{-2}m^2$	AIII
FFZSC	$f^{-2}mf$	$f^{-1}m$	BIV
MFZDC	$m^{-1}f^{-1}m^2$	$f^{-1}m$	
FZC	$f^{-1}m$	$f^{-1}m$	CI
MFZSC	$m^{-1}f^{-1}mf$	e	
Sb	e	e	DII
FMBDC	$f^{-1}m^{-1}fm$	e	
MBC	$m^{-1}f$	$m^{-1}f$	AIII
MMBDC	$m^{-2}fm$	$m^{-1}f$	
FMBSC	$f^{-1}m^{-1}f^2$	$m^{-1}f$	AIV
MMBSC	$m^{-2}f^2$	$m^{-2}f^2$	
FFZC	$f^{-2}m$	$f^{-2}m$	BI
FSb	f^{-1}	f^{-1}	
MFZC	$m^{-1}f^{-1}m$	f^{-1}	CII
FMBC	$f^{-1}m^{-1}f$	m^{-1}	
MSb	m^{-1}	m^{-1}	DIII
MMBC	$m^{-2}f$	$m^{-2}f$	
FZDC	$f^{-1}m^2$	$f^{-1}m^2$	BIII
ZC	m	m	CIV
FZSC	$f^{-1}mf$	m	
MBDC	$m^{-1}fm$	f	DI
C	f	f	
MBSC	$m^{-1}f^2$	$m^{-1}f^2$	AII
FFSb	f^{-2}	f^{-2}	AI
FMSb	$f^{-1}m^{-1}$	$f^{-1}m^{-1}$	BII
MF Sb	$m^{-1}f^{-1}$	$f^{-1}m^{-1}$	
MMSb	m^{-2}	m^{-2}	CIII
ZDC	m^2	m^2	CIII
DC	fm	mf	DIV
ZSC	mf	mf	
SC	f^2	f^2	AI

(S, h, m, f) represents a structure with *generalized* or *asymmetric* exchange if the smallest positive integer n satisfying the equation $h^n = e$ is greater than 2. The asymmetric connubium system closes after n marriages. If $h^2 = e$, i.e., if $h = h^{-1}$, the structure exhibits a rule of *direct* or *symmetric* exchange. Ego's wife's siblings are equivalent to his siblings' spouses and sister-exchange is a structural possibility.

Conditions for unilateral cross-cousin marriage are easily defined:

Patrilateral cross-cousin marriage. According to table 1.1, FZC are denoted by the kinship mapping $f^{-1}m$. They are merged with wife and wife's siblings if and only if $f^{-1}m = h = fm^{-1}$. It immediately follows that $m^2 = f^2$ and hence (see table 1.1) that SC and ZDC are structurally equivalent in systems with a rule of FZD-marriage. Conversely, $m^{-2} = f^{-2}$, and MMSb and FFSb are also merged.

Matrilateral cross-cousin marriage. According to table 1.1, MBC are denoted by $m^{-1}f$; they are merged with wife and wife's siblings if and only if $m^{-1}f = h = fm^{-1}$, i.e., if and only if m^{-1} and f commute. It can be easily shown that this condition is equivalent to $hm = mh$, and since h and m are the generators of the group (S, G) , this group will be commutative or Abelian. This important conclusion is summarized in the following theorem:

Theorem 1: a necessary and sufficient condition for an elementary kinship structure (S, h, m, f) to be compatible with matrilateral cross-cousin marriage is that the associated group (S, G) be commutative.

A partial model of a commutative structure with matrilateral cross-cousin marriage is presented in figure 1.4 (bottom); cf. Lorrain 1975:171. For a commutative structure, the kinship mappings representing

kinship relations may be given a more simple formulation, as the order of the generator mappings is not significant. Kinship mappings for commutative structures are presented in table 1.1 (column 3). Kintypes that are structurally equivalent in models with matrilinear cross-cousin marriage can also be read off directly from table 1.1. Thus, for commutative structures, FFZSC, MFZDC, and FZC are merged, as are MFZSC, Sb, and FMBDC. Note that a rule of *exclusive* matrilinear cross-cousin marriage only holds for commutative structures in which $h^n = e$ for n greater than 2. Conversely, for commutative structures with $h^2 = e$, i.e., structures with symmetric exchange, $h = h^{-1} = m^{-1}f = f^{-1}m$. Hence ego's wife and wife's siblings are merged with his MBC and his FZC and marriage with the bilateral cross-cousin is possible (cf. table 1.1).

Kinship morphisms

Following Courrège (1965, 1974), the possibility of linking kinship models and comparing invariant aspects of their structure is expressed with formal rigour by the concept of a kinship morphism.

Consider the elementary kinship structures $X = (S_1, h_1, m_1, f_1)$ and $Y = (S_2, h_2, m_2, f_2)$. A *kinship morphism* of X onto Y is a function θ of S_1 onto S_2 such that: (1) $h_1\theta = \theta h_2$; (2) $m_1\theta = \theta m_2$; (3) $f_1\theta = \theta f_2$. (In fact, if any two of these conditions hold, the remaining condition is automatically satisfied.) If θ is a one-to-one mapping of S_1 onto S_2 , then θ is a *kinship isomorphism* of X onto Y .

Let $X = (S_1, h_1, m_1, f_1)$, $Y = (S_2, h_2, m_2, f_2)$ and $Z = (S_3, h_3, m_3, f_3)$ be elementary kinship structures, and let θ and ξ be kinship morphisms of, respectively, X onto Y and Y onto Z . Then $\zeta = \theta\xi$ is a kinship morphism of X onto Z . Hence the composition of kinship morphisms is well-defined (cf. Courrège 1975:307-310).

Kinship morphisms and group homomorphisms

The following theorem summarizes the fundamental relationship between kinship morphisms and homomorphisms of the associated group structures (cf. Courrège 1975: 310-311, lemmas 2.1 and 2.2):

Theorem 2: Let $X = (S_1, h_1, m_1, f_1)$ and $Y = (S_2, h_2, m_2, f_2)$ be elementary kinship structures with a kinship morphism θ defined from X onto Y . Then there exists a unique homomorphism ψ from the group (S_1, G_1) onto the group (S_2, G_2) , i.e., ψ is a function such that $(\alpha\beta)\psi = (\alpha\psi)(\beta\psi)$ for all α and β in (S_1, G_1) .

Sketch of the proof: θ is a kinship morphism, hence $h_1\theta = \theta h_2$, $m_1\theta = \theta m_2$, $f_1\theta = \theta f_2$, and it can be verified without difficulty that $\alpha\theta = \theta\alpha'$ and α' is well-defined. In general there will be an entire class of elements α of (S_1, G_1) mapped onto each α' of (S_2, G_2) . Define the group homomorphism ψ as $(\alpha)\psi = \alpha'$. Note that $(h_1)\psi = h_1' = h_2$, $(m_1)\psi = m_1' = m_2$, and $(f_1)\psi = f_1' = f_2$. Note also that if two kinship structures are isomorphic, then their groups are also isomorphic. However, the converse relation does not hold: isomorphism of the group structures does not necessarily imply isomorphism of the kinship structures (see Courrège 1974:311-312).

Quotient structures and quotient groups

The following results on quotient structures are adaptations of Courrège's lemmas 2.3 - 2.5 (1965; 1974: 313-315). I have extended his analysis, presenting the basic concepts in a format that allows a direct comparison with the important class of models developed by Boyd (1969, 1971, 1972).⁴²

Consider the elementary kinship structure $X = (S, h, m, f)$ with the partition P on S compatible with X , i.e., P is compatible with h , m , and f . Then P induces the quotient structure $\bar{X} = (P, \bar{h}, \bar{m}, \bar{f})$, i.e.,

an elementary kinship structure defined on the blocks of the partition P . If R is the canonical equivalence relation associated with P , then $P = S/R$ and $\bar{X} = X/R$. Note that $(xR)\bar{h} = [xR]h$, $(xR)\bar{m} = [xR]m$, and $(xR)\bar{f} = [xR]f$ for all x in S .

Lemma 1: Consider the elementary kinship structures $X = (S_1, h_1, m_1, f_1)$ and $Y = (S_2, h_2, m_2, f_2)$ and let θ be a kinship morphism of X onto Y . Define the inverse $\theta^{-1} = \{(s_2, s_1) \mid (s_1)\theta = s_2\}$. Let P be the partition of S_1 defined as $P = \{(y)\theta^{-1} \mid y \text{ in } S_2\}$, with the canonical equivalence relation R on S_1 generated by P defined as $R = \{(x, z) \mid (x)\theta\theta^{-1} = z \text{ for all } x \text{ and } z \text{ in } S_1\}$.

Then P is compatible with the kinship structure X , and there exists a kinship isomorphism η of Y onto the quotient structure $\bar{X} = X/R$. η is defined as the mapping $\eta: y \mapsto (y)\theta^{-1}$ of S_2 onto the partition P . It is easily verified that η is one-one, and that $h_2\eta = \eta\bar{h}_1$, $m_2\eta = \eta\bar{m}_1$, and $f_2\eta = \eta\bar{f}_1$ for all y in S_2 . For a sketch of the proof, see Courrège (1974:313-314, lemma 2.3).

Corollary: Let $X = (S, h, m, f)$ be an elementary kinship structure with partition P on S compatible with X . Let R be the associated canonical equivalence relation defined as above. Let $\bar{X} = (P, \bar{h}, \bar{m}, \bar{f})$ be the quotient structure of X induced by the partition P .

Then the mapping θ of S onto P defined by $(x)\theta = xR$, with xR the unique block of P such that x is in xR , is a kinship morphism of X onto the quotient structure $\bar{X}$.

Furthermore, consider the associated groups $G(h, m, f) = (S, G)$ and $\bar{G}(\bar{h}, \bar{m}, \bar{f}) = (P, \bar{G})$. All elements g of G are compatible with the partition P . Then the mapping ϕ of G onto $\bar{G}$ defined as $\phi: g \mapsto \bar{g}$, with $\bar{g}$ uniquely defined by $(xR)\bar{g} = [xR]g$ for all x in S and g in G is a homomorphism of G onto $\bar{G}$.

Finally, let H be the subgroup of G consisting of all elements g of G such that $[xR]g = xR$ for all blocks xR

of the partition P on S . Then H is normal in G , and $\bar{G}$ is isomorphic to the quotient group G/H . It is easily verified that the isomorphism is defined by the mapping $\psi: Hg \rightarrow \bar{g}$ of G/H onto $\bar{G}$, with $\bar{g}$ defined as above by $(xR)\bar{g} = [xR]g$.

Theorem 2, lemma 1 and its corollaries support the following theorem which is of central importance:

Theorem 3: Consider the elementary kinship structure $X = (S, h, m, f)$. Let $\mathcal{P}$ be the set of partitions P_i on S , with each partition compatible with X . Let $\mathcal{N}$ be the set of normal subgroups N_i of the associated group $G(h, m, f) = (S, G)$. Then there is a matching of $\mathcal{P}$ and $\mathcal{N}$, i.e., a one-to-one mapping μ of $\mathcal{P}$ onto $\mathcal{N}$ such that each partition P_i on S corresponds to a unique normal subgroup N_i of G .

If R_i is the canonical equivalence relation associated with P_i , then $\mu: P_i \rightarrow N_i$ is uniquely defined by taking N_i to be the normal subgroup of G consisting of all elements g of G such that $[xR_i]g = xR_i$ for all blocks xR_i of the partition P_i . Let $\bar{X}_i = (P_i, \bar{h}_i, \bar{m}_i, \bar{f}_i)$ be the quotient structure of X induced by the partition P_i , with $\bar{G}_i(\bar{h}_i, \bar{m}_i, \bar{f}_i) = (P_i, \bar{G}_i)$ the associated group. Then $\bar{G}_i$ is isomorphic to the quotient group G/N_i .

The proof of theorem 3 is fairly direct. The results presented here for elementary kinship structures closely parallel the more familiar homomorphism theorems for groups (cf. Baumslag and Chandler 1968:114-127). Following Courrège (1974:316-317), it can be easily demonstrated that further correspondence theorems (for example, on the quotient structures of a quotient structure) may be derived. Such refinements of the basic model are, however, not essential to my analysis.

By constructing the set of quotient structures one is able to focus more clearly on specific characteristics of the encompassing kinship structure, including

'latent' possibilities implicit in the original structure which might otherwise be overlooked. In addition, any partition compatible with the encompassing structure may be realised in data from actual societies and identified with, say, a moiety structure or some other system of social groupings or categories. The extent to which such structural possibilities are actually realised is of course a matter for empirical research.

Returning now to the kinship models introduced by the prewar Leiden scholars, the structures of figure 1.1 all represent quotient structures of the basic kinship model with double descent and circulating connubium. The catalogue of possible quotient structures is, however, not complete. I now reformulate the original class of double descent models with exclusive matrilinear cross-cousin marriage as algebraic structures and derive a set of quotient structures that is indeed complete.

DOUBLE DESCENT AND MATRILINEAL CROSS-COUSIN MARRIAGE

Let I be the index set $I = \{1, 2, \dots, n\}$ and define the basic set of objects $S_n = \{O_i | i \text{ in } I\}$. Define the permutation c of S_n as $c = (O_1, O_2, \dots, O_n)$, i.e., c is the cyclic permutation of order n which maps each object O_i onto O_{i+1} , with $i+1$ reduced modulo n .

Then the identity permutation $e = c^n = c^0$ and in general, $(O_i)c^k = O_{i+k}$ with $i+k$ reduced modulo n , and the inverse of the permutation c^k is c^{-k} . It is easily verified that c generates the cyclic group C_n of order n which is commutative.

These preliminaries lead to simple definitions of two well-known kinship structures with unilineal descent and a rule of matrilinear cross-cousin marriage.

Definition 1: M_n , the n -matriline elementary kinship structure with circulating connubium, is defined as

$M_n = (S_n, c, e, c)$, i.e., $h = c$, $m = e$, $f = c$, and the associated permutation group (S_n, C_n) is commutative. In like manner, P_n , the n -patriline elementary kinship structure with circulating connubium, is defined as $P_n = (S_n, c, c^{-1}, e)$, i.e., $h = c$, $m = c^{-1}$, $f = e$, and the associated permutation group (S_n, C_n) is commutative. Note that there is no kinship isomorphism of the kinship structures M_n and P_n , although the associated groups are clearly isomorphic. Since the groups are commutative, according to theorem 1 both kinship structures are compatible with matrilineal cross-cousin marriage. Kinship models clearly isomorphic with M_n and P_n (for n equal to 2, 3 and 4) have been described by previous generations of Leiden anthropologists.⁴³

One may now obtain the class of double descent models with matrilineal cross-cousin marriage by defining the *product structure* of M_n and P_n . Definition 2 emulates and formalizes the more intuitive procedure devised by Van Wouden (1968 [1935]:90-94) and others.

Definition 2: $M_n \times P_n$, the elementary kinship structure with circulating connubium and double descent on n matrilineal and n patrilineal lines, is defined as

$M_n \times P_n = (S_n \times S_n, c \times c, e \times c^{-1}, c \times e)$, i.e., $M_n \times P_n = (S, h, m, f)$ with $S = S_n \times S_n$ and h, m and f defined by $(O_i, O_j)h = (O_i, O_j)(c \times c) = ((O_i)c, (O_j)c)$; $(O_i, O_j)m = (O_i, O_j)(e \times c^{-1}) = ((O_i)e, (O_j)c^{-1})$; and $(O_i, O_j)f = (O_i, O_j)(c \times e) = ((O_i)c, (O_j)e)$ for all elements (O_i, O_j) of $S_n \times S_n$. Furthermore, $(S, H) = (S_n \times S_n, C_n \times C_n)$ is the commutative group associated with the kinship structure $M_n \times P_n$.

For convenience of notation the elements (O_i, O_j) of S will be denoted by the ordered double index (i, j) .

Figure 1.5 shows a kinship network representing the

elementary kinship structure $M_n \times P_n$. Capital letters A, B, C, ... denote matrilineages; roman numerals I, II, III, ... denote patrilineages, and the nodes are thus differentiated by a double index and linked by the basic kinship mappings h , m , and f . For example, $(CII)_h = DIII$, $(CII)_m = CI$, and $(CII)_f = DII$, i.e., a man in CII marries a woman in DIII, his sister's children are in CI and his own children in DII. The marriage system is assumed to close back on itself after n exchanges, with n successive generations of men from one matriline marrying into different patrilineages. In fact, since $h^n = m^n = f^n = e$, the kinship structure $M_n \times P_n$ is perhaps best represented by mapping the diagram of figure 1.5 onto the surface of a torus.

The kinship structure $M_n \times P_n$, defined as the product of the matrilineal structure M_n with the patrilineal structure P_n , is a direct representation of the general double descent model introduced by Van Wouden in his thesis of 1935. (Van Wouden (1968:92-94) even applied the term 'product' to his model.) Van Wouden also extended his basic model, allowing for double descent structures with unequal numbers of matri- and patrilineages (1968:94). For example, he postulated a 'theoretical reconstruction' of Kei social structure based on the intersection of four matrilineal *fam* groups with six patrilineal clans (1968:158-160). This extension can easily be formalized by defining the general class of product structures $M_n \times P_q$ (with n unequal to q) in analogy to definition 2. (A different type of solution is developed in Chapter 3.) Nevertheless, within the Leiden tradition it is arguably the four-matriline, four-patriline model with circulating connubium formulated by Van Wouden and Held in 1935 that became the standard example, i.e., the concrete problem solution on which much subsequent research was to be modelled. I now develop and clarify the most important characteristics of this particular model.

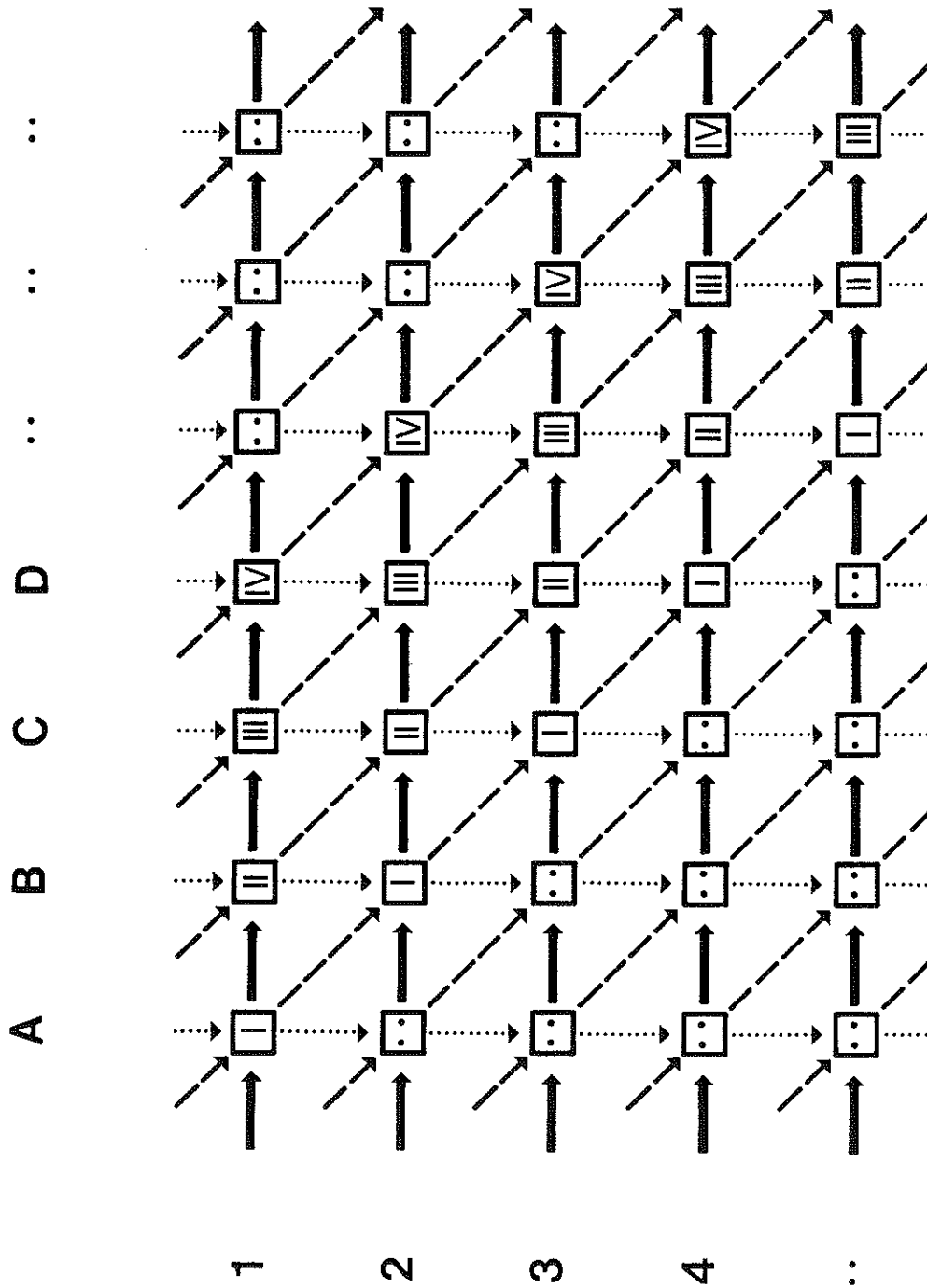


Fig. 1.5. Representation of the structure $M_n \times P_n$ with circulating connubium and double descent. Matrilines are denoted by capital letters, patriline by roman numerals. Nodes are linked by the mappings h (solid arrows), m (dotted arrows), and f (broken arrows).

$M_4 \times P_4$: partitions and quotient structures

Consider the kinship structure $M_4 \times P_4$ with circulating connubium and double descent defined as the product of the four-matriline structure M_4 with the four-patriline structure P_4 . With index set $I = \{1, 2, 3, 4\}$, the sixteen-element set $S = S_4 \times S_4 = \{(O_i, O_j) | i \text{ and } j \text{ in } I\}$ contains the objects on which $M_4 \times P_4$ is defined. They are interpreted as the nodes, sibling groups, or 'classes' in a reduced genealogical network (see figure 1.4 and figure 1.5). In condensed notation, elements of S are represented by double indices:

$$S = \{11, 12, 13, 14, \dots, 24, 31, \dots, 41, 42, 43, 44\}.$$

In figure 1.5 and in table 1.1 I introduce a slightly different notational convention, substituting capital letters A, B, C, D and roman numerals I, II, III, IV for, respectively, the values of the first and second indices. Consequently, elements of S are:

$$S = \{AI, AII, \dots, BIV, CI, \dots, DI, DII, DIII, DIV\}.$$

Both of these schemes are in direct correspondence with the system of symbols employed by Held (1935:95), P.E. de Josselin de Jong (1980 [1951]:39) and others, and also used in figure 1.1.

Let $c = (1, 2, 3, 4)$ be the basic permutation. Then $c^4 = e$, $c^{-1} = c^3 = (1, 4, 3, 2)$, $c^{-2} = c^2 = (1, 3)(2, 4)$ and the basic mappings of $M_4 \times P_4$ are defined as:

$$\begin{aligned} h &= c \times c = (11, 22, 33, 44)(12, 23, 34, 41) \\ &\quad (13, 24, 31, 42)(14, 21, 32, 43); \\ m &= e \times c^{-1} = (11, 14, 13, 12)(21, 24, 23, 22) \\ &\quad (31, 34, 33, 32)(41, 44, 43, 42); \text{ and} \\ f &= c \times e = (11, 21, 31, 41)(12, 22, 32, 42) \\ &\quad (13, 23, 33, 43)(14, 24, 34, 44). \end{aligned}$$

Note that $h^4 = m^4 = f^4 = e$, hence $h^{-1} = h^3$, $m^{-1} = m^3$, and $f^{-1} = f^3$.

The 16 elements of the associated commutative group (S, H) represent kinship mappings. They are: e (of

order 1); m^2, f^2, m^2f^2 (of order 2); $m, m^3, f, f^3, mf, f^3m^3, m^2f, f^3m^2, m^3f, f^3m, m^3f^2, f^2m$ (of order 4).

The associated set of kintypes for each kinship mapping can be obtained from table 1.1. Also, assuming that male ego is situated in node 31 (or CI) of the model, kinship mappings and kintypes are specified for all nodes of the genealogical structure in figure 1.1 (bottom, right) and figure 1.5 (for $n = 4$). For example, ego's MBC are located in node DII, since $(31)m^3f = 42 = \text{DII}$; ego's FZC are in node BIV as $(31)f^3m = 24 = \text{BIV}$.

The group (S, H) is isomorphic to the homocyclic group $C_4 \times C_4$ (the group labelled 16/3 in Thomas and Wood's (1980) summary of group tables). Since (S, H) is commutative, all subgroups are commutative. Hence it follows directly from the definition of a normal subgroup (see the appendix) that all subgroups of (S, H) are also normal in (S, H) . The normal subgroups and their elements are:

The order 1 subgroup $E = \{e\} \cong C_1$.

Three order 2 subgroups: $N_8 = \{e, f^2\}$;
 $N_{12} = \{e, m^2f^2\}$;
 $N_{13} = \{e, m^2\}$, all isomorphic to the cyclic group C_2 .

Seven order 4 subgroups: $N_4 = \{e, f, f^2, f^3\}$;
 $N_6 = \{e, m^2f, f^2, f^3m^2\}$;
 $N_7 = \{e, m^3f, m^2f^2, f^3m\}$;
 $N_9 = \{e, mf, m^2f^2, f^3m^3\}$;
 $N_{10} = \{e, f^2m, m^2, m^3f^2\}$;
 $N_{11} = \{e, m, m^2, m^3\}$, all isomorphic to C_4 , and
 $N_5 = \{e, m^2, f^2, m^2f^2\}$ isomorphic to $C_2 \times C_2$, the Klein group.

Three order 8 subgroups:

$N_1 = \{e, m^3f, m^2f^2, f^3m, f^2, m^2, mf, m^3f^3\}$;
 $N_2 = \{e, f, f^2, f^3, m^2, m^2f, m^2f^2, f^3m^2\}$;

$N_3 = \{e, m, m^2, m^3, f^2, f^2m, m^2f^2, m^3f\}$, all isomorphic to $C_2 \times C_4$.

Finally, the trivial subgroup $H = (S, H)$ of order 16.

Having obtained the complete set of normal subgroups, the next step is to derive all possible partitions compatible with the double descent structure $M_4 \times P_4$, together with the corresponding quotient structures. If H/N_i is a quotient group, then the blocks of the partition P_i on S are denoted as $P_i = \{B_1^i, B_2^i, B_3^i, \dots, B_m^i\}$, and the associated quotient structure $\bar{X}_i = (P_i, \bar{h}_i, \bar{m}_i, \bar{f}_i)$. Note that H/N_i defines the partition $\{N_i g | g \text{ in } H\}$ on H , and that the mapping $\phi: g \mapsto N_i g$ is a homomorphism of H onto H/N_i . Consequently, since $(e)\phi = N_i$, the normal subgroup N_i is the identity of the quotient group H/N_i , and $\{y | y = (x)g \text{ for } g \text{ in } N_i \text{ and some } x \text{ in } S\}$ is the equivalence class of x , i.e., a block of the partition P_i .

The complete set of quotient structures and partitions compatible with the double descent structure $M_4 \times P_4$ is listed below.

H/E : $P_E = S$, and $\bar{X}_E = (P_E, \bar{h}_E, \bar{m}_E, \bar{f}_E) = (S, h, m, f)$.

H/N_8 : $P_8 = \{B_1^8, \dots, B_8^8\}$ with

$$\begin{aligned} B_1^8 &= \{11, 31\}; B_2^8 = \{12, 32\}; B_3^8 = \{13, 33\}; \\ B_4^8 &= \{14, 34\}; B_5^8 = \{21, 41\}; B_6^8 = \{22, 42\}; \\ B_7^8 &= \{23, 43\}; B_8^8 = \{24, 44\}. \end{aligned}$$

Hence $\bar{X}_8 = (P_8, \bar{h}_8, \bar{m}_8, \bar{f}_8)$ with

$$\begin{aligned} \bar{h}_8 &= (B_1^8, B_6^8, B_3^8, B_8^8)(B_2^8, B_7^8, B_4^8, B_5^8), \\ \bar{m}_8 &= (B_1^8, B_4^8, B_3^8, B_2^8)(B_5^8, B_8^8, B_7^8, B_6^8), \\ \bar{f}_8 &= (B_1^8, B_5^8)(B_2^8, B_6^8)(B_3^8, B_7^8)(B_4^8, B_8^8), \end{aligned}$$

and $(\bar{h}_8)^4 = (\bar{m}_8)^4 = (\bar{f}_8)^2 = \bar{e}_8$.

H/N_{12} : $P_{12} = \{B_1^{12}, \dots, B_8^{12}\}$ with

$$\begin{aligned} B_1^{12} &= \{11, 33\}; B_2^{12} = \{12, 34\}; B_3^{12} = \{13, 31\}; \\ B_4^{12} &= \{14, 32\}; B_5^{12} = \{21, 43\}; B_6^{12} = \{22, 44\}; \\ B_7^{12} &= \{23, 41\}; B_8^{12} = \{24, 42\}. \end{aligned}$$

Hence $\bar{h}_{12} = (B_1^{12}, B_6^{12})(B_2^{12}, B_7^{12})(B_3^{12}, B_8^{12})(B_4^{12}, B_5^{12})$,
 $\bar{m}_{12} = (B_1^{12}, B_4^{12}, B_3^{12}, B_2^{12})(B_5^{12}, B_8^{12}, B_7^{12}, B_6^{12})$,
 $\bar{f}_{12} = (B_1^{12}, B_5^{12}, B_3^{12}, B_7^{12})(B_2^{12}, B_6^{12}, B_4^{12}, B_8^{12})$,
 and $(\bar{h}_{12})^2 = (\bar{m}_{12})^4 = (\bar{f}_{12})^4 = \bar{e}_{12}$.

H/N_{13} : $P_{13} = \{B_1^{13}, \dots, B_8^{13}\}$ with

$$\begin{aligned} B_1^{13} &= \{11, 13\}; B_2^{13} = \{12, 14\}; B_3^{13} = \{21, 23\}; \\ B_4^{13} &= \{22, 24\}; B_5^{13} = \{31, 33\}; B_6^{13} = \{32, 34\}; \\ B_7^{13} &= \{41, 43\}; B_8^{13} = \{42, 44\}. \end{aligned}$$

Hence $\bar{X}_{13} = (P_{13}, \bar{h}_{13}, \bar{m}_{13}, \bar{f}_{13})$ with

$$\begin{aligned} \bar{h}_{13} &= (B_1^{13}, B_4^{13}, B_5^{13}, B_8^{13})(B_2^{13}, B_3^{13}, B_6^{13}, B_7^{13}), \\ \bar{m}_{13} &= (B_1^{13}, B_2^{13})(B_3^{13}, B_4^{13})(B_5^{13}, B_6^{13})(B_7^{13}, B_8^{13}), \\ \bar{f}_{13} &= (B_1^{13}, B_3^{13}, B_5^{13}, B_7^{13})(B_2^{13}, B_4^{13}, B_6^{13}, B_8^{13}), \end{aligned}$$

and $(\bar{h}_{13})^4 = (\bar{m}_{13})^2 = (\bar{f}_{13})^4 = \bar{e}_{13}$.

In addition to the trivial quotient structure $\bar{X}_E$ and the three quotient structures $\bar{X}_8$, $\bar{X}_{12}$, and $\bar{X}_{13}$ associated with partitions of order 8, there are exactly seven quotient structures on partitions of order 4. They are:

H/N_4 : $P_4 = \{B_1^4, \dots, B_4^4\}$ with

$$\begin{aligned} B_1^4 &= \{11, 21, 31, 41\}; B_2^4 = \{12, 22, 32, 42\}; \\ B_3^4 &= \{13, 23, 33, 43\}; B_4^4 = \{14, 24, 34, 44\}. \end{aligned}$$

Hence $\bar{X}_4 = (P_4, \bar{h}_4, \bar{m}_4, \bar{f}_4)$ with $\bar{h}_4 = (B_1^4, B_2^4, B_3^4, B_4^4)$,
 $\bar{m}_4 = (B_1^4, B_4^4, B_3^4, B_2^4)$, and $\bar{f}_4 = \bar{e}_4 = (\bar{h}_4)^4 = (\bar{m}_4)^4$.

$$\begin{aligned}
H/N_6: \quad P_6 &= \{B_1^6, \dots, B_4^6\} \text{ with} \\
B_1^6 &= \{11, 23, 31, 43\}; \quad B_2^6 = \{12, 24, 32, 44\}; \\
B_3^6 &= \{13, 21, 33, 41\}; \quad B_4^6 = \{14, 22, 34, 42\}.
\end{aligned}$$

$$\begin{aligned}
\text{Hence } \bar{X}_6 &= (P_6, \bar{h}_6, \bar{m}_6, \bar{f}_6) \text{ with } \bar{h}_6 = \bar{m}_6 = \\
&= (B_1^6, B_4^6, B_3^6, B_2^6), \quad \bar{f}_6 = (B_1^6, B_3^6)(B_2^6, B_4^6), \\
&\text{and } (\bar{h}_6)^4 = (\bar{m}_6)^4 = (\bar{f}_6)^2 = \bar{e}_6.
\end{aligned}$$

$$\begin{aligned}
H/N_7: \quad P_7 &= \{B_1^7, \dots, B_4^7\} \text{ with} \\
B_1^7 &= \{11, 22, 33, 44\}; \quad B_2^7 = \{12, 23, 34, 41\}; \\
B_3^7 &= \{13, 24, 31, 42\}; \quad B_4^7 = \{14, 21, 32, 43\}.
\end{aligned}$$

$$\begin{aligned}
\text{Hence } \bar{X}_7 &= (\bar{P}_7, \bar{h}_7, \bar{m}_7, \bar{f}_7) \text{ with } \bar{m}_7 = \bar{f}_7 = \\
&= (B_1^7, B_4^7, B_3^7, B_2^7) \text{ and } \bar{h}_7 = \bar{e}_7 = (\bar{m}_7)^4 = (\bar{f}_7)^4.
\end{aligned}$$

$$\begin{aligned}
H/N_5: \quad P_5 &= \{B_1^5, \dots, B_4^5\} \text{ with} \\
B_1^5 &= \{11, 31, 13, 33\}; \quad B_2^5 = \{12, 32, 14, 34\}; \\
B_3^5 &= \{21, 41, 23, 43\}; \quad B_4^5 = \{22, 42, 24, 44\}.
\end{aligned}$$

$$\begin{aligned}
\text{Hence } \bar{X}_5 &= (P_5, \bar{h}_5, \bar{m}_5, \bar{f}_5) \text{ with } \bar{h}_5 = (B_1^5, B_4^5)(B_2^5, B_3^5), \\
\bar{m}_5 &= (B_1^5, B_2^5)(B_3^5, B_4^5); \quad \bar{f}_5 = (B_1^5, B_3^5)(B_2^5, B_4^5), \\
&\text{and } (\bar{h}_5)^2 = (\bar{m}_5)^2 = (\bar{f}_5)^2 = \bar{e}_5.
\end{aligned}$$

$$\begin{aligned}
H/N_9: \quad P_9 &= \{B_1^9, \dots, B_4^9\} \text{ with} \\
B_1^9 &= \{11, 24, 33, 42\}; \quad B_2^9 = \{12, 21, 34, 43\}; \\
B_3^9 &= \{13, 22, 31, 44\}; \quad B_4^9 = \{14, 23, 32, 41\}.
\end{aligned}$$

$$\begin{aligned}
\text{Hence } \bar{X}_9 &= (P_9, \bar{h}_9, \bar{m}_9, \bar{f}_9) \text{ with } \bar{h}_9 = (B_1^9, B_3^9)(B_2^9, B_4^9), \\
\bar{m}_9 &= (B_1^9, B_4^9, B_3^9, B_2^9), \quad \bar{f}_9 = (B_1^9, B_2^9, B_3^9, B_4^9), \\
&\text{and } (\bar{h}_9)^2 = (\bar{m}_9)^4 = (\bar{f}_9)^4 = \bar{e}_9.
\end{aligned}$$

$$H/N_{10}: \quad P_{10} = \{B_1^{10}, \dots, B_4^{10}\} \text{ with}$$

$$\begin{aligned} B_1^{10} &= \{11, 34, 13, 32\}; B_2^{10} = \{12, 31, 14, 33\}; \\ B_3^{10} &= \{21, 44, 23, 42\}; B_4^{10} = \{22, 41, 24, 43\}. \end{aligned}$$

Hence $\bar{X}_{10} = (P_{10}, \bar{h}_{10}, \bar{m}_{10}, \bar{f}_{10})$ with

$$\bar{h}_{10} = (B_1^{10}, B_4^{10}, B_2^{10}, B_3^{10}),$$

$$\bar{m}_{10} = (B_1^{10}, B_2^{10})(B_3^{10}, B_4^{10}),$$

$$\bar{f}_{10} = (B_1^{10}, B_3^{10}, B_2^{10}, B_4^{10}),$$

and $(\bar{h}_{10})^4 = (\bar{m}_{10})^2 = (\bar{f}_{10})^4 = \bar{e}_{10}.$

H/N_{11} : $P_{11} = \{B_1^{11}, \dots, B_4^{11}\}$ with

$$B_1^{11} = \{11, 14, 13, 12\}; B_2^{11} = \{21, 24, 23, 22\};$$

$$B_3^{11} = \{31, 34, 33, 32\}; B_4^{11} = \{41, 44, 43, 42\}.$$

Hence $\bar{X}_{11} = (P_{11}, \bar{h}_{11}, \bar{m}_{11}, \bar{f}_{11})$ with

$$\bar{h}_{11} = \bar{f}_{11} = (B_1^{11}, B_2^{11}, B_3^{11}, B_4^{11}),$$

and $\bar{m}_{11} = \bar{e}_{11} = (\bar{h}_{11})^4 = (\bar{f}_{11})^4.$

The three quotient structures compatible with a dual division of S are:

H/N_2 : $P_2 = \{B_1^2, B_2^2\}$ with

$$B_1^2 = \{11, 21, 31, 41, 13, 23, 33, 43\} \text{ and}$$

$$B_2^2 = \{12, 22, 32, 42, 14, 24, 34, 44\}.$$

Hence $\bar{X}_2 = (P_2, \bar{h}_2, \bar{m}_2, \bar{f}_2)$ with $\bar{h}_2 = \bar{m}_2 = (B_1^2, B_2^2)$

and $\bar{f}_2 = \bar{e}_2 = (\bar{h}_2)^2 = (\bar{m}_2)^2.$

H/N_1 : $P_1 = \{B_1^1, B_2^1\}$ with

$$B_1^1 = \{11, 22, 33, 44, 31, 42, 13, 24\} \text{ and}$$

$$B_2^1 = \{12, 23, 34, 41, 32, 43, 14, 21\}.$$

Hence $\bar{X}_1 = (P_1, \bar{h}_1, \bar{m}_1, \bar{f}_1)$ with $\bar{m}_1 = \bar{f}_1 = (B_1^1, B_2^1),$

and $\bar{h}_1 = \bar{e}_1 = (\bar{m}_1)^2 = (\bar{f}_1)^2.$

H/N_3 : $P_3 = \{B_1^3, B_2^3\}$ with

$B_1^3 = \{11, 14, 13, 12, 31, 34, 33, 32\}$ and

$B_2^3 = \{21, 24, 23, 22, 41, 44, 43, 42\}$.

Hence $\bar{X}_3 = (P_3, \bar{h}_3, \bar{m}_3, \bar{f}_3)$ with $\bar{h}_3 = \bar{f}_3 = (B_1^3, B_2^3)$,

and $\bar{m}_3 = \bar{e}_3 = (\bar{h}_3)^2 = (\bar{f}_3)^2$.

Finally, there is the trivial one-block quotient structure:

H/H : $P_H = \{S\}$, hence $\bar{X}_H = (P_H, \bar{h}_H, \bar{m}_H, \bar{f}_H)$ with

$$\bar{h}_H = \bar{m}_H = \bar{f}_H = \bar{e}_H.$$

This complex mass of information may be reduced to more manageable proportions by introducing the following definitions.

Consider the set $\mathcal{P}$ of all partitions P_i on S compatible with the kinship structure $M_4 \times P_4$. Then P_i is a *refinement* of P_j (notation: $P_i \leq P_j$) if each block of P_j is the union of a unique subset of blocks of P_i , i.e., if there exists a morphism of the quotient structure $\bar{X}_i$ onto the quotient structure $\bar{X}_j$. If $P_i \leq P_j$, then $\bar{X}_j$ is called an *approximation* or a *reduction* of $\bar{X}_i$, since $\bar{X}_j$ exhibits some, but not all of the structural characteristics of $\bar{X}_i$ (see also Courrège 1974:316-317 and Boyd 1969:152-153).

In the example above, the partition P_8 is a refinement of the partitions P_4 , P_6 , and P_5 , but not of P_7 , P_9 , P_{10} or P_{11} . The refinement relation is a binary relation which is reflexive, antisymmetric, and transitive. It can be shown to induce a partial ordering of the set of quotient structures of $M_4 \times P_4$ with greatest lower bound $\bar{X}_E$ and least upper bound $\bar{X}_H$, hence $\bar{X}_E \leq \bar{X}_i \leq \bar{X}_H$ for all quotient structures $\bar{X}_i$ of $M_4 \times P_4$. Finally, for $\bar{X}_i \neq \bar{X}_j$, $\bar{X}_j$ is a *cover* of $\bar{X}_i$ if $\bar{X}_i < \bar{X}_j$ and there is no $\bar{X}_k$ such that $\bar{X}_i < \bar{X}_k < \bar{X}_j$.

The system of relations between the quotient structures of $M_4 \times P_4$ (and between the corresponding quotient groups) is diagrammed in figure 1.6. Wavy arrows link each quotient structure to its covering structures. The diagram is in fact a *lattice*, i.e., a partial ordering such that a least upper bound and a greatest lower bound exist for any two quotient structures $\bar{X}_i$ and $\bar{X}_j$. Each wavy arrow also represents a kinship morphism. Hence $\bar{X}_j$ is an approximation or a reduction of $\bar{X}_i$ if there is a chain of arrows linking $\bar{X}_i$ to $\bar{X}_j$. Finally, for each particular quotient structure, the basic mappings h , m , and f are denoted by solid arrows, dotted arrows, and broken arrows.

COMPARISONS AND EXTENSIONS

I now compare the complete set of quotient structures of $M_4 \times P_4$ with the results obtained under the early Leiden double descent paradigm.

In the first place, the following 'latent' structures are clearly distinguished (see figure 1.1 and Van Wouden (1968:90-92) and P.E. de Josselin de Jong (1980:37-42, 186-188)): (i) a four-'clan' matrilineal structure, and (ii) a four-'clan' patrilineal structure, both with exclusive matrilineal cross-cousin marriage; (iii) a matrimoiety structure, (iv) a patrimoiety structure, and (v) a four-'class', double phratry structure. In these three structures sister exchange and double cross-cousin marriage are possible.

The corresponding quotient structures of $M_4 \times P_4$ are: (i) $\bar{X}_{11}$, (ii) $\bar{X}_4$, (iii) $\bar{X}_3$, (iv) $\bar{X}_2$, and (v) $\bar{X}_5$. The quotient structure $\bar{X}_7$ expresses the general principle of generation endogamy, with marriage possibilities confined to members of the same generation and the cycle repeating after four consecutive generations. $\bar{X}_1$ is the analogous dual structure with generation endogamy and alternating generations. These two quotient structures are also

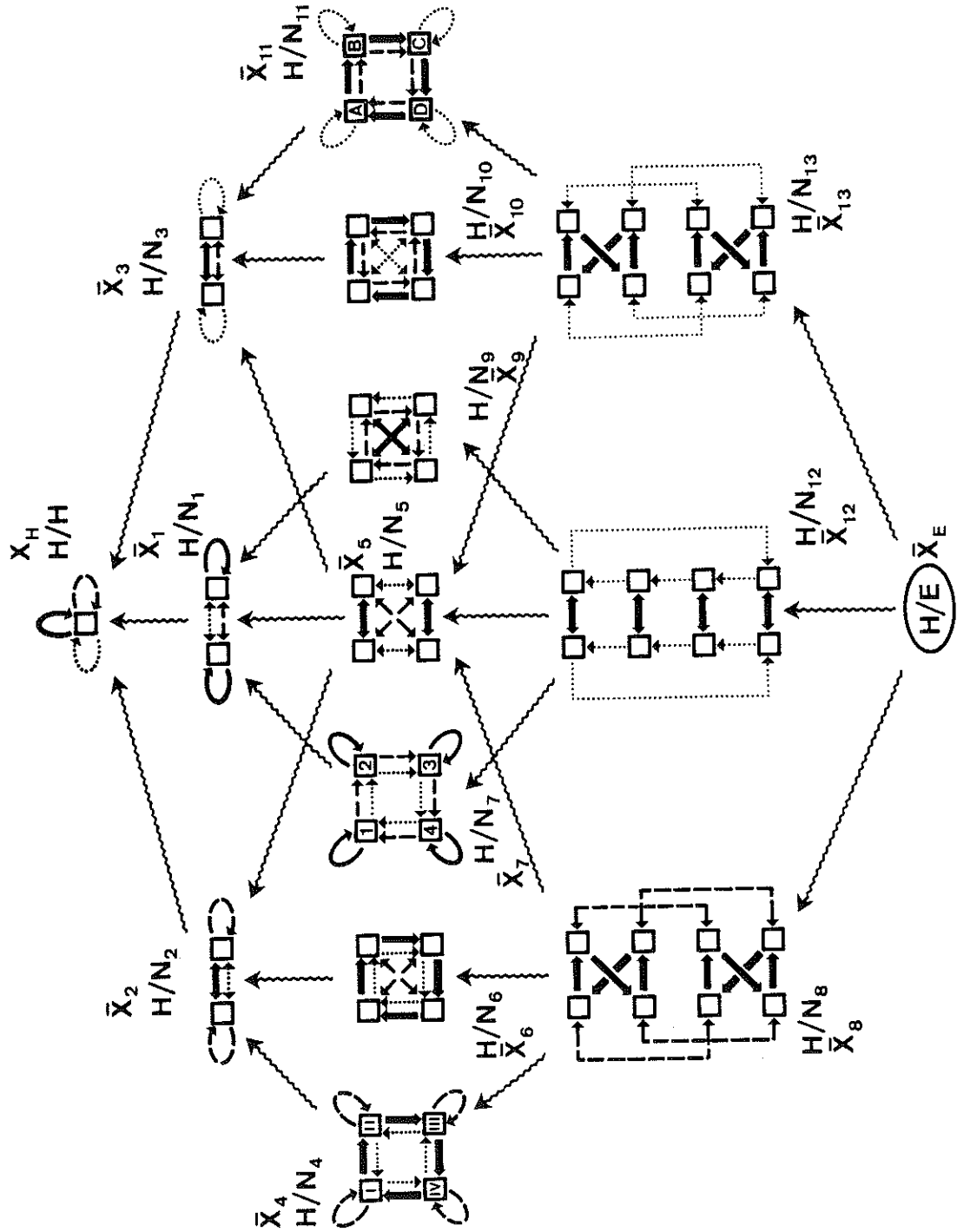


Fig. 1.6. Lattice of quotient structures of $M_4 \times P_4$. Wavy arrows point from $\bar{X}_i$ to a covering structure. See the text for further details.

identified by Van Wouden (1968:92).

Second, the idea of partitioning the sixteen double descent 'classes' into a set of larger groupings compatible with the original kinship structure is made explicit in diagrams by Van Wouden (1968:93) and P.E. de Josselin de Jong (1980:41) (see figure 1.2, adapted from kinship chart 1).⁴⁵

Third, not all of the quotient structures appear in analyses of the Indonesian material. The four-block structures $\bar{X}_6$, $\bar{X}_9$ and $\bar{X}_{10}$, and the eight-block structures $\bar{X}_8$, $\bar{X}_{12}$ and $\bar{X}_{13}$ are not derived. This omission is curious, considering the importance of fourfold and eightfold schemes of classification in many Indonesian societies (cf. P.E. de Josselin de Jong 1980:151-162). Many of these structures do, however, figure prominently in the classic discussions of kinship systems from other fields of study: Australia and China.

Thus, $\bar{X}_{12}$ is the four-generation structure with direct exchange described by Howitt (1889:43-46) for the Waramunga, by Stanner (1933:398-399) for the Nangiomeri, and by Webb (1933) for the 'Murngin'. It is isomorphic to the 'regular' and 'alternate' structures in Lévi-Strauss's figure 19 (1970:171). $\bar{X}_8$ is the two-matriline, four-patriline 'Lawrence' structure with asymmetric exchange⁴⁴ (cf. Barnes 1967:17). It is isomorphic to Lévi-Strauss's 'combined regular and alternate Murngin system' depicted in his figure 23 (1970:175) and in a variant notation, by J.P.B. de Josselin de Jong (1952:42). $\bar{X}_6$ is the hypothetical four-class asymmetrical structure reconstructed by Lévi-Strauss (1970:185-189) as a necessary stage in the development of the full 'Murngin' structure. This model is also described by Jorion (1982:77-78) for the Karadjeri. $\bar{X}_{13}$ is the 'reversed' Lawrence structure with asymmetric exchange, four matriline and two patriline described by Granet (1939:206 fig. C, 232-233 fig. G) for the Chinese system. It is reproduced by Lévi-Strauss as his figure 58 and figure 66 (right) (1970:324, 343). (Granet also

derived the 'Lawrence' structure $\bar{X}_8$ (1939:209 fig. E).)

$\bar{X}_9$, the four-class, four-generation structure with direct exchange, and $\bar{X}_{10}$, the 'reversed' version of Lévi-Strauss's hypothetical four-class 'Murngin' model, have previously only been described as theoretical structures ($\bar{X}_9$: Lorrain 1975:177, Tjon Sie Fat 1975:24-25, De Meur and Jorion 1981:16; $\bar{X}_9$ and $\bar{X}_{10}$: Kemeny et al. 1966 [1956]:432). There are no clear descriptions of these two structures as models of actual kinship systems.

As formalized above, the class of double descent models developed in the 1930s as a comprehensive scheme for the comparison of Indonesian societies raises a number of important issues. First, the crucial notion of diverse possibilities 'implicit' or 'latent' in a kinship model (Locher 1968:ix) is made more precise by the concept of a quotient structure. The mathematical procedure is exhaustive: all possible 'latent' or reduced structures of $M_4 \times P_4$ are catalogued, with the more arbitrary selection previously derived under the traditional approach included as a subset. The partial realisation of these possible structures in data from real societies is a question for empirical research.

Second, under the traditional approach, hypothetical development sequences have been proposed, leading from, say, a moiety system to an eight-class system, via a double moiety, four-class structure (cf. Granet 1939, among many others). The lattice diagram of figure 1.6 summarizes the covering relationships between the quotient structures of $M_4 \times P_4$. If one so wishes, the linking kinship morphisms may be interpreted as simple paths for systems development or collapse. There are indeed many possible trajectories linking dual structures to kinship structures with eight classes. Any realistic theory of structural change must set out criteria for choosing a specific sequence from the set of possible alternatives. Research along these lines has been initiated by Boyd (1969) and De Meur and Jorion (1981).

Finally, the mathematical formalization provides the natural framework for analysing the *coding problem* or *extension problem* in relation to kinship structures. A well-known example of coding in anthropology is the componential representation of American kin terms described by Wallace and Atkins (1960). Here a set of lexemes $T = \{\text{father, mother, ..., cousin}\}$ (i.e., the American-English consanguineal kin terms) is mapped onto the Cartesian product $A \times B \times C$ of three sets of distinctive features, where A (sex) = {MALE, FEMALE}, B (generation) = $\{G^{+2}, G^{+1}, G^0, G^{-1}, G^{-2}\}$, and C (lineality) = {LINEAL, COLINEAL, ABLINEAL}. The problem is to distinguish the kin terms by representing their 'meanings' as combinations of values on the underlying 'semantic dimensions' A , B , and C . There are, unfortunately, a number of competing componential analyses for the American-English terminology, based on different dimensions and different sets of distinctive features.

An analogous coding problem occurs in kinship theory. Thus, writers on Australian kinship have labelled the basic classes in diagrams of the same kinship system in various ways. The 'Lawrence' eight-class structure isomorphic to $\bar{X}_8$ (fig. 1.6) is defined as $L = (S, h, m, f)$ with $S = \{1, 2, \dots, 8\}$, $h = (1, 6, 5, 2)(3, 8, 7, 4)$, $m = (1, 3, 5, 7)(2, 4, 6, 8)$, and $f = (1, 8)(2, 3)(4, 5)(6, 7)$. This is the scheme put forward by Barnes (1967:20). There are variants. Lévi-Strauss introduces two additional notational schemes (1970:175, 187-189). A fourth convention is employed by J.P.B. de Josselin de Jong (1952:39); he also summarizes the last three systems of notation in his Chart A (1952:61).

These four renderings of the two-matriline, four-patriline 'Lawrence' eight-class system are set out below in table 1.2 (with the asymmetric marriage connubia and the matrilineal and patrilineal descent cycles defined by the permutations h , m , and f).

Barnes	L_0 :	1	2	3	4	5	6	7	8
Lévi-Strauss I	L_1 :	A1	B2	C2	D1	A2	B1	C1	D2
Lévi-Strauss II	L_2 :	Px	Sy	Qx	Py	Rx	Qy	Sx	Ry
De Josselin de Jong	L_3 :	1A	4B	4A	3B	3A	2B	2A	1B

Table 1.2. Alternative codings for the 'Lawrence' system.

The choice of symbols in these isomorphic representations of the 'Lawrence' structure is not arbitrary: the codings reflect different theoretical points of view.

Boyd (1969, 1971, 1972) has drawn clear parallels between the componential approach in anthropology and the more comprehensive fields of mathematical coding theory and the theory of group extensions. Roughly, the extension problem (as it applies to group theory) focusses on the question of how to describe an algebraic group as the composition of smaller structures. A key issue is the requirement that the binary operation on the group to be 'extended' be coded in terms of operations defined on the smaller structures. A more technical characterization is given in the following theorem:

Theorem 4 (adapted from Boyd 1969:158-159): Let G be a group with normal subgroup H and let K be a group isomorphic to the quotient group G/H . Let X be a left transversal of H in G , i.e., a set of elements of G containing one and only one element $\bar{x}$ from each left coset xH of H in G , with e in X . Then $\bar{x}$ is the representative of the coset and each element g of G can be uniquely described as $g = \bar{x}h$ for some $\bar{x}$ in X and some h in H . Then there is a one-to-one mapping θ from the set of elements of G onto the Cartesian product of K and H defined by $(g)\theta = (\bar{x}h)\theta = (xH, h)$, and the product of any two elements g and f of G is equivalent to $gf = (\bar{x}h)(\bar{y}k) = \bar{x}\bar{y}m = \bar{x}\bar{y}(a_{x,y}\bar{y}^{-1}h\bar{y}k)$, with h, k and m in H , and $a_{x,y}$ some element of H uniquely determined by x and y . G is said to be an extension of H by K and the complete set of elements $a_{x,y}$ is the factor

set of the group extension. (For a sketch of the proof see the appendix.)

Two important special cases to be considered are:

(1) The left transversal X (i.e., the system of representatives for G/H) forms a *subgroup* of G . Hence $\bar{x}\bar{y} = \overline{xy}$, thus $a_{x,y} = e$ and $gf = (\bar{x}h)(\bar{y}k) = \overline{xy}(\bar{y}^{-1}h\bar{y}k)$. G is called the *semi-direct product* of the subgroup H by X .

(2) The left transversal X forms a *normal subgroup* of G . Hence $\bar{x}\bar{y} = \overline{xy}$, $a_{x,y} = e$, and since $\bar{x}h = h\bar{x}$ for all $\bar{x}$ in X and h in H , $gf = (\bar{x}h)(\bar{y}k) = (\bar{x}\bar{y})(hk)$. G is then the *direct product* of its normal subgroups H and X .

Boyd (1969:156-166) has applied these findings to the groups associated with a number of the classic kinship structures (the Karia, the Karadjeri, the Ambrym, and the Aranda), deriving the group extensions and interpreting these as codings or componential definitions. His general framework can be easily reformulated and applied to the more specific Courrège-type kinship models introduced in this chapter. For reasons of space I only examine selected cases related to the double descent structure $M_4 \times P_4$. In particular, it is important to establish if a kinship structure $X = (S, h, m, f)$ with group G is isomorphic to the direct product of its quotient structures $\bar{X}_i$ and $\bar{X}_j$, and to determine whether or not such a representation is indeed unique.

Referring now to the alternative codings given for the eight classes of the 'Lawrence' structure in table 1.2, let P_2 , G_2 , M_2 , P_4 , A_4 , and K_4 be, respectively, the kinship structures isomorphic to the quotient structures $\bar{X}_2$, $\bar{X}_1$, $\bar{X}_3$, $\bar{X}_4$, $\bar{X}_6$, and $\bar{X}_5$ of figure 1.6. These are the non-trivial quotient structures of the 'Lawrence' structure L (itself isomorphic to $\bar{X}_8$). Disregarding the order of multiplication, there are exactly *four* non-trivial, isomorphic representations or codings of L in terms of its quotient structures. These are:

(1) $L_3 = P_4 \times M_2 \cong \bar{X}_4 \times \bar{X}_3$; i.e., as the product of a

four-patriline structure with generalized exchange and a matrimoiety structure. This is the double-descent coding chosen by J.P.B. de Josselin de Jong (1952:39).

(2) $L_2 = A_4 \times M_2 \cong \bar{X}_6 \times \bar{X}_3$; i.e., as the product of a four-class structure with generalized exchange and an exotic descent rule (a man's children belong to the class of their mother's brother's wife) and a matrimoiety structure. This is the second solution advocated by Lévi-Strauss (1970:187-189). The coding is not in terms of double (unilineal) descent.

(3) $L_4 = P_4 \times G_2 \cong \bar{X}_4 \times \bar{X}_1$; i.e., as the product of a four-patriline structure with generalized exchange and an alternating-generation structure.

(4) $L_5 = A_4 \times G_2 \cong \bar{X}_6 \times \bar{X}_1$; i.e., as the product of the non-unilineal structure described under (2) and an alternating-generation structure. These last two codings for the 'Lawrence' structure have not previously been presented in the literature on Australian kinship.

What, then, of L_1 , Lévi-Strauss's first solution? Let K_4 be the kinship structure on four classes A, B, C, D which is isomorphic to $\bar{X}_5$. Then $h = (A, B)(C, D)$, $m = (A, C)(B, D)$, and $f = (A, D)(B, C)$ and K_4 represents the classic 'Kariera' four-section model. The 'Kariera' structure is indeed a quotient structure of L (obtained by setting m^2 equal to e). This is apparently expressed in Lévi-Strauss's coding L_1 , and it can be retrieved by suppressing the values of his second index. In other words, the 'Kariera' partitioning of L is compatible with the original kinship permutations. This is obviously not the case with the partitioning (or dual division) induced by the values 1 and 2 of his second index. In fact, the 'Lawrence' structure cannot be coded as the direct product of the 'Kariera' structure with one of the three regular structures M_2, P_2 and G_2 . Lévi-Strauss's two codings of the 'Lawrence' structure appear to be related to the two possible evolutionary sequences leading to the 'Murngin' type of kinship system (1970:175-176, 186

-196, 216 fig. 44). However, his discussion is abstruse and his argumentation less than clear (cf. Barnes 1967: 18-21).

The 'Kariera' four-class structure K_4 is another example of a kinship model that has occasioned much contentious debate. On purely formal grounds the structure may be rendered as the direct product of its quotient structures in three distinct (though isomorphic) configurations: $K_4 \cong M_2 \times P_2 \cong G_2 \times M_2 \cong G_2 \times P_2$ (with the order of multiplication disregarded), i.e., as the product of matri- and patri-moieties, or as the product of an alternating-generation structure with either a matri-moiety or a patri-moiety structure. All three codings have had their proponents since Fison and Howitt first published their *Kamilaroi and Kurnai* in 1880. (For a recent discussion see Scheffler 1978:432-480.)

Finally, in passing from the 'Lawrence' structure to the 'Kariera' model we have arrived where we started: with kinship chart 10 (figure 1.1) and the early Leiden paradigm. The final diagram in this fundamental scheme is of course the structure $M_4 \times P_4$, the 'circulating system (patrilineal and matrilineal)' with exclusive matrilineal cross-cousin marriage. This model is obviously coded as a double-descent structure, a formulation in accordance with the then prevailing Leiden view. With hindsight, and with the results obtained by applying the formal axiomatization introduced in this chapter, the uniqueness of this coding may now be questioned.

Let P_4 , A_4 , G_4 , K_4 , B_4 , RA_4 and M_4 denote, respectively, the four element quotient structures $\bar{X}_4$, $\bar{X}_6$, $\bar{X}_7$, $\bar{X}_5$, $\bar{X}_9$, $\bar{X}_{10}$ and $\bar{X}_{11}$ of $M_4 \times P_4$. (These are in fact the only possible four element regular kinship structures.)⁴⁶ Then one can easily prove the existence of exactly eleven distinct direct product representations in addition to (1) $M_4 \times P_4$. These are: (2) $G_4 \times M_4$, (3) $G_4 \times P_4$, (4) $G_4 \times A_4$, (5) $G_4 \times RA_4$; i.e., the product of a four-generation structure with either a four-matriline or a four-patriline structure,

with Lévi-Strauss's exotic 'original Murngin structure', or with its equally exotic 'reversed' variant (a structure with generalized exchange and a non-linear descent rule allocating a woman's children to the class of their father's sister's husband); (6) $M_4 \times A_4$, (7) $P_4 \times RA_4$; i.e. the product of a four-matriline structure or a four-patriline structure with the hypothetical 'Murngin' structure or its reverse; (8) $M_4 \times B_4$, (9) $P_4 \times B_4$; i.e., the product of four-matriline or four-patriline structures with a four-generation structure with direct exchange; and the three exotic products (10) $A_4 \times B_4$, (11) $RA_4 \times B_4$, and (12) $A_4 \times RA_4$. (There are no direct products of order two with order eight structures isomorphic to $M_4 \times P_4$.)

The eleven additional representations listed above are formally possible models. However, under the standard view on kinship, not all such codings are plausible or interesting alternatives to $M_4 \times P_4$. Nevertheless, in the light of recent analyses of North Moluccan societies by a younger generation of Leiden scholars (Visser 1984, Platenkamp 1988), the possibility of alternative decompositions of the classic double-descent model represented by $M_4 \times P_4$ as the product of a four-generation structure with, say, a matri- or patri-descent structure has suddenly become interesting. Thus, Platenkamp, discussing the Tobelo and Galela conceptions of incest, exogamy and 'the severance of the origin' identifies the idea of a 'four-generation rule' as a central structural principle (1988:225-235). He has this to say on the methodological problem of extending the classic kinship models to northern Halmaheran societies (1988:257):

This fourfold division [of Tobelo society] is not generated by a system of asymmetric connubial relations between ideologically four groups, as it has been analysed in other parts of Indonesia ... It results from the limitation of the relation to the 'origin' of 'life' to four generations; and from the idea that only in a fifth descendant generation 'life' can be alienated from its 'owner'. It should be emphasized that the principle of 'periodical extinction of

exogamy' ... after the fourth descendant generation, which in western Indonesia can be integrated with matrilineal cross-cousin marriage and double-unilineal descent into one structural model ..., in northern Halmahera is a function of this rule of alienation, and not of MBD-marriage.

The traditional models of Indonesian social structure (including their formal analogues presented here) are deficient and in many respects highly unrealistic. Possible extensions and modifications can only be formulated and tested against the background of further detailed ethnographical material. As I see it, formal axiomatizations will continue to play a key role in highlighting the structural complexes and relations (or inconsistencies) all too easily neglected by less rigorous approaches.

In the following chapters a related series of more specific extensions to the conventional anthropological kinship structures is developed. In each case an attempt is made to provide an exhaustive specification of the associated class of proper kinship models.

APPENDIX

Kinship notation. Kintypes and genealogical relationships are coded by means of the following set of upper case symbols:

M = mother	D = daughter	Z = sister	W = wife
F = father	S = son	B = brother	H = husband
P = parent	C = child	Sb = sibling	Sp = spouse.

Congruences. Two integers a and b are called *congruent (modulo m)* with respect to some integer m if $(a - b)$ is an integral multiple of m . Notation: $a \equiv b \pmod{m}$. An integer a is said to be *reduced modulo m* to b if $a \equiv b \pmod{m}$ and $0 \leq b < |m|$. A positive integer p is a *prime*

if p is divisible only by 1 and p . Any two integers a and b are *coprime* if their greatest common divisor is unity.

Sets and relations. The *Cartesian product* $S \times T$ of any two sets S and T consists of all ordered pairs of elements (s, t) with s in S and t in T . A *binary relation* R is a subset of $S \times T$. If R is a binary relation then the *inverse relation* $R^{-1} = \{(b, a) | (a, b) \text{ in } R\}$. An *equivalence relation* R on S is a binary relation on S (i.e., a subset of $S \times S$) which is reflexive, symmetric, and transitive: (1) (a, a) is in R for all a in S ; (2) if (a, b) is in R , then (b, a) is also in R ; (3) if (a, b) and (b, c) are in R , then (a, c) is in R . The congruence relation $\equiv$ on the set of integers is an equivalence relation. A binary relation R is antisymmetric if (a, b) in R and (b, a) in R imply $a = b$. A *partition* P on the set S is a decomposition of S into nonempty, disjoint subsets such that every element of S belongs to exactly one subset, and S may be expressed as the union of subsets. If R is an equivalence relation on S , define the *equivalence class* of an element s of S with respect to R as $sR = \{y | (s, y) \text{ in } R\}$. Then $P = \{xR | x \text{ in } S\}$ is a partition on S called the *quotient set* or *factor set* of S with respect to R . Notation: $P = S/R$. Each equivalent class xR is also called a *block* of the partition P . Conversely, if P is a partition on the set S , then $R = \{(x, y) | x \text{ and } y \text{ in block } B \text{ of } P\}$ is the equivalence relation on S generated by P . R is called the *canonical* equivalence relation associated with P .

Mappings. A *mapping* f from a set S into a set T (notation: $f: S \rightarrow T$) is a subset of $S \times T$, i.e. a relation such that (s, t_1) and (s, t_2) are in f if and only if $t_1 = t_2$, and for each s in S there exists an element (s, t) in f . In general, $(s)f = t$ denotes the *image* of s under f , i.e. the element t in T to which the element s in S is mapped by f . The mapping f is *onto* if every t in T is an $(s)f$ for at

least one s in S . The mapping f is *one-to-one* if each element of S is mapped onto one and only one element of T , i.e., if the inverse relation f^{-1} is also a mapping. If $f: S \rightarrow T$ is onto and $g: T \rightarrow U$, then the *composition* of f with g (taken in this order) is a mapping $h: S \rightarrow U$ defined by $(s)h = (s)(fg) = ((s)f)g$ for all s in S . The term *function* will sometimes be used as a synonym for mapping. Any mapping f of $S \times S$ into S is called a *binary operation* in S . Instead of $((s_1, s_2))f = s_3$ the multiplicative notation $s_1 s_2 = s_3$ will be used for binary operations. A *permutation* p of a set S is a one-to-one mapping (or function) from S onto itself. If T is a subset of S , then $[T]p = \{(x)p \mid x \text{ in } T\}$ is the image of T under p . Let R be an equivalence relation on S defining the partition $P = S/R$ on S . a permutation p of S is *compatible* with the partition P if $[sR]p$ is a block of the partition P for all s in S ; i.e., if $[sR]p = (\{y \mid (s, y) \text{ in } R\})p = tR$ for some t in S .

Groups. A *group* G is a set provided with a binary operation which has the following properties: (1) The set is closed under the binary operation. Hence xy is in G for all x and y in G . (2) The binary operation is associative. Hence $(xy)z = x(yz)$ for all x, y , and z in G . (3) There is an identity element e such that $ex = xe = x$ for all x in G . (4) Each element x of G has an inverse. Hence $xx^{-1} = x^{-1}x = e$ for each x and some x^{-1} in G . (See also the equivalent definition given in the previous chapter.) The *order* of an element x of G is the smallest positive integer m such that $x^m = e$ (the identity element). The order of a group (notation: $|G|$) is the number of distinct elements of G . A group is *commutative* or *Abelian* if $xy = yx$ for all x and y in G . A *homomorphism* of a group G into a group H is a mapping ψ of G into H such that $(xy)\psi = (x)\psi(y)\psi$ for all x and y in G . An *endomorphism* of G is a homomorphism of G into G . G is *isomorphic* to H (notation: $G \cong H$) if and only if there is a one-to-one homomorphism of G onto H . An

automorphism of G is an isomorphism of G onto G . Under the usual composition of mappings the set $\text{Aut}(G)$ of all automorphisms of G forms a group called the *automorphism group* of G . If $\{p, q, \dots, r\} = T$ is a set of permutations on a set S , then under the normal composition of permutations (i.e., a binary operation on S) the elements of T generate the *permutation group* (S, G) . If $\{p, q, \dots, r\}$ is the set of generators, the notation $G(p, q, \dots, r)$ will also be used. The group generated by a single permutation p of order m is the *cyclic group* $C_m = G(p, p^m = e)$.

Subgroups and quotient groups. A subgroup H of a group G is a subset of G which is itself a group under the binary operation defining G . A *right coset* of H in G is a subset of the form $Hg = \{x | x = hg \text{ and } h \text{ in } H\}$ for some g in G . A *left coset* of H in G is defined as $gH = \{x | x = gh \text{ and } h \text{ in } H\}$ for some g in G . The set of right (or left) cosets of H in G constitute a partition of H in G (see Baumslag and Chandler 1968:109 for a proof). A *right transversal* of H in G is a *complete system of representatives* of the coset partition. I.e., select from each right coset Hg one element $\bar{g}$ (the representative of Hg), with $\bar{e} = e$ the representative of $He = H$. The definition of a *left transversal* follows by analogy. A subgroup H of G is *normal* or *invariant* in G (notation: $H \triangleleft G$) if and only if $Hg = gH$ for all g in G . A necessary and sufficient condition for the left and right cosets of a subgroup H of G to provide the *same* partition is that H is normal in G (see Baumslag and Chandler 1968:111). If H is normal in G , then the partition constituted by the cosets of H in G forms a group under the binary operation defined as $(Hh)(Hg) = H(hg)$ for all h and g in G . This group is called the *quotient group* or *factor group* G/H . The mapping $\psi: G \rightarrow G/H$ defined as $(g)\psi = Hg$ for all g in G is a homomorphism of G onto G/H with $(e)\psi = H$, the identity of G/H .

Sketch of the proof of theorem 4: Let $H \triangleleft G$, $G/H \cong K$, and X is a left transversal of H in G as defined above. For any g in G , there is a unique expression of the form $g = \bar{x}h$ with $\bar{x}$ in X and h in H . Then, for any two elements $g = \bar{x}h$ and $f = \bar{y}k$ of G it must be shown that there is a unique expression of the form $gf = \bar{z}m$ with $\bar{z}$ in X and m in H .

First, for any g in G and $\bar{x}$ in X , $g\bar{x}$ belongs to some coset yH of H in G with representative $\bar{y}$. Therefore $g\bar{x} = \bar{y}h$ for some h in H uniquely determined by g and $\bar{x}$. Denote h by $a_{g,x}$, i.e., $g\bar{x} = \bar{y}a_{g,x}$ (1).

Then, since the isomorphism ψ of G/H onto K is a one-to-one mapping of the set of cosets of H onto K , the representative $\bar{g}$ of the coset gH can be unambiguously defined as x_i if $(gH)\psi = i$ with i in K . Note that $x_e = e$ since $(eH)\psi = e$. Under this notation, $X = \{x_i | i \text{ in } K\}$. Also, for x_i and x_j in X , $(x_i x_j H)\psi = (x_i H)\psi (x_j H)\psi = ij$, with i and j in K . Hence the representative of the coset $x_i x_j H$ is x_{ij} . Then from (1) it follows that $x_i x_j = x_{ij} a_{x_i, x_j}$ with a_{x_i, x_j} in H . Without loss of information, write $a_{i,j}$ for a_{x_i, x_j} (2).

Every element g in G can be written uniquely as $g = x_i h$ where x_i is in X and h is in H . Then the product gf of any two elements of G can be written as $gf = (x_i h)(x_j k)$ with x_i and x_j in X and h and k in H . Hence $gf = x_i x_j x_j^{-1} h x_j k = x_i x_j (x_j^{-1} h x_j k)$ from (2). And since H is normal in G , $x_j^{-1} h x_j$ is in H , and thus $a_{i,j} (x_j^{-1} h x_j) k$ is in H .

Thus $gf = x_{ij} m$, or in an equivalent notation, $gf = (\bar{x}h)(\bar{y}k) = \bar{xy}m = \bar{xy}(a_{x,y} \bar{y}^{-1} h \bar{y} k)$, with h , k and m in H , and $a_{x,y}$ some element of H determined by x and y . (See also Baumslag and Chandler 1968:232-233.)

NOTES

- 1 First published in 1962. My references are to the second, expanded version of 1970.
- 2 For a critical discussion of theoretical reconstruction in anthropology see Barrett (1984).
- 3 The identification numbers are arbitrary codings applied recently. I am indebted to Ms. Marga van de Munt for bringing these charts to my notice.
- 4 Personal communication.
- 5 Chart number 1 may even have been the original drawing photographically reduced in Van Wouden's thesis (1935:97) and in the English translation (1968:93).
- 6 Although there are many exceptions to the rule, the 'standard' Leiden notational system employed numbers for patriline and capital letters for matriline.
- 7 J.P.B. de Josselin de Jong was appointed as part-time professor of General Anthropology at Leiden University in 1922. His first inaugural address, *Cultuurtypen en Cultuurphases* ('culture types and culture phases') was also delivered in 1922.
- 8 A 'field of ethnological study' was defined in 1935 as a certain area of the earth's surface 'with a population whose culture appears to be sufficiently homogeneous and unique to form a separate object of ethnological study, and which at the same time apparently reveals sufficient local shades of differences to make internal comparative research worthwhile' (1977:167-168). Single cultures are to be considered as variations on a common theme. For the Indonesian 'field of study', a system made up of four features (asymmetric connubium and double descent, socio-cosmic dualism, and the specific manner in which Indonesian indigenous cultures reacted to foreign influences) constituted the 'structural core' (1977:168-175).
- 9 Many of the key texts have appeared in the *Translation Series* of the *Koninklijk Instituut voor Taal-, Land- en Volkenkunde*. See in particular the reader edited by P.E. de Josselin de Jong (first published in 1977).
- 10 Cf. Barnes 1985, P.E. de Josselin de Jong 1985.
- 11 The term 'model' is employed by Durkheim and Mauss in a number of places: 'Toute classification implique un ordre hiérarchique dont ni le monde sensible ni notre conscience ne nous offrent le modèle' (1903:6). See also the use of the term on page 25, 39, 55, 67 of the 1903 article. This observation corrects a statement made previously (Tjon Sie Fat 1988:239, note 4). It remains interesting to note (Chao 1962:559) that the earliest use of the term 'model' in linguistics only dates to the 1940s.

- 12 Friedericy's thesis was also published in the *Bijdragen* (1933, volume 90:447-602).
- 13 The first French edition was published in 1949. Unless otherwise stated, I shall cite the 1970 English edition.
- 14 See also the discussion by P.E. de Josselin de Jong (1980:222-227) in the supplementary chapter to his thesis of 1951.
- 15 In their recent (1989) discussion of the development of cultural anthropology at Leiden University, P.E. de Josselin de Jong and H.F. Vermeulen apply the term 'possibilistic' to the class of models developed by the early Leiden anthropologists.
- 16 Not all of the theses prepared in the 1930s were of course devoted to the study of kinship. See the recent historical overview by Locher (1988).
- 17 I am indebted to Professor Locher for providing me with a copy of the letter written by J.P.B. de Josselin de Jong (30 August 1930) with suggestions on his (Locher's) dissertation research on Australian kinship.
- 18 Unless otherwise stated I shall refer to the third (1980) edition of his thesis.
- 19 Nicolas Bourbaki, formerly of the Royal Poldavian Academy and currently on the faculty of the University of Nancago. The collective pseudonym for a group of (largely) French mathematicians (André Weil, Henri Cartan, Jean Dieudonné, Jean Delsarte and others) who in 1934 set out to axiomatize mathematics. The name is taken from an obscure French general, Charles Denis Sauter Bourbaki (1816-1897) who, after an embarrassing retreat, tried to shoot himself in the head, but missed. He had previously been offered the Greek throne. For further details see Regis (1987:76-79), Dieudonné (1970), and Kramer (1982:700-702). For remarks on the use of the concept of 'structure' in mathematics, see the discussion in Bastide (1962:139-141). Piaget (1968) is obviously influenced by the Bourbaki. For a critique of his use of mathematical concepts, see Seltman and Seltman (1985).
- 20 Granger mentions the work of Martial Guérault, Victor Goldschmidt, Ginette Dreyfus and Jules Vuillemin.
- 21 I.e., the elements of any such system, at the level of propositions or concepts, are themselves always 'open' and only partially determined by their reciprocal relations.
- 22 *Received View*: the term introduced by Hilary Putnam (1962).
- 23 Arguably the best discussion is provided in Suppe (1977; the second edition with an extended Afterword).
- 24 See Suppe (1977), Chapter 5 and the Afterword.
- 25 See Leplin (1984). Sneed (1983) discusses the relations between structuralism and scientific realism.
- 26 Key texts are Sneed (1979; first edition, 1971), Stegmüller (1976, 1979), Feyerabend (1977), Kuhn (1976), Balzer and Sneed (1977, 1978), Niiniluoto (1984), and

- Balzer, Moulines, and Sneed (1987).
- 27 For a summary of the basic mathematical concepts, see the appendix to this chapter.
 - 28 As Sneed remarks, particular groups may have a set G composed of physical objects like bits of clay, or of non-physical objects like rotations (1979:10).
 - 29 See Balzer et al. (1987:1-35) for a technical discussion of the distinction between M and M^* . Roughly, in developed theories the fundamental laws express non-trivial connections between the non-base terms. Laws are formulas which are neither typifications nor characterizations (1987:14).
 - 30 This is the position set out by Nutini in his critique of anthropological structuralism. See Nutini (1968, 1970).
 - 31 For a technical discussion, see Balzer (1983). Some of Balzer's formal definitions are reproduced in Tjón Sie Fat (1988).
 - 32 'Ramsey sentences' (see Stegmüller (1976) and Simon and Groen (1977)) are special cases of the empirical claims introduced by Balzer.
 - 33 At the level of the formal model, 'marriage classes' are codified in terms of a double index which usually denotes the intersection of a system of matriline with a system of patriline. See figure 1.1. However, a double index coding may also indicate the intersection of other, non-unilineal types of kinship structure. Examples are discussed below.
 - 34 Lévi-Strauss (1970:496), in the final paragraphs of his book, does indeed mention the paradox of women 'at once a sign and a value' in communication. However, his focus throughout is clearly on 'a universal fact, that the relationship of reciprocity which is the basis of marriage is not established between men and women, but between men by means of women, who are merely the occasion of this relationship' (1970:116).
 - 35 For a classic description of the exchange value of specific types of gifts in Indonesia see Onvlee (1949). The vital importance of recognizing specific 'images' of women in marriage transactions has, since 1952, been stressed by many other researchers. See Kolenda (1984) and Postel-Coster (1985). Curiously, Postel-Coster does not refer to J.P.B. de Josselin de Jong's early critique of Lévi-Strauss.
 - 36 Lévi-Strauss (personal communication, April 18, 1989) has kindly provided the following comments on an early draft of this chapter: 'I agree that my basic point of disagreement with the Leiden school (to which I owe so much) is a reluctance to admit double descent as an explanatory model when and where it cannot be shown to exist. On the other hand it is not quite correct to write as you do ... that according to me "asymmetric marriage always presupposes a unilineal mode of descent". On the contrary I have emphasized in *Les Structures*

- that "le mariage des cousins croisés ne requiert pour son existence aucune théorie unilinéaire de la filiation" (p. 506 of the 1967 edition. See also p. 139, 154; *Anthropologie structurale*, p. 50; *Anthropologie structurale deux*, p. 126; *Le Regard éloigné*, p. 90, 133-136). It may sometimes be helpful to employ unilineal filiation as a graphic convention which simplifies the diagrams. However I always tried to think in terms of relations, not of classes either unilineally or bilineally determined.' The question of double-unilineal or non-unilineal codings of exchange units or 'classes' is discussed briefly in the final section of this chapter.
- 37 For important developments of the classic Leiden view after 1952, see Van Wouden (1977) and P.E. de Josselin de Jong (1977), first published in the special issue of the *Bijdragen* serving as a *Festschrift* for J.P.B. de Josselin de Jong on his retirement in 1956. See also P.E. de Josselin de Jong (1985).
- 38 See Lévi-Strauss (1970:464-465) and J.P.B. de Josselin de Jong (1952:29).
- 39 As realized in actual kinship systems, a rule of exclusive patrilateral cross-cousin marriage may indeed exhibit certain disadvantages in functioning as an all-embracing, 'total' exchange system for the ordering of affinal relations between social groups or categories, a point recognized long ago by Van Wouden in his 1935 dissertation (1968:90) and acknowledged by J.P.B. de Josselin de Jong (1952:49). However, at the level of the structural model 'long cycles' are essential characteristics of the structure of relations. Lévi-Strauss's reference to 'optical illusions' or 'delusions' in such kinship diagrams is therefore not correct (1970:xxxiv): if a failure to close even the shortest cycle is seen as the mark of a patrilateral system (1970:xxxiv-xxxv), this feature can only refer to the partial realization of certain aspects of the full model in specific empirical systems of exchange. The diagram is part of the model; both are distinct from empirical reality. Here again Lévi-Strauss's perspective on patrilateral models appears to be less structural than that of his Leiden predecessors.
- 40 See P.E. de Josselin de Jong (1977). Apart from the work of Durkheim and Mauss, the influence of Radcliffe-Brown is important. See also P.E. de Josselin de Jong (1984), Locher (1988), and P.E. de Josselin de Jong and Vermeulen (1989). Arguably the most fundamental difference between the Leiden and Lévi-Straussian approaches is not examined in the 1952 essay: the focus on regional comparison and the systematic analysis of local variations (versus the emphasis on universal generative principles) as a means of articulating theory and empirical research.

- 41 The literature on algebraic kinship models alone is fairly extensive. See Courrège (1965, 1974), White (1963), Lorrain (1975), Boyd (1969, 1971, 1972), Jorion and De Meur (1980), De Meur (1986), Read (1984), Ballonoff (1974a), Tjon Sie Fat (1981, 1983a), Liu (1986) and Lucich (1987) for a relevant cross-section.
- 42 Boyd's work, linking the classic permutation models with coding theory, componential analysis and the mathematics of inverse semigroups, has unfortunately been neglected by anthropologists.
- 43 Duyvendak (1926:127): P_4 ; Friedericy (1933:141-143): M_2, P_2, M_3, P_3 (see also fig. 1.2, top); Held (1935: 54-59, 63, 95): P_2, P_3, P_4 (see also fig. 1.2, middle diagram); Van Wouden (1968:91), and P.E. de Josselin de Jong's 1951 dissertation (1980:37-39, 185-186).
- 44 See Lawrence (1937).
- 45 For another clear example, see Chart A in J.P.B. de Josselin de Jong's 1952 essay. Here codings of many distinct partitions are superimposed on a genealogical grid representing exclusive MBD-marriage and the allocation of kin terms.
- 46 There are exactly seven such four-element structures, not six (as claimed by Kemeny et al. (1966:432) and White (1963:82)). All seven are associated with a commutative group and hence are compatible with MBD-marriage.

2. RECURSIVE DEFINITIONS: MORE COMPLEX FORMULAE OF GENERALIZED EXCHANGE¹

The fundamental proposition developed by Lévi-Strauss in his first major theoretical work, *Les Structures élémentaires de la parenté* (1970)², is that of the limited range of possible social structures (p. 493):

We have thus established that superficially complicated and arbitrary rules may be reduced to a small number. There are only three possible elementary kinship structures; these three structures are constructed by means of two forms of exchange; and these two forms of exchange themselves depend upon a single differential characteristic, namely the harmonic or disharmonic character of the regime considered.

The dominant theme of the book is exchange, restricted and generalized, and the rules of preferential marriage with a certain type of relative which define elementary kinship structures are interpreted as a function of these two simple exchange formulae.³

Lévi-Strauss's aim of a general theory of kinship and marriage (to which *Les Structures* was to have provided only the introduction) has yet to be realized. A companion volume on complex structures (i.e., systems which, as far as marriage is concerned, are based on rules or preferences not directly expressed in terms of kinship) was never written. In Lévi-Strauss's opinion, any solution to the problems that arise in attempting to extend his theory of marriage as exchange to complex structures or even to the so-called Crow-Omaha systems must await the development of suitable mathematical techniques (1966:18-21; 1970:xxxvi-xlii). Hence the only viable option currently available is the use of computer methods and statistical analyses to tease out any

regularities of the essentially probabilistic structure of exchange events (1966:16, 20-21; 1970:x1-xlii).

In my opinion, Lévi-Strauss is being unduly pessimistic. Non-statistical models for extending the analysis of elementary structures outwards to (and possibly to include) the Crow-Omaha systems are already available. This is an issue separate from the anthropologically important question of whether or not any general theory of systems of kinship and marriage should indeed be based on exchange (in all its forms).

This chapter is concerned with the formal derivation of a family of such models that include as special cases the elementary structures with unilateral cross-cousin marriage. After making a few preliminary observations on the problems presented by hybrid structures, I shall formulate an extended model of generalized exchange structures. Then I shall combine the model with an existing mathematical model of elementary kinship structures to derive a further series of kinship models.

HYBRID STRUCTURES AND ALTERNATIVE MARRIAGES

The elementary/complex distinction is a heuristic device and cannot be the sole criterion for defining a kinship system. All systems contain an 'elementary' core derived from the incest prohibitions, while even in the strictest elementary structure the positive marriage rules allow some freedom of choice (Lévi-Strauss 1966:18; 1970:xxiii-xxiv). Lévi-Strauss also acknowledges (1970:xxiv) the existence of 'certain hybrid forms, where economic privileges allow a secondary choice within a prescribed category (marriage by purchase combined with marriage by exchange), or where there are several preferential solutions.'

The Gilyak system is one such hybrid kinship structure. The Gilyak are divided into patrilineal exogamous lineages or clans. In Lévi-Strauss's analysis the

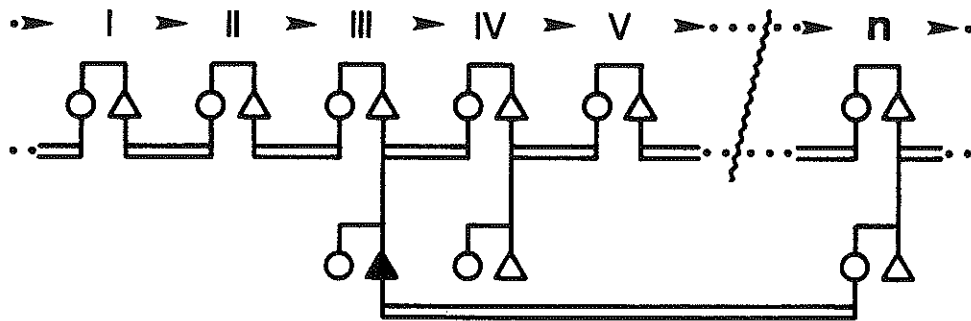


Fig. 2.1. Alternative marriage within a system of generalized exchange. Ego is represented by the dark triangle. Lines IV and II are his 'close fathers-in-law' and 'close sons-in-law'; lines V and I are his 'distant fathers-in-law' and 'distant sons-in-law'.

preferred marriage is with male ego's matrilineal cross-cousin. Members of male ego's family obtain women from their *axmalk* ('fathers-in-law') and give women to their *imgi* ('sons-in-law'). Each wife-giver is in a subordinate position relative to his wife-taker. The *axmalk/imgi* relation is extended to the wife-givers of the wife-givers and the wife-takers of the wife-takers, so that each family or clan is linked to at least four others in an asymmetric system of exchange. There is also a convention which places a two-generation limit on the asymmetrical rule of exogamy: after two generations the exchange cycle may be reversed or discontinued (Lévi-Strauss 1970:292-96, 303).

The Gilyak system also allows an alternative form, 'marriage by purchase', based on a complex system of prestations and counter-prestations. Bride-price is extremely high, so that the poor are obliged to follow the rule of cross-cousin marriage. However, 'marriage by purchase' is always a function of the kin-type marriage rule. It involves the renunciation of an old alliance partnership, creating at the same time a new cycle of families linked by the extended *axmalk/imgi* relations

(fig. 2.1). Moreover, part of the bride-price owed to the new 'distant fathers-in-law' (mother's brothers of the bride) may be paid in women, creating yet another linked cycle of asymmetric exchange (Lévi-Strauss 1970: 303-6). Hence, in contrast with the basic model of matrilineal cross-cousin marriage, the Gilyak exchange cycles do not constitute an invariant series of alliances between descent groups. What marriages are possible or prohibited is determined at any point by the marriages that have occurred in preceding generations. Given a 'rule of exogamy limited to two generations' (Lévi-Strauss 1970:296) and a finite number of groups (families, lineages, clans), the emergence of a periodic structure of a certain type after a few generations might be expected.

Another hybrid structure is the Iatmul system. In an attack on the consistency and applicability of the concepts developed in *Les Structures*, Korn (1971) describes the Iatmul system as a closed 'asymmetric prescriptive system with five lines and alternation by genealogical level' on the basis of her analysis of the 'relationship terminology' and the alliances between patrilineal descent groups. She attempts to resolve a number of conflicts in the rules of marriage and exchange. The rules are (1) 'a woman should climb the same ladder that her father's father's sister climbed': a man marries his *iai* (FMBSD; in general, a woman of FM's clan); (2) 'the daughter goes in payment for the mother': marriage with *na* (FZD); (3) 'women should be exchanged': interpreted as exchange of sisters; and (4) '*laua*'s son will marry *wau*'s daughter': FMBD marriage. In addition, alternate generations in patrilineal clans are equated; also, certain pairs of clans are in a reciprocal relationship resting upon the tradition that they exchange women with each other (pp. 103-8). Korn's solution is to postulate an alliance model for the patrilineal groups with two kinds of closed cycles,

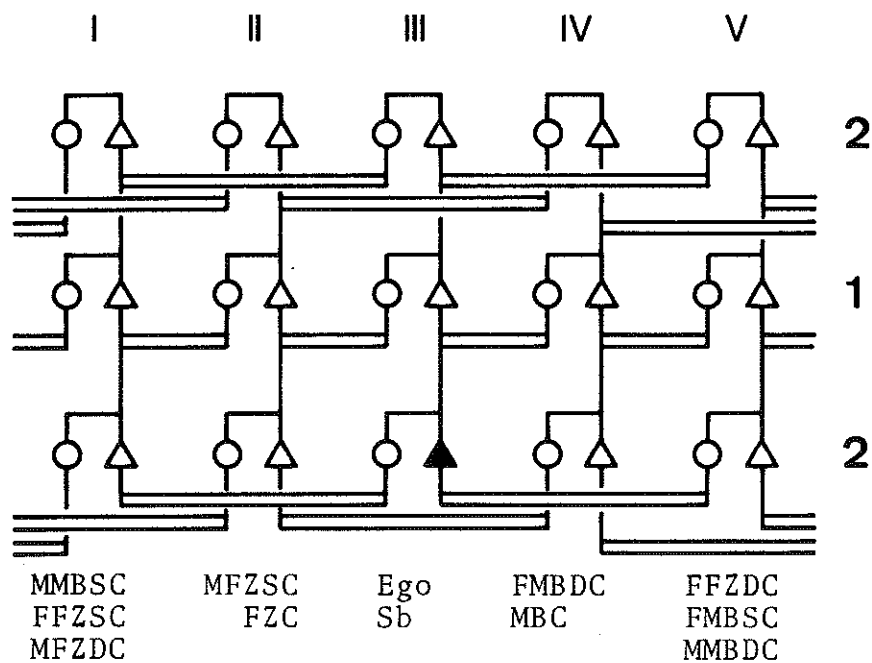
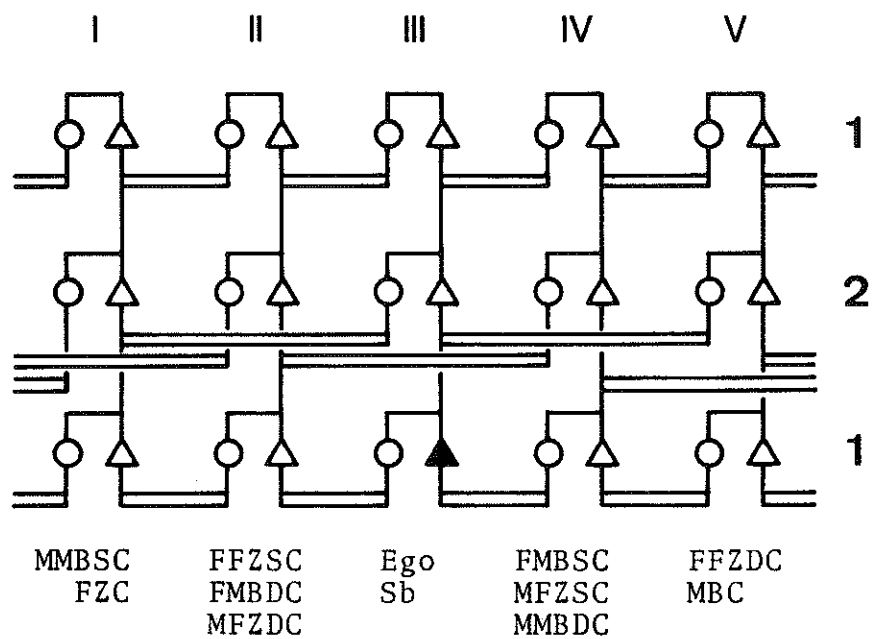


Fig. 2.2. Structural inconsistencies in an asymmetric prescriptive system with five lines and alternation by genealogical levels (after Korn 1971:116).

exhibited in alternate generations: a direct cycle (1) connecting adjacent descent lines and an indirect cycle (2) connecting alternate descent lines (p. 111; see my fig. 2.2, top). The 'relationship terminology' may then be consistently allocated to the social categories (Korn 1971: figs. 8 and 10).

Apart from the theoretical problems that arise when one is attempting to evaluate a model premised on the antigenealogical and antiextensional analysis of systems of *kin* classification as a relationship terminology of *social* categories, it can be shown that Korn's results are not internally consistent. Figure 2.2 (top) is a direct representation of the information in Korn's figures 8 and 10. Now, presumably, for a male ego in group III who marries according to cycle 1, his spouses are to be found in his FM's group IV. Hence Wife = FMBSD = MMBDD = MFZSD = *iai*. Also, ego's matrilateral cross-cousins are in his MF's group V, i.e., MBD = FFZDD = *mbuambo*. Now, if we proceed to the following generational level and look at the 'relationship terminology' from the point of view of ego's son, who is also in group III but who marries according to cycle 2, we find (fig. 2.2, bottom) that ego's spouse is taken from group V, and Wife = FMBSD = MMBDD = FFZDD. Hence FFZDD is denoted by two distinct terms, *iai* and *mbuambo*. Numerous other discrepancies can be found by comparing the two sections of figure 2.2. The conclusion is obvious: either we accept the fact that Korn's model is logically inconsistent, or we adjust the definition of 'relationship terms' so as to exclude any kind of genealogical referent, including the very data upon which the model was based in the first place. Korn's Iatmul model is an example of a non-homogeneous structure: ego's location determines the specification of kin type equivalences.⁴

Another hybrid system is described by Etienne (1975) for the Baule. Here, in addition to principles of

descent and alliance, there is a system of 'alliance rivalry'; certain relatives are designated as 'rivals' and serve as focal points in defining incompatible alliances. Not only do marriage exchanges determine marriage possibilities and prohibitions in succeeding generations, but a new marriage may result in the prescribed dissolution of previous alliances (pp. 6-8, 21-24). Etienne shows that the diverse explanations propounded by the Baule derive from a more fundamental principle of 'nonreduplication of marriage bonds'. The most economical description of the Baule system is a model with eight nodes comprising extended sibling groups, each related twice, as givers and as takers of women, to a total of four other groups (pp. 11-13, 23-26). This eight-class model, with its multiple alliances, is similar to the hypothetical eight-class model constructed by Granet (1939:238-42) for the 'original' Kachin system and subsequently demolished by Lévi-Strauss (1970:249-51), Leach (1971:73-74), and Needham (1961: 104-6).

The patrilineal Swazi described by Kuper (1978) combine a ranked social system with a modified system of asymmetric exchange. Kinship-defined marriage rules operate alongside of status-marriage rules and the payment of bridewealth. Kuper presents an open seven-line model of marriage with male ego's FMBS. Spouses are obtained in alternate generations from FM's line and MF's line, but the direction of exchanges is not reversed, even in alternate generations: male ego marries a woman of the wife-giver's wife-giver's line of his father. The purely kin-marriage rules appear to be rooted, or reflected, in the developmental cycle of the domestic group and are partially distinct from the status-marriage rules, which relate to the politico-jural domain (pp. 573-75, 577).

All of these hybrid cases are essentially variations on the same theme. 'Secondary' marriages and multiple

exchange strategies, though of course associated with an elementary core of kin-marriage rules, are described in terms of the relations seen to hold within some other domain (social classification, economic prestations, bridewealth transactions, rivalry, etc.). To the extent that such explanations serve to elucidate the interrelationships and multi-levelled complexity that can be found within a social system, they are clearly relevant. However, if too strictly adhered to, the distinction of 'primary' and 'secondary' marriage rules and their functional correlates tends to direct one's attention away from the possibility of a more fundamental or encompassing structure of exchange to which both the elementary aspects of the system and its 'hybrid' characteristics might be reduced.

A similar observation can be made with respect to the cross-cultural comparison of alliance systems. According to Kuper (1982), local variations in Southern Bantu marriage and bridewealth arrangements (reflected in different folk alliance models) differ greatly, and yet represent direct transformations of each other. The organization of bridewealth systems, predicated upon specific local combinations of pastoralism and agriculture, varying forms of political stratification, and four modes of marriage alliance constitute an ordered series. Repetitive as well as non-repetitive exchange strategies are modelled as transformations of a single alliance structure (pp. 157-162).

A family of such alliance structures will be developed here. Starting with Lévi-Strauss's conclusion, cited above, the basic model will be premised on exchange. Aspects of 'complexity' will then be introduced by formulating a recursive definition of the cycles of generalized exchange. Hence exchanges or marriages which occur at a particular moment will be used to determine the sequence of possible exchanges in the following generations or exchange cycles.

THE FORMAL MODEL

The main variables of any modelling process may be conveniently summarized as an ordered quadruple $R(S, P, M, T)$: the subject S takes, in view of the purpose P , the entity M as a model for the prototype T . My purpose may be glossed as *system restructuration* or *system extension* (Apostel 1961:5-7). If Lévi-Strauss's theory of elementary kinship structures is denoted by T_0 , my goal is to extend his analysis to include various hybrid systems and possibly also Crow-Omaha systems. Initially, one specific principle - generalized exchange - is selected, and the strategy followed is to attempt to construct a formal model or a related family of models which, while remaining compatible with the theory developed in *Les Structures*, allows analysis of data from nonelementary sources. Although the choice of extensions is large, it is only infinite (as claimed by Buchler and Fischer (1981)) in a trivial sense: not every arbitrary extension is an interesting or viable reconstruction of the set of proper models of T_0 . In developing more complex formulae of generalized exchange I choose to vary just one assumption of T_0 . My choice is explicitly motivated by an important issue raised in the preface to the second edition of *Les Structures élémentaires*, where the difficulties in determining the combinative possibilities of Crow-Omaha systems either by vector algebra or by computer simulation are discussed (1970:xli):

... in order to commence operations, an initial state would have to be determined. The danger then would be that of being trapped in a vicious circle, because in a Crow-Omaha system the state of the possible or prohibited marriages is constantly determined by the marriages which have occurred in preceding generations. The only solution to the problem of determining an initial state which does not violate for certain one of the rules of the system would be a regression to infinity, unless one were prepared to wager that, despite its aleatory appearance, a Crow-Omaha system returns on itself periodically in

such a way that, taking any initial state whatsoever, after a few generations a structure of a certain type must necessarily emerge.

My extension of T_0 does exactly as Lévi-Strauss suggests: it assumes an arbitrary initial state with generalized exchange and defines all subsequent exchange cycles *recursively*. The result is not an infinite regression (as Lévi-Strauss feared), but the necessary emergence of a periodic exchange structure of a certain type. This extension of T_0 follows directly from Lévi-Strauss's comments cited above, with the problem of vicious circularity and infinite regression solved by the recursive definition of exchange cycles.

The formal model chosen is algebraical, building on the seminal work of Weil (see the mathematical appendix to Lévi-Strauss 1970), White (1963), Courrège (1965), and Lorrain (1975), among others. The mathematical concepts it employs are taken from group theory and elementary number theory. In this section the formal set of objects (which will be interpreted anthropologically as primary exchange units, i.e., individuals or social groups, lineages, etc.) is introduced. Although the model is algebraical, it may be helpful to visualize the structure of its elements and relations geometrically. The initial set of formal objects maps out a flat, two-dimensional grid, horizontal rows corresponding to exchange circuits while each vertical column represents the trajectory (as it were) of a particular exchange unit through time. Alternatively, rows may be interpreted as generations, columns as descent lines. 'Structure' will subsequently be imposed on this basic grid through further constraints on the defining relations, resulting in a closed cylindrical model of exchange. Within each horizontal layer of this model, a specific exchange cycle linking exchange units is defined recursively as the projection of a previous exchange cycle. A structure of generalized exchange will consist of the entire set

of such exchange cycles and will repeat itself after a finite number of cycles.

Objects and permutations

Let Z be the set of integers and let I be the index set $I = \{1, 2, \dots, n\}$. Then Obj , the set of objects, is defined as $\{O_{ij} | i \text{ in } Z \text{ and } j \text{ in } I\}$.

Let $Obj(i)$ be the subset of Obj defined as $\{O_{ij} | j \text{ in } I\}$ for some i in Z . For any i , $|Obj(i)| = n$, and $Gen = \{Obj(i) | i \text{ in } Z\}$ is a partition of Obj , i.e., a decomposition of Obj into disjoint subsets such that every element O_{ij} of Obj belongs to some subset.

Let $O(j)$ be the subset of Obj defined as $\{O_{ij} | i \text{ in } Z\}$

Table 2.1. Values of Euler's function $\phi(n)$ and elements of K for n less than 15.

n	$\phi(n)$	K
1	0	-
2	1	1
3	2	1, 2
4	2	1, 3
5	4	1, 2, 3, 4
6	2	1, 5
7	6	1, 2, 3, 4, 5, 6
8	4	1, 3, 5, 7
9	6	1, 2, 4, 5, 7, 8
10	4	1, 3, 7, 9
11	10	1, 2, 3, 4, 5, 6, 7, 8, 9, 10
12	4	1, 5, 7, 11
13	12	1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12
14	6	1, 3, 5, 9, 11, 13

for some j in I . $Lin = \{O(j) | j \text{ in } I\}$ is a partition of Obj into n nonoverlapping subsets of infinite order.

Consider any $Obj(i)$ in Gen . Let c be the permutation defined by $c = (O_{i1}, O_{i2}, \dots, O_{i, n-1}, O_{in})$; i.e., c is the cyclic permutation of order n which maps O_{ij} onto $O_{i, j+1}$ with $j+1$ reduced modulo n .

Lemma 1: Let K be the set of integers defined by $K = \{k | 1 \leq k < n, k \text{ and } n \text{ coprime}\}$. It can easily be shown that K is a group under multiplication (modulo n). The identity is 1, and for any k in K , the inverse k^{-1} is defined. The order of K is given by the Euler function $\phi(n)$ for any n . For n a prime, all integers $1 \leq k < n$ are coprime to n so that $K = \{1, 2, \dots, n-1\}$ and $|K| = n-1$.

The values of $\phi(n)$ and the elements of K for $n < 15$ are given in table 2.1.

Lemma 2: Consider K and let c be the cyclic permutation of order n defined above. c generates the cyclic group $G(c)$ of order n whose elements are $\{c^x | 1 \leq x \leq n \text{ and } c^n = e\}$. Note that $c^{-x} = c^{n-x}$. Then the order of any element c^x of $G(c)$ equals n if x is in K . For any x not in K , the order of c^x equals m/x , where m is the least common multiple of x and n .⁵

Lemma 3: Consider K as defined above. For any k in K , let $p(k)$ be the order of k . Then for any n , 1 and $n-1$ are elements of K with $p(1) = 1$, and $p(n-1) = 2$ for all $n > 2$.

Consider any $O(j)$ in Lin . Let s be the one-to-one mapping of $O(j)$ onto itself defined by $(O_{ij})s = O_{i+1, j}$ for all O_{ij} . s is a translation or a permutation of infinite order and generates the group $G(s)$ of infinite order whose elements are $\{s^x | x \text{ any integer and } s^0 = e\}$. Any O_{ij} is mapped by s^x onto $O_{i+x, j}$. For any positive integer x , $O_{i+x, j}$ is called the x th successor of O_{ij} . For x a negative integer, $O_{i+x, j}$ is called the x th

predecessor of O_{ij} .

Consider any two elements $Obj(a)$ and $Obj(b)$ of Gen for which $a < b$, i.e., $b = a + x$. Then $Obj(b)$ is called the x th successor of $Obj(a)$ and $Obj(a)$ the x th predecessor of $Obj(b)$.

Structures of generalized exchange

Consider the set of objects Obj with partitions Gen and Lin . Let the permutations c and s and the group K be defined as above.

Then, for some $Obj(a)$ in Gen , define the permutation w_0 by $O_{aj}w_0 = O_{aj}c$ for all O_{aj} in $Obj(a)$.

Now, for some k in K , define the permutation w_1 of $Obj(a+1)$ by $O_{aj}sw_1 = O_{aj}(w_0)^k s$. Note that w_1 is of order n .

Continuing in this manner, define further permutations w_2, w_3 , etc. of, respectively, $Obj(a+2), Obj(a+3)$, etc., as $O_{a+1j}sw_2 = O_{a+1j}(w_1)^k s$, $O_{a+2j}sw_3 = O_{a+2j}(w_2)^k s$, etc.

In general, for any positive integer x the permutation w_x of $Obj(a+x)$ may be recursively defined by

$O_{a+x-1j}sw_x = O_{a+x-1j}(w_{x-1})^k s$, with $w_0 = c$. That is, we specify the initial permutation w_0 of $Obj(a)$ and give a rule for finding the permutation w_x of $Obj(a+x)$ once the permutation w_{x-1} of $Obj(a+x-1)$ (the immediate predecessor of $Obj(a+x)$) is known. $Obj(a)$ is called the origin of Obj .

An equivalent nonrecursive definition for w_x is given by $O_{aj}s^x w_x = O_{aj}(w_0)^{k^x} s^x = O_{aj}c^{k^x} s^x$. Note that c^{k^x} , and hence w_x , is of order n .

The definition can be extended to include negative integers $-x$ by taking for k^{-x} the inverse of k^x from the group generated by k (under multiplication modulo n). That is, $k^{-x} = k^{p(k)-x}$ with $p(k)$ the order of k .

Hence for any integer x the permutation w_x is called the x th exchange cycle with respect to the origin $Obj(a)$.

The structure of generalized exchange with origin $Obj(a)$, induced on Obj by the permutations c and s for n and some k in K , is defined by $W(a, n, k) =$

$\{w_x | O_{aj} s^x w_x = O_{aj} c^{k^x} s^x \text{ for all } O_{aj} \text{ and any integer } x\}$.

The period of $W(a, n, k)$ is the smallest positive integer x such that $w_x = c$. The period of $W(a, n, k)$ is

given by $p(k)$, the order of k , since $O_{aj} s^{p(k)} w_{p(k)} = O_{aj} c^{k^{p(k)}} s^{p(k)} = O_{aj} c s^{p(k)} = O_{a+j+1} s^{p(k)} = O_{a+p(k)+j+1}$,

so that $O_{a+p(k)+j} w_{p(k)} = O_{a+p(k)+j} c$.

Direct accessibility and consecutive symmetry. Having formally defined a structure of generalized exchange $W(a, n, k)$, we are now able to derive a number of more specific properties which will enable us to differentiate between the various possible structures. These properties are (a) direct accessibility, i.e., whether a particular exchange unit is linked directly (as giver or receiver) to some other specified unit in an exchange cycle; (b) consecutive symmetry, i.e., whether the flow of exchanges is entirely reversed within a finite number of cycles; (c) continuity, i.e., whether the flow of exchanges in directly consecutive cycles proceeds in roughly the same direction.

Let $W(a, n, k)$ be defined as above. For any integer x , w_x is generated by c^{k^x} , hence w_x is a cyclic permutation of order n . The inverse permutation w_x^{-1} exists. Then

$O_{aj} s^x w_x = O_{aj} c^{k^x} s^x = O_{a+x} j+k^x = O_{a+x} t$, with $j + k^x \equiv t \pmod{n}$, and

$O_{aj} s^{x(w_x^{-1})} = O_{aj} (c^{-1})^{k^x} s^x = O_{a+x} j-k^x = O_{a+x} g$,

with $j + k^x \equiv g \pmod{n}$. Let $O(j)$, $O(t)$ and $O(g)$ be members of the partition Lin defined earlier. Then $O(j)$ has direct access to $O(t)$ if for some O_{aj} in $O(j)$ there

is an $O_{a+x} t$ in $O(t)$ such that $O_{aj} s^x w_x = O_{a+x} t$.
 Conversely, $O(j)$ is directly accessible from $O(g)$ if for
 O_{aj} in $O(j)$ there is an $O_{a+x} g$ in $O(g)$ such that
 $O_{aj} s^x (w_x^{-1}) = O_{a+x} g$.

Lemma 4: Let $TO(j)$ denote those elements of Lin to which $O(j)$ has direct access, and let $GO(j)$ denote the elements of Lin from which $O(j)$ is directly accessible. Then $|TO(j)| = |GO(j)| = p(k)$, the period of $W(a, n, k)$, for all $O(j)$ in Lin .

Let $TO(j)$ and $GO(j)$ be defined as above. Then the degree of $O(j)$ is the number of elements of Lin contained in the union of $TO(j)$ and $GO(j)$, i.e., $|TO(j) \cup GO(j)|$. The degree of every $O(j)$ in Lin is the same, so that it is possible to define the degree of $W(a, n, k)$ as the degree of any $O(j)$ in Lin .

Let the intersection of $TO(j)$ and $GO(j)$ be nonempty for some $O(j)$ in Lin . Suppose $TO(j) \cap GO(j) = O(z)$. Then for some x and y , $O_{aj} s^x w_x = O_{a+x} z$, and $O_{aj} s^y (w_y^{-1}) = O_{a+y} z$. Any exchange structure $W(a, n, k)$ for which this is true is said to exhibit *consecutive symmetry*. This definition formalizes the concept of consecutive symmetry introduced in de Josselin de Jong (1962:46).

Continuous and discontinuous structures. According to Lemma 3, for any n , 1 and $n-1$ are both coprime to n and thus elements of K . Hence $W(a, n, 1)$ and $W(a, n, n-1)$ are always structures of generalized exchange. The period of $W(a, n, 1)$ is $p(1) = 1$, so that $w_x = w_0 = c$ for all x . All exchange cycles are identical and $W(a, n, 1)$ is said to be *continuous*. For $n > 2$ the period of $W(a, n, n-1)$ is $p(n-1) = 2$, so that $w_x = w_0 = c$ for even integers x and $w_x = w_1 = c^{-1}$ for uneven integers x . Exchange cycles reverse completely in successive circuits and $W(a, n, n-1)$ is said to be *discontinuous*. The continuity/discontinuity

Table 2.2. Structures of generalized exchange for n less than 15.

Structure	Period	Degree	Gloss
$W(a, n, 1)^* \dots\dots$	1	2	continuous
$W(a, n, n-1)^* \dots\dots$	2	2	discontinuous; c.sym.
$W(a, 5, 2) \dots\dots$	4	4	continuous; c.sym.
$W(a, 5, 3) \dots\dots$	4	4	discontinuous; c.sym.
$W(a, 7, 2) \dots\dots$	3	6	continuous
$W(a, 7, 3) \dots\dots$	6	6	continuous; c.sym.
$W(a, 7, 4) \dots\dots$	3	6	discontinuous
$W(a, 7, 5) \dots\dots$	6	6	discontinuous; c.sym.
$W(a, 8, 3) \dots\dots$	2	4	continuous
$W(a, 8, 5) \dots\dots$	2	4	discontinuous
$W(a, 9, 2) \dots\dots$	6	6	continuous; c.sym.
$W(a, 9, 4) \dots\dots$	3	6	continuous
$W(a, 9, 5) \dots\dots$	6	6	discontinuous; c.sym.
$W(a, 9, 7) \dots\dots$	3	6	discontinuous
$W(a, 10, 3) \dots\dots$	4	4	continuous; c.sym.
$W(a, 10, 7) \dots\dots$	4	4	discontinuous; c.sym.
$W(a, 11, 2) \dots\dots$	10	10	continuous; c.sym.
$W(a, 11, 3) \dots\dots$	5	10	continuous
$W(a, 11, 4) \dots\dots$	5	10	continuous
$W(a, 11, 5) \dots\dots$	5	10	continuous
$W(a, 11, 6) \dots\dots$	10	10	discontinuous; c.sym.
$W(a, 11, 7) \dots\dots$	10	10	discontinuous; c.sym.
$W(a, 11, 8) \dots\dots$	10	10	discontinuous; c.sym.
$W(a, 11, 9) \dots\dots$	5	10	discontinuous
$W(a, 12, 5) \dots\dots$	2	4	continuous
$W(a, 12, 7) \dots\dots$	2	4	discontinuous

Table 2.2. *Continued.*

Structure	Period	Degree	Gloss
$W(a, 13, 2)$	12	12	continuous; c.sym.
$W(a, 13, 3)$	3	6	continuous
$W(a, 13, 4)$	6	6	continuous; c.sym.
$W(a, 13, 5)$	4	4	continuous; c.sym.
$W(a, 13, 6)$	12	12	continuous; c.sym.
$W(a, 13, 7)$	12	12	discontinuous; c.sym.
$W(a, 13, 8)$	4	4	discontinuous; c.sym.
$W(a, 13, 9)$	3	6	discontinuous
$W(a, 13, 10)$	6	6	discontinuous; c.sym.
$W(a, 13, 11)$	12	12	discontinuous; c.sym.
$W(a, 14, 3)$	6	6	continuous; c.sym.
$W(a, 14, 5)$	6	6	continuous; c.sym.
$W(a, 14, 9)$	3	6	discontinuous
$W(a, 14, 11)$	3	6	discontinuous

* Defined for $2 < n$. c.sym. = consecutive symmetry

is mentioned by Lévi-Strauss (1970:478, 445). It can be extended to any exchange structure $W(a, n, k)$ as follows.

Let $W(a, n, k)$ be any structure of generalized exchange. For some n , K is the set of integers k such that $1 \leq k < n$, with k and n coprime. If n is even, then n can be written as $n = 2r$. If n is uneven, then $n = 2r + 1$. Then $W(a, n, k)$ is said to be *continuous* for values of k such that $1 \leq k \leq r$. For all other values of k (i.e., for $r < k < n$), $W(a, n, k)$ is said to be *discontinuous*.

An exchange cycle with *alternating cycles*, i.e., with period $p(k) = 2$, is not necessarily discontinuous. For example, $W(a, 8, 3)$, $W(a, 8, 5)$ and $W(a, 8, 7)$ are all of

period 2, i.e., $3^2 \equiv 5^2 \equiv 7^2 \equiv 1 \pmod{8}$, but $3 < 4 = \frac{1}{2}n$ while 5 and $7 > 4 = \frac{1}{2}n$, hence $W(a, 8, 3)$ is continuous while $W(a, 8, 5)$ and $W(a, 8, 7)$ are discontinuous, and only this last structure exhibits the property of consecutive symmetry.

Table 2.2 summarizes the properties of all possible structures of generalized exchange with $2 < n < 15$. Further examples may be added to this list for other combinations of n and k if required.

Graphic representations and reduced structures

The structure of generalized exchange $W(a, n, k)$ may perhaps be better visualized by referring to the graph of the infinite group $G(c, s)$ generated by the permutations c and s (fig. 2.3). c is a cyclic permutation of order n ; the order of s is infinite. The elements O_{ij} of the set Obj are represented by squares. Columns denote elements $O(j)$ of the partition Lin ; horizontal planes represent elements $Obj(i)$ of the partition Gen . $G(c, s)$ is commutative, i.e., for any x and y in $G(c, s)$, $xy = yx$, and any x may be written as a product of powers of c and s . With no further constraints imposed, $G(c, s)$ is the group underlying the exchange structure $W(a, n, 1)$: all exchange cycles w_x are invariant and identical to c . For any other exchange structure the associated group is not commutative.

We now choose some $Obj(a)$ as origin and define the exchange cycle w_0 on the origin by $w_0 = c$. Then for any $Obj(a + x)$ the associated exchange cycle w_x is defined with respect to the origin by the equation

$O_{aj}s^x w_x = O_{aj}(w_0)^{k^x} s^x$. Hence w_x is the projection of the permutation c^{k^x} of $Obj(a)$ onto $Obj(a + x)$ (see fig. 2.4). Then $W(a, n, k)$ consists of disjoint exchange cycles, with each $Obj(a + x)$ containing exactly one permutation w_x of order n . Then $G(w_x, s; W)$ denotes the group

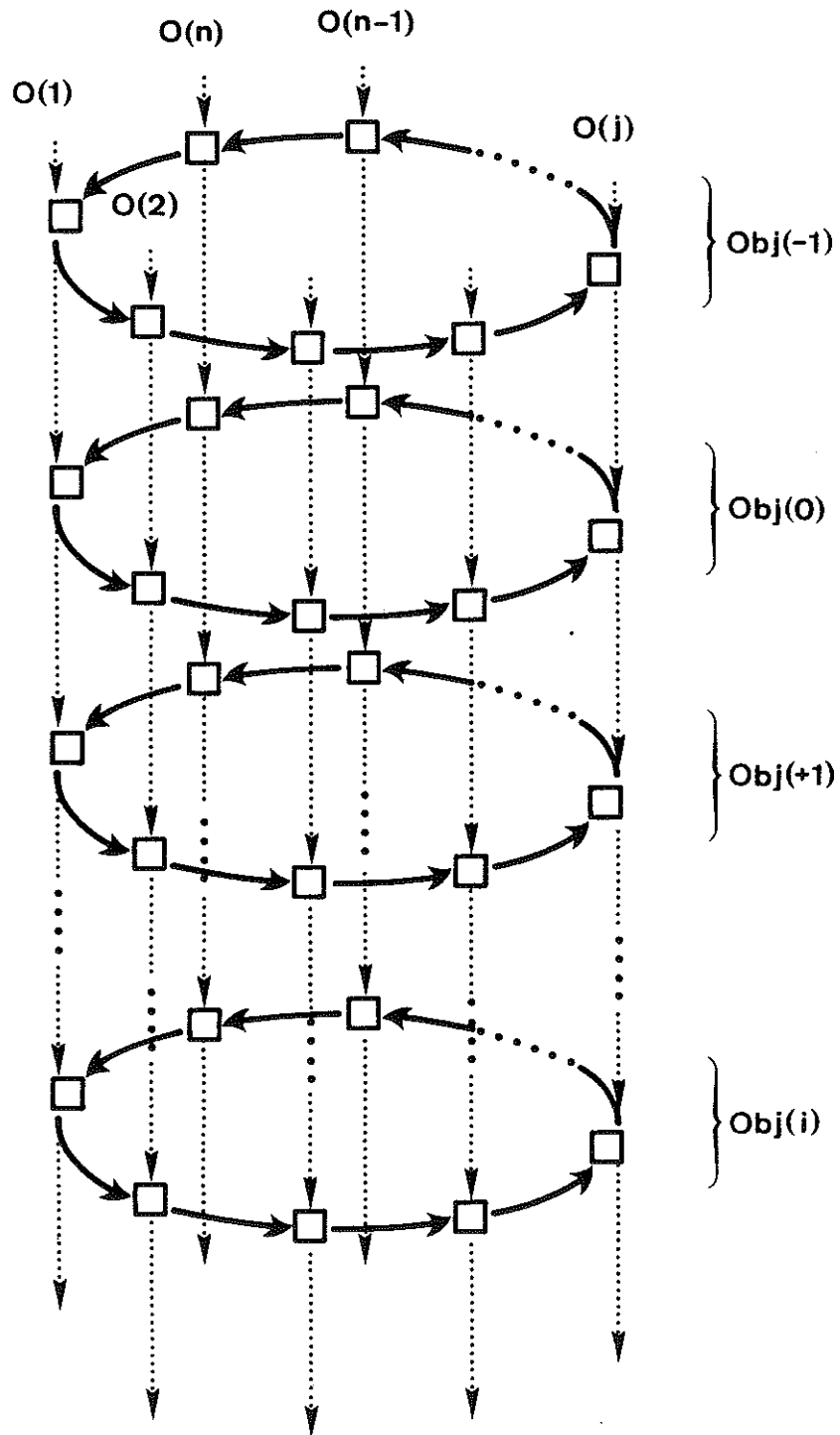


Fig. 2.3. Graph of the commutative group $G(c, s)$. The generators of $G(c, s)$ are shown as arrows: solid arrows represent c , dotted arrows s .

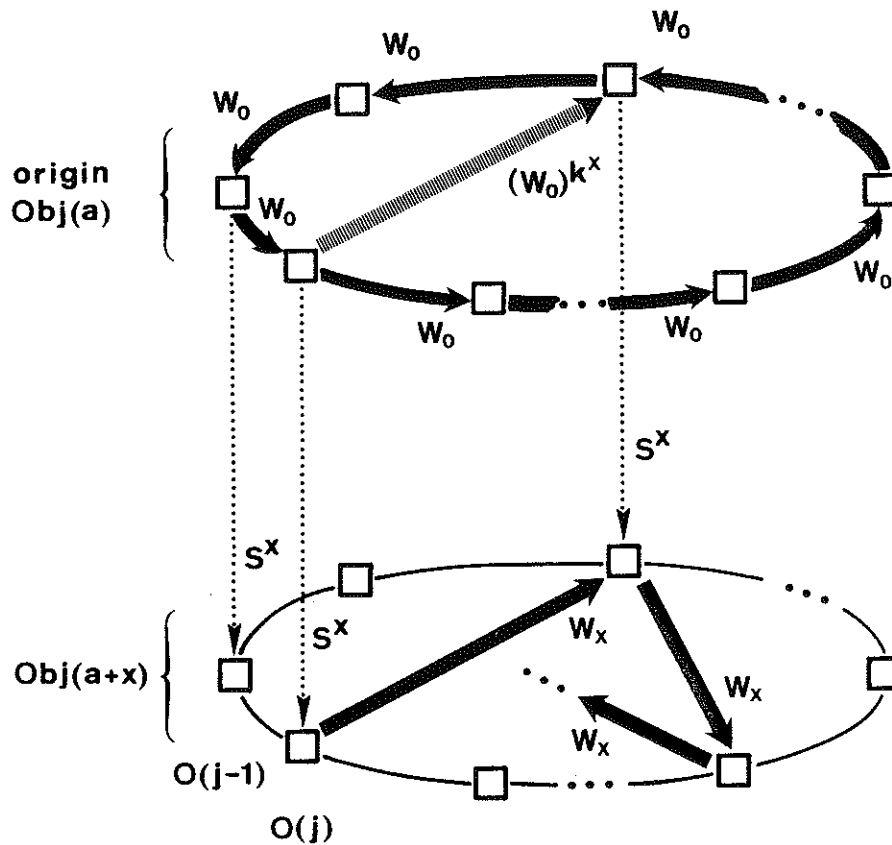


Fig. 2.4. Definition of the exchange cycle w_x as a projection from $Obj(a)$ onto $Obj(a+x)$.

associated with the exchange structure $W(a, n, k)$.

Any $W(a, n, k)$ and its related group can be mapped onto simpler, 'reduced' structures that retain certain features considered essential (cf. Lévi-Strauss 1970: 362, 364 n.3, 365, 411). 'Reductions' are essentially homomorphisms: many-to-one mappings that preserve products and specific properties of the original exchange structure. For some $W(a, n, k)$ with period $p(k)$ let ρ be the equivalence relation partitioning the integers into equivalence classes defined by $x \equiv r \pmod{p(k)}$. ρ induces three related mappings: of the

elements of Obj , of the exchange cycles, and of the integral powers of the permutation s .

Let $\hat{r}$ denote the equivalence class of r . Then $R = \{\hat{r} | r \in \{0, 1, \dots, p(k)-1\}\}$ is the set of all equivalence classes, i.e., the factor set of the integers with respect to p .

Then (1) $\rho: O_{a+x \ j} \rightarrow O_{a+r \ j}$; (2) $\rho: w_x \rightarrow w_r$; and (3) $\rho: s^x \rightarrow s^r$. In general, since ρ preserves the product of permutations, $(xy)\rho = (x)\rho(y)\rho$ for all x and y in $G(w_x, s; W)$.

Under ρ , the permutation s of infinite order is reduced to a cyclical permutation of order $p(k)$, i.e., for any O_{ij} of Obj , $O_{ij}s^x = O_{i+x \ j}$, with $i+x$ reduced modulo $p(k)$. Hence the $p(k)$ th successor of any $Obj(a+r)$ in Gen is $Obj(a+r)$ itself.

ρ defines the factor structure or reduced structure $W/p(a, n, k)$ and the associated factor group $G/p(w_x, s; W)$ of finite order. Geometrically, the effect of ρ is to map the cylindrical representation of the exchange model onto the surface of a torus by identifying the elements of Gen modulo $p(k)$.

The effect of ρ is equivalent to finding the factor structures induced by the normal subgroup $N(s; s^{p(k)} = e)$ of $G(w_x, s; W)$. One may obtain the complete set of reduced exchange structures by considering the complete set of normal subgroups associated with any $W(a, n, k)$. Although this is a fairly straightforward exercise, it goes beyond the scope of this chapter.

As an example, the reduced structures induced by ρ for the exchange structures $W(a, 8, 1)$, $W(a, 8, 3)$, $W(a, 8, 5)$ and $W(a, 8, 7)$ are presented in figures 2.5, 2.6, 2.7 and 2.8 (the associated kinship structures are discussed below).

The mathematical model of more complex formulae of exchange is now complete. The family of structures $W(a, n, k)$ is essentially a base onto which a variety of other anthropological structures may be mapped. One

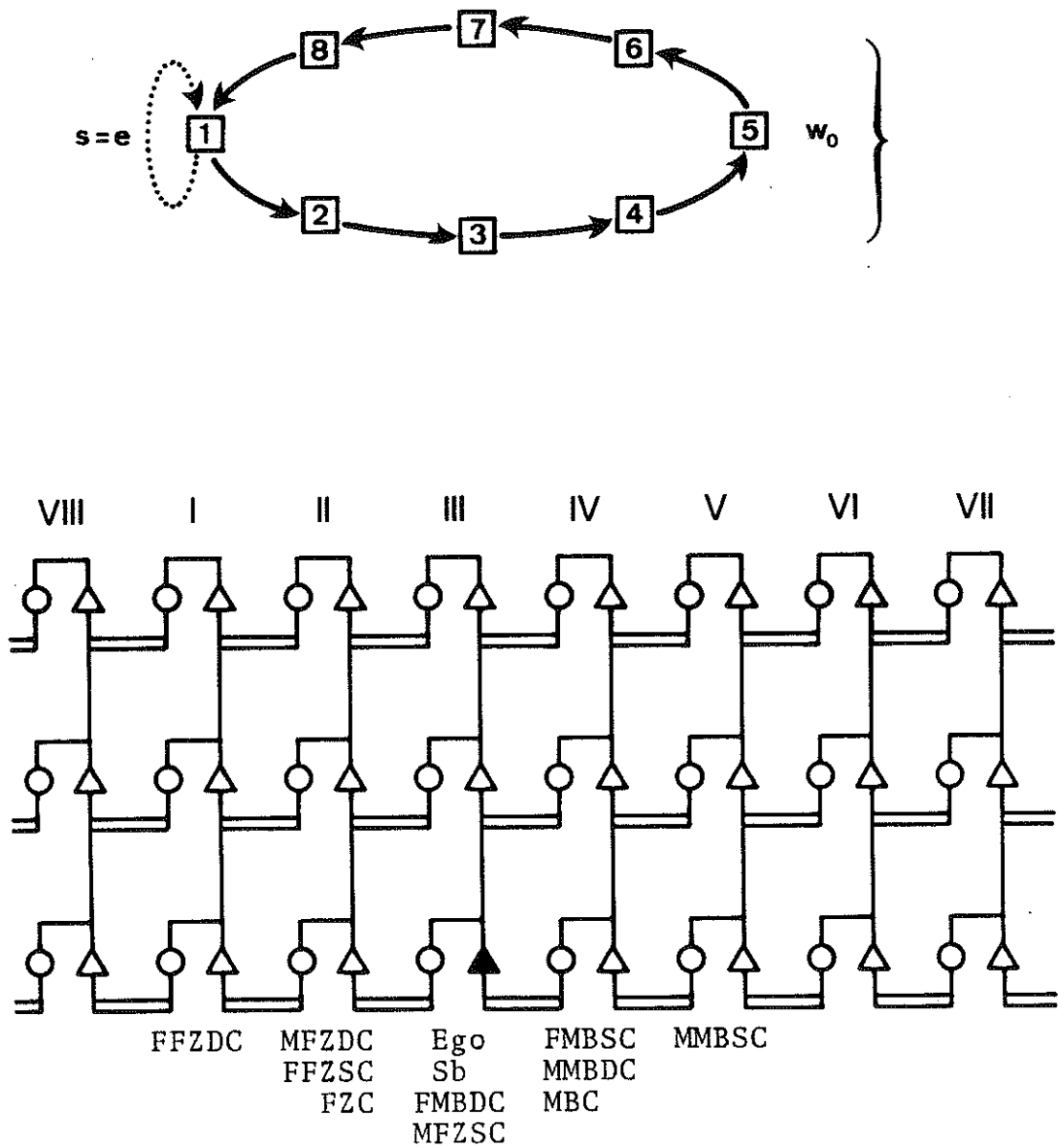


Fig. 2.5. Reduced structure and kinship structure associated with $W(a, 8, 1)$.

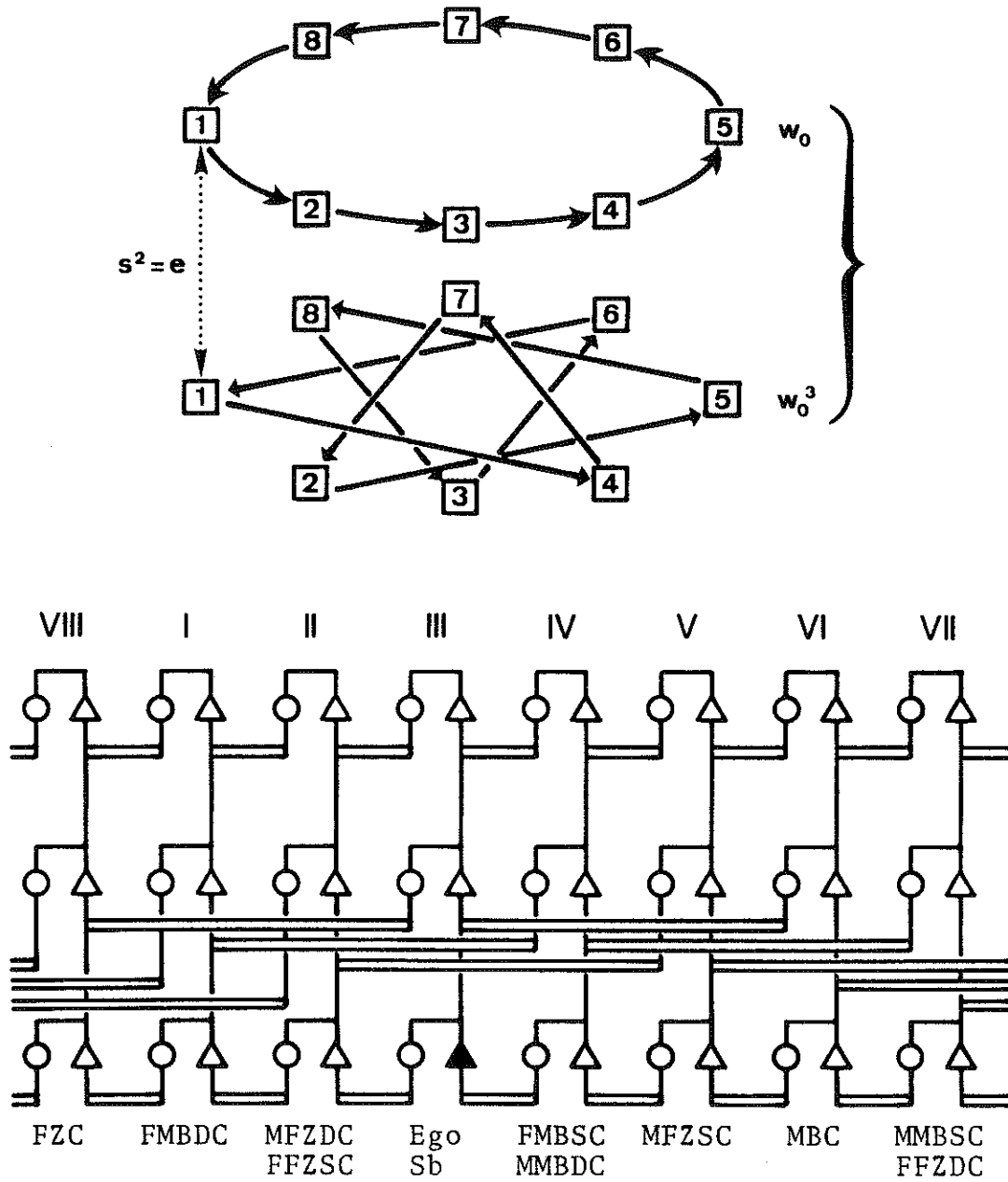


Fig. 2.6. Reduced structure and kinship structure associated with $W(a, 8, 3)$.

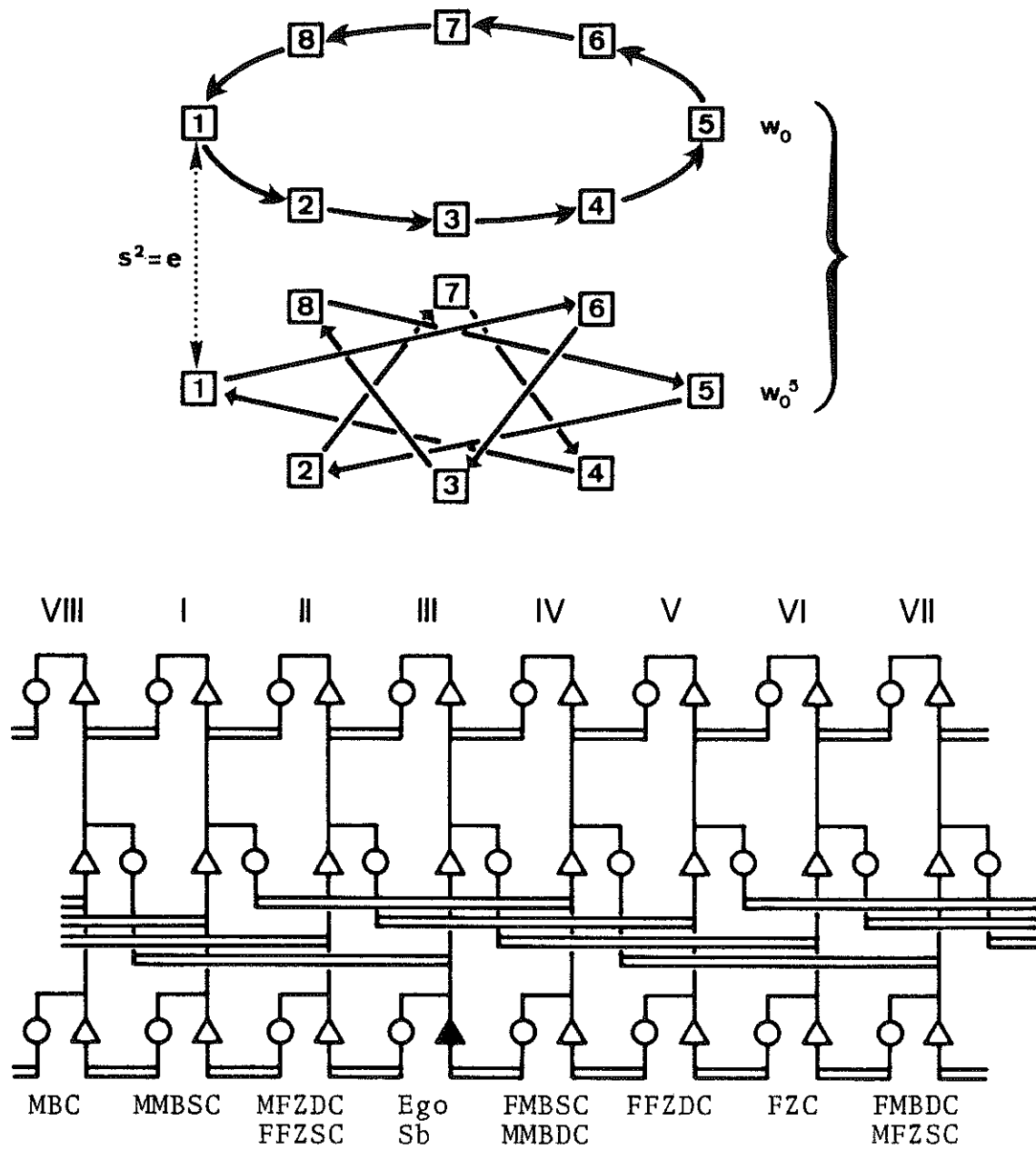


Fig. 2.7. Reduced structure and kinship structure associated with $W(a, 8, 5)$.

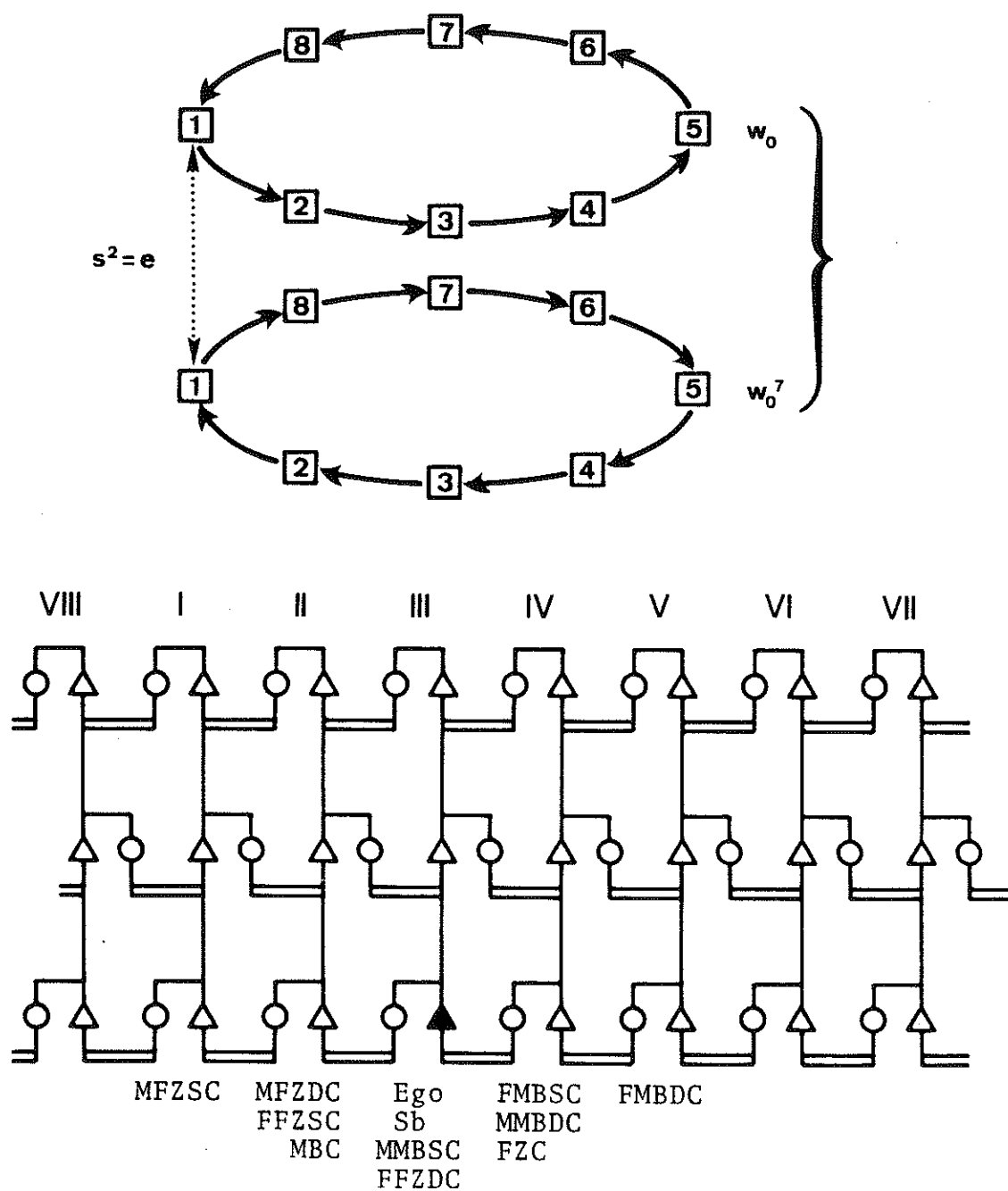


Fig. 2.8. Reduced structure and kinship structure associated with $W(a, 8, 7)$.

such mapping is considered in detail in the following section.

KINSHIP STRUCTURES AND GENERALIZED EXCHANGE

The elegant mathematical model constructed by Courrège to represent Lévi-Strauss's elementary kinship structures was introduced in the previous chapter. Basically, an *elementary kinship structure* (S, h, m, f) is represented as a set-theoretic structure, with the permutations h , m and f of S satisfying the general equation $f = hm$. Under the standard interpretation, the set S represents a partitioning of the genealogical network into non-overlapping 'classes', descent lines, or sibling groups. The permutations h , m and f represent the conjugal mapping, the maternal mapping, and the paternal mapping, while the basic equation $f = hm$ models the assumption that the children of a man of 'class' x who is married to a woman of 'class' $(x)h$ will be allocated to 'class' $(x)f = ((x)h)m$.⁶

Courrège's model has been extensively modified by Lorrain (1974, 1975) and recast in terms of the mathematical theory of categories and functors. One important result of Lorrain's researches is that the classic permutation models of kinship systems as well as certain other types of genealogical structures may be studied as representations of a more general class of mathematical structures termed *quasi-homogeneous spaces*.⁷ It is a tempting speculation that further research along these lines might provide a unitary theoretical framework for a direct comparison of, say, the Courrège-type kinship models (articulating descent principles and marriage rules) with the Lounsbury-type transformational models generating the structure of kinship terminologies.

For example, under the standard interpretation provided in Chapter 1, the genealogical networks

underlying Courrège's regular kinship structures are assumed to be reduced according to two principles: (a) same-sex siblings and parallel cousins are considered structurally equivalent and hence are not distinguished from each other, and (b) marriage prescription, i.e., if two persons of the same sex are structurally equivalent, then their spouses are also considered structurally equivalent. (A kinship network reduced according to these principles is graphically represented in Chapter 1, figure 1.4 (top).) Now, for all practical reasons, the merging of kintypes implied by principles (a) and (b) may also be obtained by the application of Lounsbury's 'same-sex sibling rule' in combination with his 'half-sibling rule' (Lounsbury 1964:360-361). However, although a system of Lounsbury-type merging rules may produce comparable results to a system of algebraic 'functorial' reductions, the two families of models are not based on the same formal principles. Lorrain (1975:263-268) has indicated important differences: equivalences due to merging rules are unidirectional, not symmetric (i.e., $X \rightarrow Y$ does not imply $Y \rightarrow X$), and a system of merging rules is sometimes hierarchically ordered, while 'functorial' reductions form an unordered set of rules. Further development of the algebraic models is necessary if one requires a range of structural reductions similar to that which can be derived from the more specific properties of a system of merging rules.

Indeed, principles (a) and (b) are unduly restrictive, a point to be taken up in following chapters. For present purposes, however, I employ the standard Courrège-Lorrain model introduced in Chapter 1 (cf. table 1.1). The underlying mathematical structure is regular and homogeneous. This expresses the fact that no node in the reduced genealogical network is privileged with respect to any other of the elements of S in terms of kinship relations.

The structure of generalized exchange $W(a, n, k)$ can be combined with the Courrège-Lorrain model of an elementary kinship structure denoted by (h, m, f) in two steps:

1. Let O_{ij} , the elements of the infinite set Obj on which $W(a, n, k)$ is defined, be identified as nodes in the reduced kinship network S of (h, m, f) .

2. Let the set of exchange cycles w_x of $W(a, n, k)$ be identified with h , the conjugal mapping relating husband to wife. Unilineal descent rules are specified by identifying either the matrilineal or the patrilineal mapping with s , the successor mapping defined on Obj . Hence $M(h_x, s, f)$ with $f = h_x s$ is the *matrilineal kinship structure* and $P(h_x, m, s)$ with $s = h_x m$ the *patrilineal kinship structure*, both induced by the exchange structure $W(a, n, k)$.

The mapping, essentially an isomorphism from $W(a, n, k)$ onto (h, m, f) , preserves the product of relations and the regular structure of $W(a, n, k)$. Features of the exchange structure correspond with properties of the kinship model. For example, the partition Gen of the set Obj may be interpreted as a classification of the nodes of the kinship network into discrete generation levels. For matrilineal (respectively, patrilineal) structures the partition Lin of Obj represents the n matri- (patri-) lines of the kinship model. The formal concept of continuity corresponds to a system of unidirectional marriage exchanges. Conversely, in discontinuous structures the direction of the exchange cycle varies from generation to generation. In kinship structures which exhibit consecutive symmetry, the same line is both wife-giver and wife-taker to men in ego's own line in different (not necessarily alternating) generations. Most important, the recursive definition of exchange provides a simple formula for describing marriages in terms of the alliances that have taken place in preceding generations. Hence the *global* exchange

structure can effectively be generated by applying a recursively defined marriage rule, phrased in terms of *local* constraints and possibilities.

The mapping of the elements $Obj(j)$ of Gen onto generation levels is defined relative to some ego in generation G^0 by $G^{-j} = Obj(j)$. Hence $G^{+2} = Obj(-2)$, $G^{+1} = Obj(-1)$, $G^0 = Obj(0)$, $G^{-1} = Obj(+1)$, etc. As a consequence, when one is comparing the allocation of kin types to nodes and descent lines from the point of view of some ego and, say, his sister's son, the origin of the generating structure must be suitably adapted. Ego's kinship network is generated by $W(a, n, k)$. Considered from the point of view of the sister's son, who is situated in the following generation, the origin has receded, and the corresponding generating structure is now $W(a-1, n, k)$.

The patrilineal kinship structures associated with the exchange structures $W(a, 8, 1)$, $W(a, 8, 3)$, $W(a, 8, 5)$ and $W(a, 8, 7)$ are shown in figures 2.5 - 2.8. Roman numerals identify patriline (I = $O(1)$, II = $O(2)$, etc.). In these four examples the origin $Obj(a)$ is situated at the G^{+2} level, with exchange cycle $w_0 = c$. Ego is in line III in node O_{03} . All kin types relative to ego can be derived by applying the composite mappings described earlier. For example, taking the patrilineal structure generated by $W(-2, 8, 3)$ (fig. 2.6), $MBC: (O_{03})_m^{-1}f = (O_{03})_f^{-1}h_1f = (O_{03})_s^{-1}w_1s = (O_{03})_c^3 = O_{06}$, i.e., in generation G^0 and line VI. In like manner, all G^0 kin types have been allocated to the structures of figures 2.5 - 2.8.

Figures 2.5 and 2.8 represent two elementary structures which have been the subject of some of the major theoretical discussions in anthropology during the last half-century: generalized exchange structures with matrilateral and patrilateral cross-cousin marriage. The literature on unilateral cross-cousin marriage is extensive, and the formal properties of both types of

exchange structure are presumed to be common knowledge.

The study of alliance structures has been muddled by the existence of numerous analyses based on conflicting and partially incommensurable theoretical assumptions. For example, systems with exclusive matrilineal cross-cousin marriage have been glossed as structures in which (1) 'a man marries (or should marry) his MBD', i.e., a particular genealogically defined relation; (2) 'there is one specific group (or marriage class, clan, lineage, local line, etc.) II from which men of group III obtain their spouses; conversely, group III (cf. fig. 2.5) invariably gives women to group IV'; (3) 'a man marries (or should marry) a woman of a particular social category or status (not exclusively or necessarily determined by kinship) denoted by a particular relationship term'. Any of these positions may be amended by defining marriage possibilities with respect to female ego or by formulating additional prohibitions or negative rules. The formal mathematical model developed here is compatible with theories predicated either on the exchange of spouses between sociological groups or categories, or with kin-defined marriage rules. The model may be used to represent the structure of actual alliance networks, the participants' marriage ideology, or the theoretical constructs of anthropologists.

Marriage rules expressed in kin type notation can also be defined in terms of wife-givers or wife-takers of ego's group or line. Thus a MBD-marriage rule holds in a system organized so that the wife-givers in any generation $i + 1$ are the same as in the preceding (i th) generation, e.g., $wg(i + 1) = wg(i)$. A system with a FZD-marriage rule implies that the wife-givers in generation $i + 1$ are identical to the wife-takers in the preceding (i th) generation, e.g., $wg(i + 1) = wt(i)$ and the exchange cycles are completely reversed in consecutive generations.

The alternating generation structures of figures 2.6

and 2.7 both specify marriage with male ego's MMBDD (structurally equivalent to FMBSD). Moreover, lines in both structures are linked to two wife-giving and two wife-taking lines. However, the exchange rule expressed in figure 2.6 is recursively defined as $wg(i + 1) = wg^3(i)$ and the structure is continuous. In contrast, figure 2.7 depicts a structure for which $wg(i + 1) = wt^3(i)$ and the flow of spouses changes direction when consecutive cycles are compared.

Kinship terminologies have traditionally been characterized by classifying first cousins as 'cross' or 'parallel' relatives. A number of attempts have been made to extend the cross/parallel distinction beyond the range of close kin. I make use of Scheffler's Iroquois and Dravidian cross/parallel extensions as a diagnostic feature of the equivalence class structure induced by the different exchange models on the genealogical grid:⁸

1. A G^0 kin type is *Iroquois-cross* if the linking kin at G^{+1} (parents of ego and alter) are of the opposite sex; otherwise it is *Iroquois-parallel*.

2. A G^0 kin type is *Dravidian-cross* if either the linking kin at G^{+1} (ego's and alter's parents) or the sibling pair at G^{+2} (ego's and alter's grandparents), but not both, are of the opposite sex; otherwise the kin type is *Dravidian-parallel*.

The classification of all first- and second-cousin kin types is given in table 2.3. A kinship structure is *Iroquois (Dravidian)-compatible* if and only if cross kin types are merged with cross and parallel kin types with parallel. For example, the four kinship structures of figures 2.5 - 2.8 are all Dravidian-compatible with respect to G^0 kin types, but only the structures of figures 2.6 and 2.7 are also Iroquois-compatible.

When the definitions and theorems derived in preceding sections are applied to the exchange structures of table 2.2, the complete family of kinship models with generalized exchange and less than fifteen lines is obtained. The

Table 2.3. Cross/parallel classification of kin types.

G ⁰ kin type	Dravidian	Iroquois
MFZSC	//	X
MFZDC	X	//
MMBSC	//	X
MMBDC	X	//
{ MBC	X	X
{ Sb	//	//
{ FZC	X	X
FFZSC	X	//
FFZDC	//	X
FMBSC	X	//
FMBDC	//	X
{ MFBSC	X	X
{ MFBDC	//	//
{ MMZSC	X	X
{ MMZDC	//	//
{ MZC	//	//
{ FBC	//	//
{ FFBSC	//	//
{ FFBDC	X	X
{ FMZSC	//	//
{ FMZDC	X	X

Note: Adapted from Scheffler (1971).

results are given in table 2.4. Patrilineal descent is assumed and for purposes of comparison male ego is consistently situated in line III. Arrows point from husband to wife (reversing them gives the direction of the flow of women). The period of each kinship structure

(i.e., the number of generations in which the original exchange cycle is renewed) is listed, together with the wife-givers and wife-takers of line III, the exchange rule, a list of kintypes merged with ego's spouse, and the type of G^0 cross/parallel compatibility. Other properties of the kinship structure - degree, continuity of exchange, consecutive symmetry - have already been summarized in table 2.2.

To sum up: (1) Only the first two structures allow unilateral first-cousin marriage (with MBD, respectively FZD). These are two of the elementary structures described by Lévi-Strauss and others. (2) Expanding the cycle of generalized exchange (i.e., increasing n , the number of exchange units or descent lines) will not necessarily result in a corresponding increase in a structure's period. (3) For exchange cycles of length greater than eight, marriage with relatives structurally distinguished from both first and second cousins becomes possible. However, there is no simple relationship between the number of descent lines and the various types of second-cousin marriage. (4) A marriage rule (defined either in genealogical terms or by reference to wife-givers and wife-takers) is not always a sufficient criterion for establishing unique structures. For example, four structures in table 2.4 have 'FFZDD marriage', and there are five in which the wife-givers of the wife-givers in a previous exchange cycle provide a spouse for male ego (i.e., $wg(i+1) = wg^2(i)$). A marriage rule is only one of the relevant structural features which, in different combinations, account for the variation between the models. (5) Finally, there is no direct association between Iroquois or Dravidian compatibility and other features of the kinship structure (descent, marriage rule, periodicity, etc.). Only the first two structures are consistently Dravidian compatible (for any number of lines greater than three).

With the exception of the elementary structure with

Table 2.4. Kinship structures with generalized exchange.

Generating structure	Period	Exchange cycles		
		Wife-takers	Male ego	Wife-givers
W(a , n , 1)	1	II	=> III =>	IV
W(a , n , n -1)	2	II, IV	=> III =>	IV, II
W(a , 5 , 2)	4	II, I, IV, V	=> III =>	IV, V, II, I
W(a , 5 , 3)	4	II, V, IV, I	=> III =>	IV, I, II, V
W(a , 7 , 2)	3	II, I, VI	=> III =>	IV, V, VII
W(a , 7 , 3)	6	II, VII, I IV, VI, V	=> III =>	IV, VI, V, II, VII, I
W(a , 7 , 4)	3	II, VI, I	=> III =>	IV, VII, V
W(a , 7 , 5)	6	II, V, VI, IV, I, VII	=> III =>	IV, I, VII, II, V, VI
W(a , 8 , 3)	2	II, VIII	=> III =>	IV, VI
W(a , 8 , 5)	2	II, VI	=> III =>	IV, VIII
W(a , 9 , 2)	6	II, I, VIII, IV, V, VII	=> III =>	IV, V, VII, II, I, VIII
W(a , 9 , 4)	3	II, VIII, V	=> III =>	IV, VII, I
W(a , 9 , 5)	6	II, VII, V, IV, VIII, I	=> III =>	IV, VIII, I, II, VII, V
W(a , 9 , 7)	3	II, V, VIII	=> III =>	IV, I, VII
W(a , 10 , 3)	4	II, X, IV, VI	=> III =>	IV, VI, II, X
W(a , 10 , 7)	4	II, VI, IV, X	=> III =>	IV, X, II, VI

Exchange rule	Spouse merged with	Cross/parallel compatibility
$wg(i+1) = wg(i)$	MBD, MMBDD, FMBS (FFZDD for $n = 3$)	Dravidian (for $n > 3$)
$wg(i+1) = wt(i)$	FZD, MMBDD, FMBS (MFZSD for $n = 3$)	Dravidian (for $n > 3$)
$wg(i+1) = wg^2(i)$	FMBDD, FFZSD, MFZDD	
$wg(i+1) = wt^2(i)$	MMBSD, FFZSD, MFZDD	
$wg(i+1) = wg^2(i)$	FFZDD	
$wg(i+1) = wg^3(i)$	MFZSD	Iroquois
$wg(i+1) = wt^3(i)$	FFZDD	Iroquois
$wg(i+1) = wt^2(i)$	MFZSD	
$wg(i+1) = wg^3(i)$	MMBDD, FMBS	Iroquois and Dravidian
$wg(i+1) = wt^3(i)$	MMBDD, FMBS	Iroquois and Dravidian
$wg(i+1) = wg^2(i)$	no first or second cousin	
$wg(i+1) = wg^4(i)$	no first or second cousin	Iroquois
$wg(i+1) = wt^4(i)$	no first or second cousin	Iroquois
$wg(i+1) = wt^2(i)$	no first or second cousin	
$wg(i+1) = wg^3(i)$	FFZSD, MFZDD	Iroquois and Dravidian
$wg(i+1) = wt^3(i)$	FFZSD, MFZDD	Iroquois and Dravidian

Table 2.4 (continued).

Generating structure	Period	Exchange cycles		
		Wife-takers	Male ego	Wife-givers
W(a, 11, 2)	10	II, I, X, VI, IX, IV, V, VII, XI, VIII	=> III =>	IV, V, VII, XI, VIII, II, I, X, VI, IX
W(a, 11, 3)	5	II, XI, V, IX, X	=> III =>	IV, VI, I, VIII, VII
W(a, 11, 4)	5	II, X, IX, V, XI	=> III =>	IV, VII, VIII, I, VI
W(a, 11, 5)	5	II, IX, XI, X, V	=> III =>	IV, VIII, VI, VII, I
W(a, 11, 6)	10	II, VIII, XI, VII, V, IV, IX, VI, X, I	=> III =>	IV, IX, VI, X, I, II, VIII, XI, VII, V
W(a, 11, 7)	10	II, VII, IX, I, XI, IV, X, VIII, V, VI	=> III =>	IV, X, VIII, V, VI, II, VII, IX, I, XI
W(a, 11, 8)	10	II, VI, V, VIII, X, IV, XI, I, IX, VII	=> III =>	IV, XI, I, IX, VII, II, VI, V, VIII, X
W(a, 11, 9)	5	II, V, X, XI, IX	=> III =>	IV, I, VII, VI, VIII
W(a, 12, 5)	2	II, X	=> III =>	IV, VIII
W(a, 12, 7)	2	II, VIII	=> III =>	IV, X
W(a, 13, 2)	12	II, I, XII, VIII, XIII, X, IV, V, VII, XI, VI, IX	=> III =>	IV, V, VII, XI, VI, IX, II, I, XII, VIII, XIII, X
W(a, 13, 3)	3	II, XIII, VII	=> III =>	IV, VI, XII

Exchange rule	Spouse merged with	Cross/parallel compatibility
$wg(i+1) = wg^2(i)$	no first or second cousin	
$wg(i+1) = wg^3(i)$	FMBDD	Iroquois and Dravidian
$wg(i+1) = wg^4(i)$	MMBSD	Iroquois and Dravidian
$wg(i+1) = wg^5(i)$	no first or second cousin	Iroquois
$wg(i+1) = wt^5(i)$	no first or second cousin	Iroquois
$wg(i+1) = wt^4(i)$	FMBDD	Iroquois and Dravidian
$wg(i+1) = wt^3(i)$	MMBSD	Iroquois and Dravidian
$wg(i+1) = wt^2(i)$	no first or second cousin	
$wg(i+1) = wg^5(i)$	MMBDD, FMBSD	Iroquois and Dravidian
$wg(i+1) = wt^5(i)$	MMBDD, FMBSD	Iroquois and Dravidian
$wg(i+1) = wg^2(i)$	no first or second cousin	
$wg(i+1) = wg^3(i)$	FFZDD	Iroquois and Dravidian

Table 2.4 (*continued*).

Generating structure	Period	Exchange cycles		
		Wife-takers	Male ego	Wife-givers
W(a, 13, 4)	6	II, XII, XIII, IV, VII, VI	=> III =>	IV, VII, VI, II, XII, XIII
W(a, 13, 5)	4	II, XI, IV, VIII	=> III =>	IV, VIII, II, XI
W(a, 13, 6)	12	II, X, VI, VIII, VII, I, IV, IX, VIII, XI, XII, V	=> III =>	IV, IX, VIII, XI, XII, V, II, X, VI, VIII, VII, I
W(a, 13, 7)	12	II, IX, VI, XI, VII, V, IV, X, XIII, VIII, XII, I	=> III =>	IV, X, VIII, VIII, XII, I, II, IX, VI, XI, VII, V
W(a, 13, 8)	4	II, VIII, IV, XI	=> III =>	IV, XI, II, VIII
W(a, 13, 9)	3	II, VII, XIII	=> III =>	IV, XII, VI
W(a, 13, 10)	6	II, VI, VII, IV, XIII, XII	=> III =>	IV, XIII, XII, II, VI, VII
W(a, 13, 11)	12	II, V, XII, XI, XIII, IX, IV, I, VII, VIII, VI, X	=> III =>	IV, I, VII, VIII, VI, X, II, V, XII, XI, XIII, IX
W(a, 14, 3)	6	II, XIV, VIII, IV, VI, XII	=> III =>	IV, VI, XII, II, XIV, VIII
W(a, 14, 5)	6	II, XII, VI, IV, VIII, XIV	=> III =>	IV, VIII, XIV, II, XII, VI
W(a, 14, 9)	3	II, VIII, VI	=> III =>	IV, XII, XIV
W(a, 14, 11)	3	II, VI, VIII	=> III =>	IV, XIV, XII

Exchange rule	Spouse merged with	Cross/parallel compatibility
$wg(i+1) = wg^4(i)$	MFZSD	Iroquois and Dravidian
$wg(i+1) = wg^5(i)$	FFZSD, MFZDD	Iroquois and Dravidian
$wg(i+1) = wg^6(i)$	no first or second cousins	Iroquois
$wg(i+1) = wt^6(i)$	no first or second cousins	Iroquois
$wg(i+1) = wt^5(i)$	FFZSD, MFZDD	Iroquois and Dravidian
$wg(i+1) = wt^4(i)$	FFZDD	Iroquois and Dravidian
$wg(i+1) = wt^3(i)$	MFZSD	Iroquois and Dravidian
$wg(i+1) = wt^2(i)$	no first or second cousins	
$wg(i+1) = wg^3(i)$	no first or second cousins	Iroquois and Dravidian
$wg(i+1) = wg^5(i)$	no first or second cousins	Iroquois and Dravidian
$wg(i+1) = wt^5(i)$	no first or second cousins	Iroquois and Dravidian
$wg(i+1) = wt^3(i)$	no first or second cousins	Iroquois and Dravidian

matrilateral cross-cousin marriage, none of the kinship structures is strictly time-invariant. This follows, of course, from the recursive definition of exchange cycles as a function of the alliances occurring in the preceding generation or cycle. Exchange cycles are invariant in the sense that they all belong to the group of *automorphisms* of the initial cycle $w_0 = c$ of $W(a, n, k)$.⁹

If one wishes to interpret this in terms of intergroup alliances, then male ego and his successors (i.e., men in succeeding generations of the same descent line) marry into different lines, with the same alliance cycle recurring only after a finite number of generations. However, as the structures are homogeneous, in terms of marriage rules all men of ego's descent line contract marriages of the same type. In structures with consecutive symmetry bride-givers become bride-takers (and vice versa) at some stage before the initial alliance cycle repeats. Structures with continuous exchange, in which the flow of exchanges proceeds in roughly the same direction (relative to the previous cycle), may be glossed as 'more invariant' when compared with discontinuous structures.

INTENDED APPLICATIONS AND EMPIRICAL CLAIMS

I must stress that the correspondence established above between my exchange model and the Courrège-Lorrain model of elementary kinship structures does not provide the only set of intended applications. Paraphrasing Sneed (1979:28), one might claim that the formal, mathematical core of the theory - characterized by the predicate '*is a structure of generalized exchange $W(a, n, k)$* ' - applies to a particular cultural system or domain, or to all cultural systems of a certain kind. The individual domains of different applications may overlap.

For example, the successor mapping s might designate not only matrilineal or patrilineal descent, but, say,

some order of precedence holding between siblings. If marriage possibilities are a function of birth order, the type of marriage contracted by the first brother (or sister) could determine (either absolutely or recursively) the marriages of succeeding siblings. Under such an interpretation of the exchange model, the elements of the partition *Gen* of the set *Obj* would correspond not to separate generations in a kinship network, but to consecutive exchange cycles *within* each generation. The co-existence of such multiple marriage models with cousin exchange rules based on birth order of siblings has recently been reported for the Beng in Ivory Coast (Gottlieb 1986). The formal model is a possible framework for the systematic analysis of birth-order dependent marriage strategies, a theme neglected in alliance studies. Other possible applications of the exchange model include the *kula* and other Melanesian systems (cf. Damon 1980), and the basic structure of many of the gift reproduction systems described by Gregory (1982).

Adopting the Suppes-Sneed-Stegmüller 'non-statement' or 'structuralist' programme summarized in Chapter 1, a *theory-element* $T = \langle K, I \rangle$ is the smallest unit which can be used to formulate empirical claims. $K = \langle M_p, M, M_{pp}, C \rangle$ is a *theory-element core*, with M_p the class of potential models, M the class of models, M_{pp} the class of partial potential models, and C the class of constraints. On this view, the 'conceptual apparatus' K of the theory-element consists of certain categories of set-theoretic structures that are used to say something about some array of 'things', the class of *intended applications* I (Sneed 1984:96). In general, I is understood to be a pragmatically determined, open set, grounded in a subset I_0 of a theory-element's paradigmatically successful applications.¹⁰

Following the modifications introduced by Balzer (1983), partial potential models are considered as arbitrary 'substructures' of a theory-element's potential

models, and I is a subset of M_{pp} .¹¹ Thus (Balzer 1983: 10-11):

... intended applications are regarded as substructures of potential models, that is, as 'real systems partly exhibiting T 's concepts'. The idea is to take into account only those parts of the real systems that are known, i.e. actually observed, identified and measured. Other 'parts' which eventually could be determined but actually are not, do not occur in members of I . An intended application thus describes or captures the known 'data' about some system. The empirical claim we can formulate with some theory-element core K and a corresponding set $I \subseteq M_{pp}$ of intended applications then simply says that the substructures in I are in fact 'parts' (i.e. substructures) of proper models, and that these models satisfy the constraints. ... Informally, the empirical claim of T reads as follows: *There exists a set X of extensions of intended applications so that X is a set of models and also satisfies the constraints.*

Kinship data collected by anthropologists include a variety of indigenous models. Such 'home-made' or 'local' models are often selective or partial constructions, embodying only those structural principles generally recognized or deemed important by the participants. The anthropologist, interpolating certain theoretical assumptions and constraints, attempts to formulate a more general series of explanatory constructs. Taking some specific cultural system as an intended application of a formalized kinship theory, the fragmentary or selective information represented by participants' models is pooled with other kinds of data and the set of partial structural relations determined. One is then able to make an empirical claim by positing the existence of suitable extensions to such partial data structures so that they then constitute a set of proper models which also satisfy the constraints of the formalized kinship theory.

From the non-statement or structuralist perspective, the selection of relevant kinship data is particularly straightforward: *none* of the various types of data traditionally identified by anthropologists (cf. Kuper

1980:24-26) are specifically excluded, at least in the first instance. Broadly speaking, the only important restriction is that the type of data one uses should be directly amenable to the formulation of partial models as set-theoretic structures.

The inherent flexibility of the non-statement approach is illustrated by the following example. Consider the exchange structure $W(a, 7, 2)$. The standard kinship representation and the associated reduced structure are given in figure 2.9. As a proper model it describes a homogeneous, global structure with seven patriline and alliances repeated in every third generation. Marriage is with male ego's FFZDD (merged with his FEMBSSD) and the structure does not exhibit consecutive symmetry. Hence the three wife-giving lines (FFM, MFF, FMF) are distinct from the three wife-taking lines (MMF, FMM, MFF) and from ego's own patriline (FFF, merged with MMM's patriline). The recursive exchange rule is $wg(i+1) = wg^2(i)$, i.e. ego marries into his MBW's line (his father's WBW's line). G^0 kintypes are neither Dravidian nor Iroquois compatible (as defined previously).

Interestingly, isomorphic versions of this structure have been put forward by several anthropologists as the most consistent and parsimonious scheme for organizing entirely different sets of data from distinct cultural systems. $W(a, 7, 2)$ is the model proposed by Kuper (1982, 1983, 1987) for the Tsonga, a Southern Bantu people. To begin with, the general cultural formula for all Southern Bantu cultures is summarized: (a) women must farm, men ideally keep cattle. (b) To marry a wife a man must pay bridewealth (*lobolo*) in cattle. Kuper then argues that the general exchange formula (wives for cattle) is systematically adjusted and transformed in the context of distinct local conditions. The Tsonga analysis hinges on the articulation of the *lobolo* exchange system with the well-documented data on

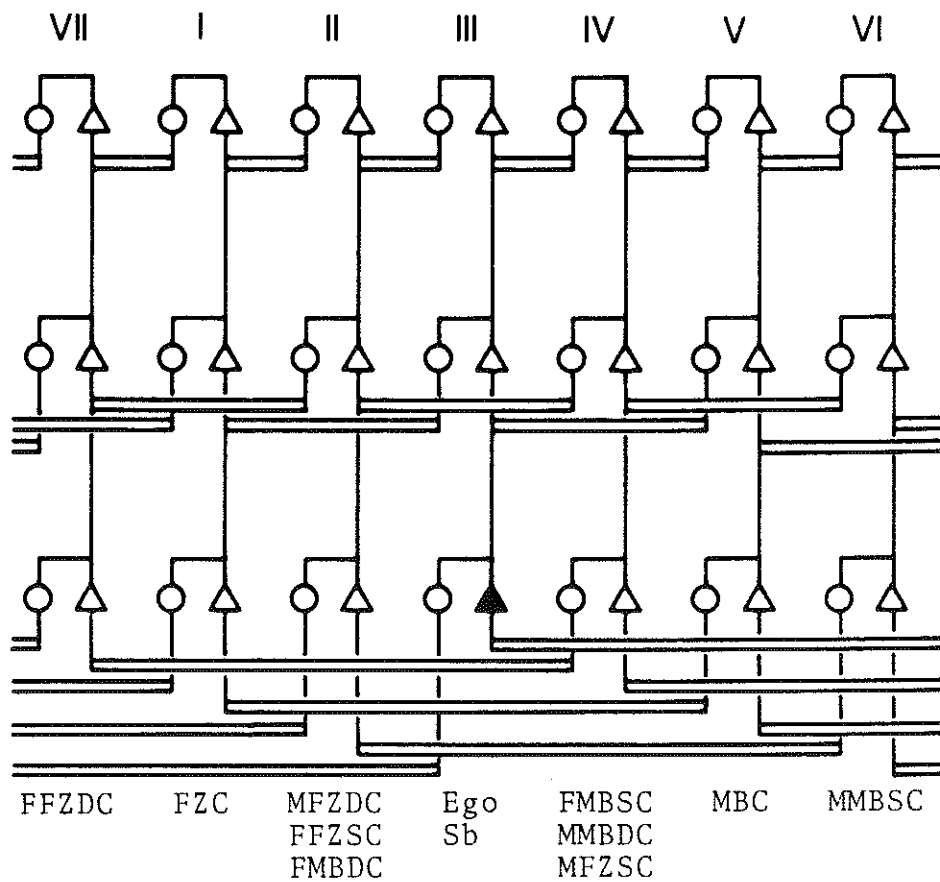
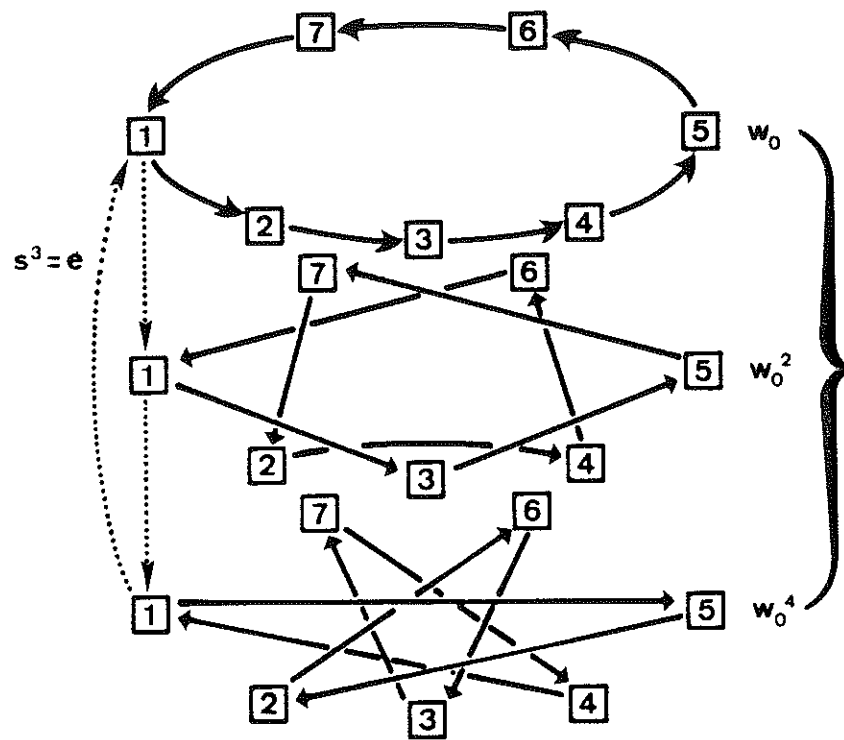


Fig. 2.9 (opposite). Reduced structure and kinship structure associated with $W(a, 7, 2)$.

joking and avoidance relationships. In sum (Kuper 1987:130, 131):

... the *lobolo* system dominates Tsonga social relations. In this 'Omaha' system, it is the determining institution in the field of marriage. The rule which underlies the institution is that *lobolo* brings a wife; but there is an ambiguity at its heart. Who provides the *lobolo*: a man or his sister? Who receives it: a woman's brother or this brother's wife's brother? ... The WBW represents the ultimate surety for a man's marriage. Should his own marriage fail, then unless he is given a substitute wife, or recovers his bridewealth from his brother-in-law, he may in the last instance claim his brother-in-law's wife. Conversely, when a woman's husband dies she may be passed on to the ultimate *lobolo* provider, her husband's sister, and disposed of to her son. ... The logic of the *lobolo* system therefore explains both the avoidance of the WBW and the joking relationship with the MB and MBW. The rule says that a *lobolo* payment brings a wife: but the rule is ambiguous, for the 'same' payment may marry several wives, or have several sources. The contradiction is crystallized in the relationships with the MBW and the WBW, and it is resolved (in the way that Radcliffe-Brown explained) by avoidance or joking.

Although fragmentary, the evidence on marriage preferences and prohibitions tends to support the existence of preferential marriage with a woman of the line into which a man's MB, MMB, MMBB, and his FFF (but not his F or FF) married (Kuper 1982:121). The seven-line structure $W(a, 7, 2)$ with exchange rule $wg(i+1) = wg^2(i)$ is indeed the simplest proper model (cf. table 2.4) to which the partial structures in the Tsonga data can be extended.

Lounsbury's modern classic, 'A semantic analysis of the Pawnee kinship usage' (1956), provides the second example of partial data structures extended to the proper model $W(a, 7, 2)$. In contrast with the mixed collection of data underlying Kuper's Tsonga model, Lounsbury's basic information is taken from one lexical domain: the Republican Pawnee kinship terminology originally published by Lewis Henry Morgan in 1871.

The analysis is primarily concerned with the semantics of reference. Lounsbury's goal is to provide an adequate componential description of the kinship data in terms of distinctive features, i.e. to derive the underlying semantic structure of the Pawnee terminology set.

The Pawnee terminology system exhibits Crow-type skewing, with (among other diagnostic features) male ego's patrilineal cross-cousins classified upwards as 'father' and 'mother', and matrilineal cross-cousins downwards as 'children' (Lounsbury 1956:164-165). The range of meaning of each term is given by listing all kin types known to be included as referents. Lounsbury then proposes five principal dimensions, the categories of which suffice for a componential definition of the structure of the Pawnee terminology. The dimensions are: (1) *Sex of ego*, with categories MALE vs. FEMALE. (2) *Sex of kinsman*, with MALE vs. FEMALE, or relative to ego, as SAME vs. OPPOSITE SEX. (3) *Generation*, with the usual categories G^n in ascending, zero, and descending generations. (4) *Agnatic generation*, with categories A^n , i.e. agnatic kinsmen of the n th generation. (5) *Uterine generation*, with categories U^+ , U^0 , U^- , together with their totality U , i.e. uterine kinsmen of ascending generations, own generation, descending generations, and the entire class of uterine kin (Lounsbury 1956:170-171).¹²

However, the analysis is still not complete. There are a number of curious three-generation cycles in the data commented on by Morgan. Thus (Lounsbury 1956:173):

Morgan describes with some explicitness how one calls one's father 'father', and his father in turn 'grandfather', and the latter's father 'uncle' (same term as for the mother's brother), and his father 'father' again, and so on indefinitely in a three-generation cycle. Similarly he describes a three-generation cycle in descending generations, going 'son', 'grandson', 'nephew' (same as sister's son), 'son', etc. indefinitely.

In fact, simple componential definitions of the terminological classes can only be obtained by positing additional equivalences: $A^{n+3} = A^n U = U A^{n-3}$ (where n is a positive integer or zero). However reluctantly, Lounsbury is obliged to explain these necessary special identities at the level of the semantic structure of the Pawnee terminology by a hypothesis regarding social behaviour. His solution is to postulate the existence of a rule prescribing marriage with a second cousin: ego's wife is equivalent to his FFZDD (1956:181-184). Lounsbury's conclusion (mentioned in a brief note) is significant: 'It is obvious that if such a rule were consistently in operation, the society could consist of seven and only seven lineages. Perhaps it is not fortuitous that the Cherokee have seven sibs' (1956:182). Here again the seven-line exchange structure $W(a, 7, 2)$ of figure 2.9 is the simplest proper model encompassing the underlying semantic structure of the Pawnee terminology, the three-generation cycling, and the second-cousin marriage rule.¹³

My final example is taken from Zuidema's (1965) analysis of the structural similarities of specific social systems from North and South America. Referring to Lounsbury's 1956 article as his starting point, Zuidema proposes a 'theoretical scheme' in which seven groups (lineages) are linked in asymmetric connubium. Marriage is with male ego's FFZDD and the scheme exhibits a three-generation cycle. Zuidema's 'scheme Ia and b' (1965:109) is clearly isomorphic to the structure in figure 2.9, i.e., the exchange structure $W(a, 7, 2)$ is a proper model for his American data. Significantly, Zuidema is at pains to show that his formal scheme is compatible with asymmetric exchange and matrilineal descent, patrilineal descent, or parallel descent - thus extending the classic models of his Leiden predecessors to American connubial systems.¹⁴

In all three of the above examples (the Tsonga, the

Pawnee, other North and South American systems) the anthropological data are interpreted as partial models, i.e., as set-theoretic structures belonging to the theory's class of intended applications. The specific empirical claim is then: *There exists* a set X of extensions of these intended applications so that X is a set of proper models (isomorphic to $W(a, 7, 2)$) satisfying the constraints of the formal kinship theory.

MARRIAGE PROHIBITIONS AND THE LIMITS OF GENERALIZED EXCHANGE

In Lévi-Strauss's theory of kinship, the various realisations of 'elementary' kinship structures are based on the direct or generalized exchange of spouses, with alliances between exogamous units coded in terms of preferential first cross-cousin marriage rules. The main thrust of my argument has been to demonstrate the existence of an entire family of generalized exchange structures compatible with assorted second and third cross-cousin marriage formulae, as well as with 'dispersed alliance' and exogamy rules coded as marriage prohibitions with the 'clan' of one's grandparents or great-grandparents. 'Elementary' kinship structures are derived as special cases. This line of research is similar to earlier theoretical work by Ackerman (1976; see Tjon Sie Fat 1981:389-390). It complements the important case studies presented by Kuper (1978, 1982, 1987) for Southern African systems and by Héritier (1974, 1975, 1981) for the Samo of Upper Volta.¹⁵ In the remaining sections of this chapter I demonstrate how my model may be applied to recent analyses of systems with marriage prohibitions, yielding an encompassing 'limiting' structure of regular alliances.

The first cluster of phenomena relates to particular indigenous conceptions of reproduction, focussing on

native theories regarding the transmission and sharing of 'blood', 'substance', 'image', etc. which account for and set bounds to the inheritance of characteristics and the articulation of marriage prohibitions. In many cases transmission is from father to son and from mother to daughter, with distinct 'substances' shared or encoded along lines of parallel descent.

Du Boulay's recent publications on Greek kinship (based on extensive fieldwork in North Euboea) open a fresh perspective on the symbolic relationships between descent, marriage, incest prohibitions and spiritual kinship (1982, 1984). The central, pervasive idea is that of shared 'blood'. Thus (du Boulay 1984:535):

... blood is thought to be inherited equally from both parents, with both blood-lines being equally merged in the offspring. ... The phrases 'They are blood' (*eínai aíma*) or 'They are not blood' (*dhén eínai aíma*), are common ways of saying that people are or are not in the same kindred, may or may not marry. 'Blood' in this context is used in a sense indistinguishable from that which is understood by the word 'kindred' (*sóí*) - a word which also means 'stock' and which is used ... in respect not only of the kin group but also of plants and animals. The proscription on marriages within the kindred is, then, identical with a proscription on the intermingling of the same blood, and this understanding finds succinct expression in the statement, 'Blood that resembles itself should not be joined' ... and its corollary, 'They are not blood: they can marry' Thus the concepts of 'one blood', 'kin', 'not being available in marriage', and certain degrees of relationship such as ... those up to and including second cousin, are all synonymous with one another; while 'not blood' (or 'strange blood', *xéno aíma*), 'not kin' (or 'strangers', *xénoi*), 'permission to marry', and certain degrees of relationship such as all those beyond second cousin, form the reverse images.

Ideally, 'blood' is seen as a unidirectional vital force, enduring and allowing the inheritance of characteristics up to the seventh *zinári* (turn of the belt; essentially a metaphor for father/son and mother/daughter links). Marriage however, is only strictly prohibited for three *zinária*: a man may marry his FFFFZDDDD, a fourth cousin. Thus, at the point at which marriage is permitted, 'the

blood changes'; in those cases in which a marriage then actually occurs, 'the circle closes' (du Boulay 1984:542). Du Boulay's figure 5 (1984:544), illustrating the marriage prohibitions and the completed circle is identical to my figure 2.10 (top, left) for p equal to 4 (p is the number of father/son and mother/daughter links, reckoned from a common ancestor). Du Boulay is at pains to stress the cyclic symbolism inherent in the unidirectional pattern of movement coming full circle (as opposed to a functioning system of actual marriage cycles). Thus (du Boulay 1984:342):

... in Greece, where no evidence has emerged of well-defined wife-giving and wife-taking groups, and where descent groups as such have no more than a shadowy existence in a certain patrilineal bias reflected in naming, residence patterns etc., such a 'circle' of marriages would seem at best to be feasible only in a very weak sense, and would appear simply to be a way of conceiving the 'outward' movement generated by the rule of exogamy being permitted to 'return' after a prescribed number of generations, when two suitably different descendants of the same original set of siblings are married, and 'the circle is closed'.

I claim that the type of data exemplified by du Boulay's analysis can be extended to a proper exchange model of the type $W(a, n, k)$.

The image of a consistent and irreversible movement of both sexes 'outwards' in opposite directions, coming full circle and joining in marriage only after a prescribed number of generations is of course also compatible with the classic analyses of asymmetric connubium, matrilinear first cross-cousin marriage, and double-unilinear descent. This is the model developed in detail in P.E. de Josselin de Jong's Minangkabau monograph (1980; first publication 1951. See the discussion in Chapter 1.). Given a system with four matri-clans (cross-cut by four patri-clans) taking part in asymmetrical connubium, the identical combination of matri- and patri-clans is reconstituted after four generations and four linked marriage exchanges,

providing for the periodical extinction of exogamy. For the Minangkabau, 'The woman of the fifth generation equals the ancestress of five generations back, and only she may therefore assume the rôle of becoming an ancestress of a new independent unit, a *parui*' (P.E. de Josselin de Jong 1980:89). In genealogical terms, for the Minangkabau model ego's wife (his MBD) is also equivalent to his FFFZDDD. This situation is represented in figure 2.10 (top), for n equal to 4 (and p equal to 3). (The Minangkabau model is the converse of figure 2.10 (top)).

A similar model has also been described for societies proscribing matrilateral first cross-cousin marriage. For example, the Yafar of the Western Sepik prohibit all forms of first and second-cousin marriage. The ideal form of alliance is formulated as an asymmetrical exchange cycle linking four patriline (cross-cut by a line of women) in which ego marries his *taway* (his FFFZDDD; Juillerat 1981. See his figure 6.).

On Malo Island in the New Hebrides, the indigenous conceptualization of social structure is formulated in terms of lines of men 'transacted by men', complemented by a line of women 'born of women'. In the Malo theory of consanguinity a father and his children share 'blood'; a woman and her children do not. Reckoned from an initial sister/brother sibling pair, the line of women is called a *raca* (root) or *raca imbaeca* (breadfruit tree root). In the fourth generation, the preferred marriage is for the male descendant of a line of men to marry the woman descended from a line of women (the man's FFFZDDD), described as 'taking back your breadfruit tree root'. 'At this point the notion of incest, closeness or impurity in sexual relations between the male and female lines ends' (Rubinstein 1981:328). Rubinstein's figure 16 is again identical to my figure 2.10 (top, right).

An identical model is developed in Héritier's reanalysis of Zuidema's Inca material. Here the locus of the structure is the *panaca*, a group of 'brothers' and

'sisters'. Consanguinity is reckoned by a rule of parallel descent from a common ancestor and the male and female lines merge after four generations, ego marrying his FFFZDDD (Héritier 1981:138-146; cf. her figures 46, 47, 48, 50).

I now formalize my claim. Let p denote the number of parent/child links, reckoned from an initial sibling pair. For any integer $p > 0$, the structures of figure 2.10 (top) can be glossed, without loss of information, as representing a positive marriage rule with a man's $F^p Z D^p$. I.e., for $p = 1, 2, 3, \dots$ ego marries his FZD, his FFZDD, his FFFZDDD, etc. Under the assumptions of the formal model $W(a, n, k)$ set out earlier, these genealogical chains may be unambiguously translated into chains generated by the mappings w_x and s^y . Then for some p the equation $W = F^p Z D^p$ is equivalent to

$$w_p = s^{-p}(w_0^{-1}s)(w_1^{-1}s) \dots (w_{p-1}^{-1}s).$$

And since $w_x = c^{k^x}$, this is equivalent to solving the

$$\text{congruence} \quad \sum_{i=0}^p k^i \equiv 0 \pmod{n} \quad (1)$$

for some $p > 0$, k and n coprime. The solutions to (1) define a set of proper models, limiting structures for the partial models of figure 2.10 (top).

Before solving congruence (1) I introduce two other, related series of partial structures that figure in the recent literature.

Bowden's analysis of the Kwoma of Papua New Guinea (1983) is a paradigmatic example. According to Bowden, the Kwoma (like most other Papua New Guinea societies) do not formulate positive marriage rules, and do not conceptualize marriages, symmetrical or asymmetrical, as taking place between relatively stable wife-giving and wife-taking groups. Affinal ties are in fact widely dispersed, rather than concentrated, between groups. However, a modified 'alliance' approach leads to

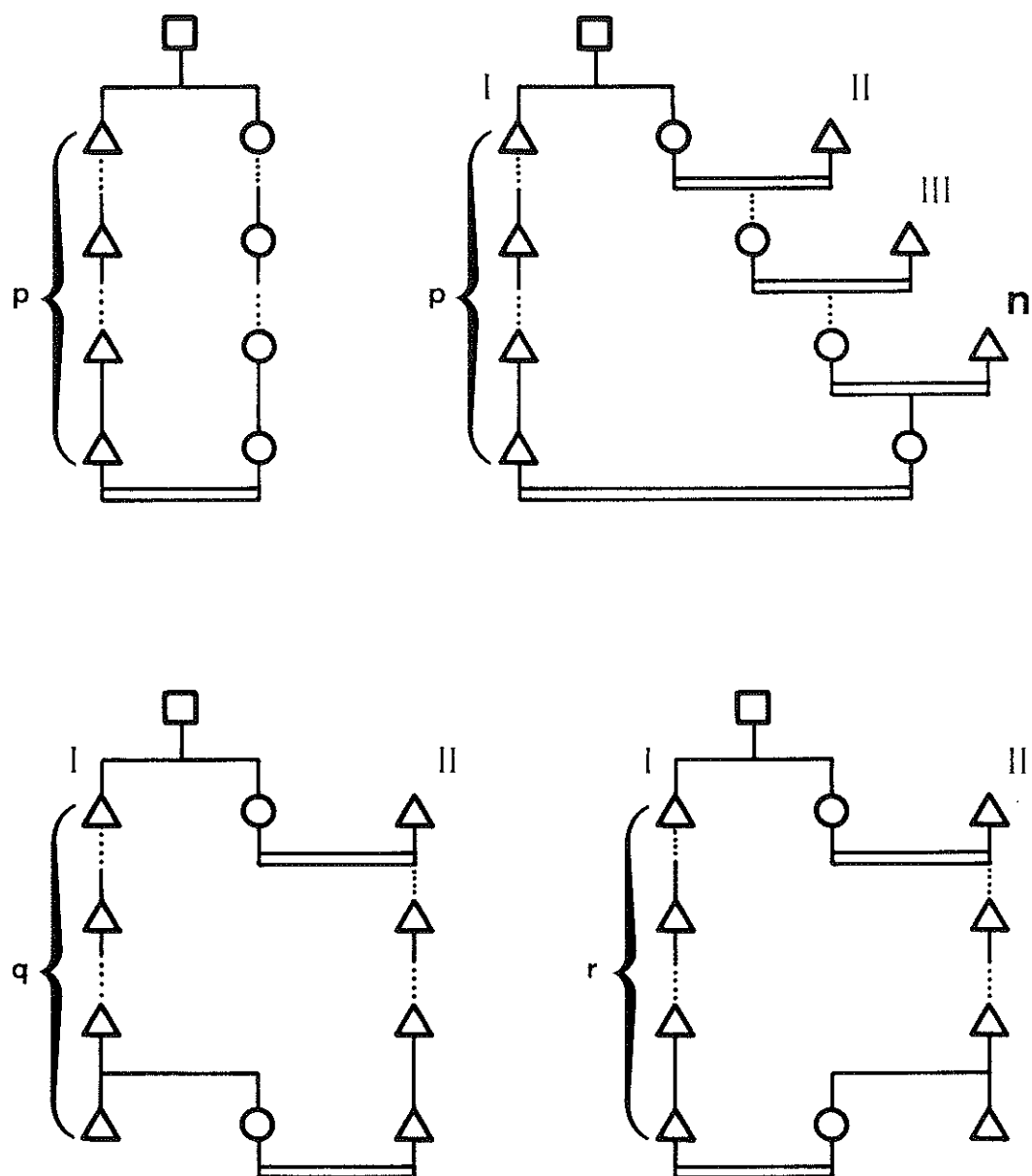


Fig. 2.10. Partial models representing marriage with the $F^p Z D^p$ (top), the $F^{q-1} M B S^{q-1} D$ (bottom, left), and with the $F^r Z S^{r-1} D$ (bottom, right). This last structure exhibits consecutive symmetry.

fruitful results if one focusses on individual Kwoma marriages and the enduring series of exchanges between *patriline*s, not descent groups or clans (1983:748).

Thus (Bowden 1983:749, 750):

All fertile marriages establish asymmetrical exchange relationships, and wider social and political alliances, between the male members of wife-giving and wife-taking *patriline*s that endure for up to four generations In all four generations the alliance is underpinned and maintained by an asymmetrical exchange of food and wealth-objects - food going to members of the wife-taking line, and wealth to the wife-giving line - and a symmetrical exchange of various domestic, social and political services. For the duration of an alliance, furthermore, no additional marriages may take place between the same two lines. This entails, for male ego, marriage that is prohibited with a member of WB's (and BWB's), MB's, FMB's and FFMB's lines, as well as with the husband's sister, daughter, son's daughter, son's son's daughter and son's son's son's daughter of same and ascending generation female members of own line

The Kwoma have an 'Omaha-type' terminology, with extensive cross-generational and lateral mergings. Bowden argues in most convincing detail that the series of enduring exchange relationships established between men of two marriage-linked *patriline*s and extending on down the lines to their male descendants is correlated with, and seen to be expressed by, the cross-generational mergings in the terminology (1983:750-759). Finally (Bowden 1983:756):

Following the death of third descending generation male members of the original wife-taking line ..., all formal and informal exchange relationships between the two lines come to an end. At this level they are now said to be 'unrelated' (*akiira ma*), and fourth (and lower) descending generation members of the wife-taking line employ no relationship terms for surviving members of the wife-giving line (and vice versa). It is only at this level, moreover, that a further marriage may take place between the two lines. Men say that such a marriage should ideally take place in the opposite direction from the original marriage, to 'balance' the exchange of women between the two lines.

Bowden's figure 1 (1983:751), summarizing his analysis, is identical to my figure 2.10 (bottom, right) for *r* (the

number of father/son links reckoned from the initial marriage exchange) equal to 4. (In my figure the terminology has of course been omitted.) In terms of my model, the Kwoma exhibit a system of consecutive symmetry linking patriline. Again, this partial structure can be extended to a full model of type $W(a, n, k)$.

Let r denote the number of father/son links, reckoned from an initial marriage linking two patriline. For any $r > 0$, the partial exchange structure of figure 2.10 (bottom, right) can be described as representing a positive marriage rule with a man marrying his $F^r Z S^{r-1} D$. I.e., for $r = 1, 2, 3, \dots$ ego marries his FZD, his FFZSD, his FFFZSSD, etc. Translating the kintype equation $W = F^r Z S^{r-1} D$ into its algebraic equivalent leads to the following congruence

$$(k^r + 1) \equiv 0 \pmod{n}, \quad (2)$$

to be solved for some $r > 0$, k and n coprime. Any such solutions then define the simplest proper models compatible with the partial structure of consecutive asymmetrical exchange and the extinction of exogamy in r generations.

The final partial exchange structure with limiting prohibitions on marriage is exemplified by Visser's (1984) Sahu analysis.

The Sahu conception of consanguinity (*ngaunu re ma la'eme*, of one blood and one flesh) is expressed bilaterally. Both parents contribute blood and flesh to their offspring, and to all further descendants up to and including their great-grandchildren. Marriage within the category of shared blood and flesh is forbidden; this proscribed category includes male ego's first cousins (*gia-bi'danga*, 'sisters'), and his second cousins (*gia-ngowa'a*, 'children'). By extension, ego also shares blood and flesh with all persons situated at

his generation level and tracing descent from at least one of his eight great-grandparents. Relations between affines-of-affines, in particular between WBW and HZH, are also rigorously proscribed.

In Sahu the basic social, ritual, residential and *adat* unit is the *fam*, with membership ideally circumscribed by the criterion of patrilineal descent reckoning. However, in-marrying women as well as cognatic kin of the core members may be adopted into the *fam*. According to Visser, the most important characteristic of the *fam* grouping is the right to transfer title to land on to male members of following generations (1984:124-127).

Hence, in Visser's model (1984:179-181), if a man from *fam* A marries a woman from *fam* B he acquires, through his wife, rights to land of *fam* B which may be passed on to his male descendants. After this initial marriage, men of A are prohibited from marrying women from B for three consecutive generations by the rule of *sou ro'ange supu* (three times outside - a variant of dispersed alliance). By compelling one's sons, grandsons and great-grandsons (i.e., male descendants of the same blood and flesh) to marry out, further rights in other *fam*'s land are secured. Finally, in the fourth generation, a great-great-grandson may again marry a woman from B, who, as a fourth cousin (a FFFMBSSSD), is not a member of the proscribed blood and flesh category. This marriage formula - for which some preference is expressed - is called *ma-si-dibo ino*, 'bringing it back': the original claim to land belonging to *fam* B, following from the marriage of the great-great-grandfather, has now been renewed.

Thus, in opposition to the Kwoma model, the Sahu exchange strategy is strictly unidirectional and premised on the idea of renewal, not on consecutive symmetry and 'balanced' reciprocity. Visser's figure 9 (1984:180) is identical to my figure 2.10 (bottom, left)

for q , the number of father/son links reckoned from the initial marriage, equal to 4.

Generalizing for $q > 0$, the partial exchange structure of figure 2.10 (bottom, left) represents marriage with a man's $F^{q-1}MBS^{q-1}D$, i.e. for $q = 1, 2, 3, \dots$ ego marries his MBD, his FMBSD, his FFMBSSD, etc. Substituting the corresponding algebraic mappings in the equation $W = F^{q-1}MBS^{q-1}D$ then allows for the derivation of the congruence

$$(k^q - 1) \equiv 0 \pmod{n}, \quad (3)$$

to be solved for some $q > 0$, n and k coprime.

The following lemma is stated without proof.

Lemma 5; If there is a solution to the congruence (1) for some integer $p > 0$, then $q = p + 1 = p(k)$ is a solution to the congruence (3), with $p(k)$ the period of the exchange structure $W(a, n, k)$.

Lemma 5 lends itself to an intriguing interpretation. Partial structures of the type described in figure 2.10 (top) occur in societies recognizing double-unilinear descent principles (double descent, with matriline bisecting patriline or vice versa, or parallel descent systems with single-sex lines), but also in societies with linear principles operating in the context of a manifestly cognatic type of reckoning. (In accordance with the Leiden view I use 'descent' as referring to a society's recognition of a *principle* for the inter-generational classification of categories of persons, whether or not there are organized, corporate descent groups in that society. See P.E. de Josselin de Jong 1984:251; 1985:202-204.) According to lemma 5, under the constraints of my model such partial structures are formally compatible with and may be articulated as a structure of unidirectional marriage exchanges linking two patriline.¹⁶ In such a structure (cf. fig. 2.10 bottom, left) a rule on 'marrying out' ensures that the

original marriage is renewable only after a prescribed number of generations have passed.

The lemma is more specific: the periodic extinction of exogamy, conceptualized or encoded as the permitted, preferred, or prescribed merging of same-sex lines in p generations or after p linked marriages, is compatible with an englobing or limiting structure of generalized exchange with period $p + 1$.

The consequences of lemma 5 are clearly illustrated in Héritier's summary diagram of the Inca structure (1981:141): for $p = 3$, wife equals FFFZDDD, merging male and female lines. In terms of unidirectional alliances between patriline, the period of the exchange cycle is four, with a marriage in generation five duplicating the initial marriage.

For a particular combination of values of p , n and k , either type of partial structure may occur as possible realizations of the same exchange model $W(a, n, k)$. Alternatively, both types may be recognized by the participants as manifest properties of their social structure. This appears to be the case in Tongan kinship where a system of patrilineages bisected by matriline (Rogers 1977) is complemented by a unidirectional exchange structure in which 'elder brother' lines take wives from 'younger brother' lines (Biersack 1982). Also, it is perhaps not entirely fortuitous that a variant of du Boulay's Greek model for the cyclic progression of 'blood' closing the circle after a prescribed number of generations (see above) is combined in Corfu with a conception of marriages as alliances between two lineages (1984:542-543). Conversely, it may be worthwhile to search for some evidence of a native conception of marriage with the FFFZDDD in Visser's Sahu data. Even as a purely latent structural feature of the Sahu model, such a partial structure should be noted for its possible wider significance: it also appears in the classical models with double descent and

matrilateral cross-cousin marriage (cf. P.E. de Josselin de Jong 1980, Van Wouden 1968). Lemma 5 points out the existence of structural invariants, features common to both 'elementary' models and 'more complex' structures of exchange.

I now provide a list of solutions to congruences (1), (2) and (3), i.e. extensions to proper models $W(a, n, k)$. Unless specified otherwise, all solutions are derived for $n < 15$ and for p, q and $r < 5$. For each class of solutions a genealogical gloss is given for the marriage rule. The authors cited either refer to the occurrence of the partial structures in their data or provide a proper exchange model.

Solutions to congruence (2): structures with consecutive symmetry. For $r = 1, 3, 5, 7, \dots$ the general solution is $W(a, n, n-1)$, the elementary structures with n lines and $W = \text{FZD}$. For $r = 2$ and $W = \text{FFZSD}$; partial structure described by Lipuma (1983) for the Maring of Highland Papua New Guinea.¹⁷ Also described by Rosman and Rubel (1975) as an alternative, unilateral exchange structure for the Maring, the Manga and the Wogeo, all Highland societies: 'In the idiom of these three societies, this marriage preference is conceptualized as the return of a woman for one given two generations earlier' (1975:123). However, contrary to Rosman and Rubel's expectations, a proper model does not require a minimum of eight groups. Rubel and Rosman (1978:252-258) add the Kuma as a fourth example. The same partial structure is described by Zuidema (1965:111-113) for American Indian social systems. However, his global model with seven lines is not a homogeneous solution. Van Dijk and De Jonge have recently (1987:63) put forward the proper model $W(a, 5, 2)$ with $W = \text{FFZSD}$ for the people of Marsela Island, South-east Moluccas.

Proper solutions for $r = 2$ are $W(a, 5, 2)$ (marriage with FFZSD , MFZDD and FMBDD), $W(a, 5, 3)$ (marriage with FFZSD , MFZDD and MMBSD), $W(a, 10, 3)$, $W(a, 10, 7)$,

$W(a, 13, 5)$ and $W(a, 13, 8)$, all with marriage to FFZSD and MFZDD.

Solutions for $r = 3$ and $W = \text{FFFZSSD}$. Described by Bowden (1988: fig. 5, 283-287) for the Daribi of Papua New Guinea. Proper models are $W(a, 7, 3)$, $W(a, 7, 5)$, $W(a, 13, 4)$ and $W(a, 13, 10)$ (with marriage to MFZSD); $W(a, 9, 2)$, $W(a, 9, 5)$, $W(a, 14, 3)$ and $W(a, 14, 5)$ (no first or second cousin marriage).

Solutions for $r = 4$ and $W = \text{FFFFZSSSD}$. Described by Bowden (1983) for the Kwoma of Papua New Guinea. The simplest proper model is $W(a, 17, 2)$. This 17-line model is fully compatible with a prohibition on marriage with first, second and third cousins (i.e., with any same generation descendant of ego's eight great-grandparents and their siblings). Ego and his sister marry fourth cousins: $W = \text{FFFFZSSSD}$, $H = \text{FFFMBSSES}$. Alternatively, the marriage rule $wg(i+1) = wg^2(i)$ may be conceptualized as $W = \text{MBWBD}$, $H = \text{FZHVS}$. (See Haenen 1988 on the Moi.)¹⁸

Solutions to congruences (1) and (3). For $q = 1, 2, 3, 4, \dots$ the general solution is $W(a, n, 1)$, the elementary structure with n lines and $W = \text{MBD}$, FMBS , MMBD , etc. For $p = 1$ and $q = 2$ the general solution is $W(a, n, n-1)$, the elementary structure with n lines and $W = \text{FZD}$, FMBS , MMBD (here r equals 1). For $p = 2$ and $q = 3$, $W(a, 3, 1)$ is the elementary structure with 3 lines, $W = \text{MBD}$, FFZDD , FMBS , MMBD and FFMBSSD . Described by Lounsbury (1978) for the Inca and by Clamagirand (1980:142) as the alternative three-partner *bei bei* marriage practiced by the Ema of Timor. For $p = 2$, $q = 3$ and $W = \text{FFZDD}$ and FFMBSSD but not MBD , proper models $W(a, 7, 2)$ have been proposed by Kuper (1982, 1987) for the Tsonga, and by Zuidema (1965:109) for American Indian systems. Lounsbury's (1956) Pawnee analysis extends a partial model of the terminology to a more encompassing proper model of the same type. The class of solutions includes $W(a, 7, 2)$, $W(a, 7, 4)$ and $W(a, 13, 3)$.

For $p = 3$, $q = 4$ and $W = \text{MBD}$, FMBS , MMBD , FFFZDDD

and FFFMBSSSD, $W(a, 4, 1)$ is the elementary structure with four lines. Described as a general model by Van Wouden (1968:91) and by P.E. de Josselin de Jong (1980) for the Minangkabau of Sumatra. (See Chapter 1.)

For $p = 3$, $q = 4$, $W = \text{FFFZDDD}$ and FFFMBSSSD but not MBD, partial structures have been described by H  ritier (1981:141) for the Inca and by Visser (1984) for the Sahu of North Halmahera.¹⁹ The simplest proper model prohibiting marriage with first and second cousins is $W(a, 15, 2)$.

For $p = 4$, $q = 5$, $W = \text{FFFFZDDDD}$ and FFFFMBSSSD but not MBD, a partial structure has been described by du Boulay for the Greek system (1984). The simplest proper model prohibiting marriage with first and second cousins is $W(a, 11, 5)$. If further prohibitions are required the structure must be extended to $W(a, 31, 2)$.

Finally, for $q = 2$ but $p \neq 1$, $W = \text{FMBS}$ D and MMBDD but not FZD or MBD. FMBSD-models have been described by Kuper (1978, 1982:94-107, 1987:122-124) for the Swazi, by Van Dijk and De Jonge (1987:64) for Marsela Island, and by Korn (1971) and Rubel and Rosman (1978:36) for the Iatmul. However, none of these analyses is strictly homogeneous. Turner's (1980:103) eight-line Ungarinyin model with $W = \text{FMBS}$ D, MMBDD is isomorphic to $W(a, 8, 5)$, extending Elkin's earlier (1964:81)²⁰ and partial Ungarinyin structure. A similar homogeneous eight-line model has recently been presented by Vuyk (1987:202) for the matrilineal Lele, extending L  vi-Strauss's (1973) partial model; De Heusch's earlier (1964:101) Lele model is not homogeneous. Blundell and Layton (1978) describe a 12-'clan' exchange model for certain West Kimberley societies (including the Worora, the Winawidjagu, the Wunambal and the Ungarinyin) with alternating generations and prescriptive FMBSD-marriage, each clan linked to two wife-giving and two wife-taking clans or clusters of clans. The class of proper models for $q = 2$ but $p \neq 1$ includes $W(a, 8, 3)$, $W(a, 8, 5)$,

$W(a, 12, 5)$ and $W(a, 12, 7)$ for $n < 15$. Either 12-line model appears to fit the West Kimberley data.

SUMMARY AND CONCLUSION

The essence of this chapter can be summarized in one short paragraph. Let C_n be the cyclic group generated by the permutation $c = (1, 2, 3, \dots, n)$ of order n . Let $\text{Aut}(C_n)$ denote the automorphism group of C_n . The order of $\text{Aut}(C_n)$ is equal to $\Phi(n)$. Hence for some n there are $\Phi(n)$ elements g which are one-to-one mappings of C_n onto C_n . For each such element g of $\text{Aut}(C_n)$, the permutation c is mapped onto another element of C_n of identical order n , i.e. $(c)g = c^k$ for some integer k that is coprime with n . (Table 2.1 gives the values of $\Phi(n)$ and the integers k coprime with n .) Then for each g in $\text{Aut}(C_n)$ and some k , $(c)g = c^k$, $((c)g)g = (c^k)^k$, etc. Let the order of g be denoted by $p(k)$, i.e., $g^{p(k)} = 1$ (the identity automorphism). Then the ordered p -tuple $W = (c, cg, cg^2, \dots, cg^{p-1})$, recursively defined, is interpreted as a closed structure of generalized exchange, repeating in $p = p(k)$ generations.

This approach, in which I define 'more complex' structures of generalized exchange by means of automorphisms of a basic exchange network will be applied in Chapter 4 to structures of direct (symmetric) exchange. In Chapter 5 I introduce a slightly different formal model, representing exchange structures as discrete dynamical systems.

A few brief remarks in conclusion: first, while retaining the basic idea of closed exchange cycles and the periodic repetition of alliance between descent lines, my models do not assume the necessary existence of corporate descent groups. They apply equally to participants' representations of their 'thought-of' orders. (Examples, however summary, have been provided above.) Second, while concerned with possible correspondences between terminologies, descent principles, and marriage rules and prohibitions, I have

not predicated my analysis on any particular theory of Crow-Omaha systems. My framework appears to be compatible with the major theories.²¹

One possibility for extending the scope of the present work lies in a comparison of the algebraic models with the type of statistical analysis undertaken by Héritier (1974, 1975, 1981) for her Samo material. Finally, if the family of models is seen as a range of possible social structures (encompassing the partial or selective constructs of the participants), a further detailed comparison of their possible transformations, contextualized within a suitable 'field of anthropological research', may lead to a more sophisticated and realistic solution to outstanding problems in kinship analysis.

APPENDIX

Proof of lemma 2: The order of any c^x is the least positive integer p such that $(c^x)^p = e$. Thus $(c^x)^p = c^x c^x \dots c^x$ (p factors) $= c^{px}$. The order of c is n , hence $px \equiv n \pmod{n}$. For all x and n coprime, $p = n$. For all other x (i.e., for all x not in K), a solution is obtained for $px = m$, the least common multiple of x and n . Hence $p = m|x$.

Proof of lemma 3: According to lemma 1, K is a group under multiplication (modulo n). Hence, for any k in K , $kk \dots k$ (i factors) $= k^i$ is also in K , i.e., k^i and n are coprime. $p(k)$ is the order of k , hence $k^{p(k)} = 1$. Now, for any n , 1 and $n-1$ are always coprime to n and thus in K . $1^{p(1)} \equiv 1 \pmod{n}$, hence $p(1) = 1$. Let $(n-1)^{p(n-1)} \equiv 1 \pmod{n}$. For $p(n-1) = 1$, $(n-1) \equiv 1 \pmod{n}$ only if n equals 1 or 2. For $p(n-1) = 2$, $(n-1)^2 \equiv n^2 - 2n + 1 \equiv 1 \pmod{n}$ for all n . Hence $p(n-1) = 2$ for $n > 2$. Each k in K generates a cyclic group of order

$p(k)$ under multiplication (modulo n).

Proof of lemma 4: The order of $TO(j)$ is equal to the number of distinct integers t to which $j + k^x$ is reduced modulo n . The period of $W(a, n, k)$ is $p(k)$, the order of k . Hence there are exactly $p(k)$ integers t such that $j + k^x \equiv t \pmod{n}$ for any j and therefore $|TO(j)| = p(k)$. The proof that $|GO(j)| = p(k)$ is entirely analogous.

NOTES

- 1 This chapter, completely rewritten for the present volume, develops and modifies the substance of an earlier paper 'More complex formulae of generalized exchange' in *Current Anthropology* (1981) 22(4):377-399.
- 2 The original French edition was published in 1949. Unless otherwise stated, I refer to the 1970 English translation of the second edition.
- 3 A summary of the general argument is given by J.P.B. de Josselin de Jong (1952). Scheffler (1970) provides a critical review of the revised edition.
- 4 Indeed, a five-line model with FZD-marriage would seem to be a plausible, homogeneous, solution to most of the problems in Korn's data.
- 5 A sketch of the proof of this and subsequent lemmas is given in the chapter appendix.
- 6 See Chapter 1. Courrège actually employs the Greek symbols ω , μ and π , and defines the composition of mappings in reverse order ($\pi = \mu\omega$).
- 7 For a definition see Lorrain (1975:49). He also gives a definition of a 'regular space' (1975:137).
- 8 A review of other cross/parallel distinctions (some of which coincide with Scheffler's criteria) is given in Buchler and Selby (1968:230-246). See also Kay (1967) and Héritier (1981).
- 9 This idea is indeed the the rationale for the formal model. See also the model of symmetric exchange in Chapter 4.
- 10 Stegmüller (1976:170-190) concludes that in order to determine I , pragmatic concepts analogous to Wittgenstein's 'family resemblances' must be used.
- 11 Balzer (1983) describes the structure of M_{pp} as a complete lattice. See Chapter 1.
- 12 A simplification of the componential definitions is achieved by introducing a fourth, derived dimension of *agnatic rank* (Lounsbury 1956:178) with categories $A^{n+1}UA^{-n}$, A^nUA^{-n} , A^nUA^{-n-1} .

- 13 A decade later, Lounsbury's position on the inclusion of marriage rules in formal semantic analyses appears to have hardened (Lounsbury 1965:185). But also compare his controversial 1978 analysis of the Inca terminology and marriage system! Scheffler completely rejects the inclusion of marriage rules (1971, 1972a, 1972b, 1978, 1982, 1984).
- 14 In the same paper Zuidema also develops other schemes based on 7-, 9-, and 13-partitions for the comparison of American social structures.
- 15 Prior to 1976 the formulation of simple homogeneous kinship models with generalized exchange and exclusive second or third cousin marriage rules is a neglected theme. Apart from the work of Lounsbury (1956), Zuidema (1965), Korn (1971) and Etienne (1975) already mentioned, there are important remarks by Elkin (1964:78-84; first published in 1938), P.E. de Josselin de Jong (1962), De Heusch (1964, 1974), Festinger (1970a,b) and Lévi-Strauss (1973). See also the paper by Ruhemann (1967).
- 16 An analogous formulation in terms of matriline is of course also possible.
- 17 See also the correspondence with Brown in *Man* (Brown 1985).
- 18 Haenen (1988) presents an extremely significant analysis of the system of marriage alliances among the Moi of Irian Jaya (Indonesia). The Moi have formulated a number of exchange rules, including marriage with the matrilinear cross cousin. There is also a rule for 'hauling back' a wife, reversing the circulation of women in the fourth descending generation (Haenen 1988:473-475). This rule corresponds to the formal principle of consecutive exchange for $r = 4$ (i.e., wife = FFFFZSSSD) described for the Kwoma. See in particular Haenen's figure 4 (1988:473). A full discussion of this important paper goes beyond the scope of this chapter.
- 19 A proper kinship model with *sister exchange*, $p = 3$, $q = 4$, $W = \text{FFFZDDD}$ and FFFMBSSTD is developed in Chapter 4. This is presented as a proper model for Héritier's Samo alliance structure.
- 20 First published in 1938.
- 21 Although not intended as a general theory of Crow-Omaha systems, Kettel's (1982) formulation of exchange rules of the type $wg(i+1) = wg^x(i)$, for x equal to 1, 2 or 3 parallels my research. His analysis of the Tugen, the Dan, Tetela and Tsonga is also compatible with the models developed in Kuper's more detailed studies of African systems (1982, 1987).

3. AGE METRICS AND TWISTED CYLINDERS: PREDICTIONS FROM A STRUCTURAL MODEL¹

A century has now gone by since the publication of Macfarlane's (1882) 'Analysis of Relationships of Consanguinity and Affinity'. Macfarlane was an examiner in mathematics at Edinburgh, not an anthropologist. Urged by Tylor to develop a systematic notation for expressing the exact degree of compound genealogical relationships, he proposed a solution analogous to the notational scheme of chemical analysis. Unfortunately, this first ingenious experiment in mathematical modelling was virtually ignored by anthropologists.²

A similar fate befell Greenberg's 1949 paper 'The Logical Analysis of Kinship', an attempt to provide a positivist axiomatic system for the analysis of kinship.³ That same year also saw the publication of another paper on the formal analysis of kinship: Weil's appendix to *The Elementary Structures of Kinship* (Lévi-Strauss 1970 [1949]).⁴ Weil's brief treatise, introducing the group-theoretic approach, was to acquire the status of a new paradigm, stimulating the development of an entire literature on the algebra of kinship.⁵

Evidence is now accumulating that challenges the anthropological assumptions underlying the standard group-theoretic models of kinship phenomena. Such models are apparently too conservative, consistently ignoring 'oblique' exchanges and the effects of age bias and systematic mean age differences on the connubial system. Hence the classic family of such models is essentially incomplete with respect to the range of actual kinship phenomena. Recent research underscores the difficulties involved in modelling age-biased kinship systems. Fieldwork among the Wanindiljaugwa (Rose 1960), the

Walbiri (Meggitt 1965), and the Alyawara (Denham, McDaniel, and Atkins 1979) in particular clearly demonstrates the incompatibility of large mean age differences between spouses with simple models of simultaneous sister exchange or bilateral cross-cousin marriage. The structural implications of systematic age disparities also were investigated in detail for matrilineal cross-cousin marriage. Computer simulations (Hammel and Hutchinson 1974; Hammel 1976) as well as statistical methods (Reid 1974) were applied. Martin (1981) extends the scope of these investigations by arguing that gender-dependent birth order, and hence systematic age differences between opposite-sex siblings, are additional factors delimiting the marriage possibilities.⁶

Since Schurtz (1902), anthropologists have traditionally focussed on the role of age in the context of 'age class' systems of social organization (for a survey of the literature I refer to Stewart (1977) and Bernardi (1985)). There is now heightened interest in the implications of age as a stimulus to the development of anthropological theory. Significantly, the recent collection of papers edited by Kertzer and Keith (1984) is titled *Age and Anthropological Theory*.

My purpose in this chapter is more limited. My aim is to extend the field of application of elementary kinship models, in particular those developed for generalized exchange. The problem is to devise a more comprehensive model of matrilineal cross-cousin marriage for systems with large systematic mean age differences. My solution is to merge a metrical structure similar to that originally proposed by Macfarlane (1882) with a Weil-type representation of kinship structure as a commutative group. This new model incorporates a finite set of open, helical marriage cycles and intergenerational age spirals instead of an infinite number of closed exchange circuits and discrete generations. I then derive a series of

helical kinship structures, many of which occur in discussions of age-biased kinship systems. My model is intended as a rigorous generalization of the helical scheme introduced by Denham et al. (1979) for the Alyawara, and earlier, by McConnel (1940, 1950, 1951) for the Wikmunkan and other Cape York societies.

THE PROBLEM

With few exceptions, anthropologists' 'mechanical' models of marriage and descent (including simple genealogical diagrams) embody variants of the following assumptions:

a. *Unity of the sibling group*, that is, same-sex siblings and parallel cousins are considered structurally equivalent. This principle is sometimes applied irrespective of gender, conceptually merging all siblings and parallel cousins.

b. *Marriage prescription*, that is, if persons of the same sex are structurally equivalent, then so are their spouses.

c. *Generational closure*, that is, there is an infinite or open series of genealogically defined generations, each of which is not only discrete but closed. Hence, in systems with generalized exchange, genealogical chains of the type WBWBWB ... or ZHZHZH ... may in principle cycle back to ego after a finite number of marriages.⁷

d. *Homogeneity of marriage type*, that is, within each generation all persons of the same sex contract the same type of primary marriage.

e. *Homogeneous partitioning of the kinship universe*, that is, individual positions in the genealogical network are merged according to principles a, b, and c into disjoint 'sibling groups', none of which occupy a privileged position. Alternatively, the entire society is partitioned into a nonoverlapping set of clans, lineages,

descent lines, local groups, sections, subsections, or marriage classes. Whatever the principle of division, all fundamental relationships among the basic units are defined exclusively in terms of marriage and descent. Homogeneity assumptions are often implicit in anthropologists' formulations.

In the new age-constrained models developed here I modify the principle of generational closure.⁸ To summarize the classic algebraic approach to kinship modelling, I consider Lorrain's (1975) adaptation of Courrège's work (1965) as the prototype. Lorrain reduces a genealogical network by means of the principles mentioned above to a basic set S of disjoint nodes ('sibling groups'). Three suitable formal mappings h , m , and f are then defined on the elements x of S . Hence, h , m , and f are one-to-one mappings of S onto itself, such that $(x)f = ((x)h)m = (x)hm$ for all x in S , and the inverse mappings h^{-1} , m^{-1} , and f^{-1} exist. Under the standard anthropological interpretation, h is the conjugal mapping linking a man to his wife, m is the matrilineal mapping linking a woman to her children, and f is the patrilineal mapping linking a man to his children. Marriage and descent are articulated through the equation $f = hm$. This implies that any man's children are identical with the children of the woman to whom he is married. Composite mappings are defined on S by taking different combinations of integral powers of h , m , and f . Each such composite mapping is interpreted as denoting a particular set of kintypes; conversely, any kintype can be denoted by some composite mapping (see Chapter 1).

If male ego is situated in node x of the reduced kin network, his siblings and parallel cousins are thus denoted by $(x)f^{-1}f = (x)m^{-1}m = (x)e$, the identity mapping e , and are consequently also in the same node as ego. Ego's matrilineal cross-cousins (MBC) are denoted by $(x)m^{-1}f$, his patrilineal cross-cousins (FZC) by $(x)f^{-1}m$, and his spouse and spouse's siblings by $(x)h$.

The mappings h , m , and f generate a mathematical group $G(h, m, f)$ not necessarily of finite order. Specific types of kinship structures are modelled by imposing additional constraints on the group $G(h, m, f)$. For instance, if MBD is the preferred or prescribed spouse, $(x)h$ should invariably equal $(x)m^{-1}f$. A fundamental theory of algebraic kinship states that a necessary and sufficient condition for a kinship structure to be compatible with matrilinear cross-cousin marriage is that the group $G(h, m, f)$ be *commutative* (see Chapter 1; a proof is provided by Lorrain 1975:141-142). This important theorem remains equally valid for my helical extension of generalized exchange structures.

Assumptions of a more statistical nature are employed in predicting the relative availability and the age distributions of spouses of a particular genealogical kintype. My prototype here is Reid's (1974) stochastic analysis of cross-cousin marriage, an extremely relevant application of techniques developed by geneticists and population demographers (Hajnal 1963; Cavalli-Sforza, Kimura, and Barrai 1966; Keyfitz 1977; Jagers 1982; Pison 1982). Suppose that it is in principle possible to determine the exact genealogical relationship between any two individuals in a real kinship system. Let j and i be the paired reciprocal kintypes denoting ego's relationship to alter and, conversely, alter's relationship to ego. Then define d_{ij} as the *mean age difference*⁹ for all such pairs of individuals denoted by the reciprocal kintypes i and j , with d_{ij} defined as the (mean age of persons denoted by i) - (mean age of persons denoted by j).

Under the additional assumption that relative ages computed along a genealogical path are mutually independent, the following properties hold for all mean age differences d_{ij} :

$$d_{ii} = 0 \quad d_{ij} = -d_{ji} \quad d_{ij} + d_{jk} = d_{ik}.$$

It follows, then, that mean age differences d_{BB} and d_{ZZ} computed for same-sex siblings always equal zero. Mean age differences $d_{BZ} = -d_{ZB}$ between opposite-sex siblings may of course depart significantly from zero, due to variations in the sex ratio with birth order (Cavalli-Sforza et al. 1966:47; Reid 1974:260; Martin 1981). I further delimit my model by assuming that it applies only to societies for which $d_{BZ} = -d_{ZB} = 0$. Hence the mean age difference between siblings, irrespective of gender, necessarily equals zero. This statistical requirement ensures that the average male age at a node, or 'sibling group', is the same as the mean age of females at that node (a node comprises siblings and their parallel cousins).

Let d_{HW} , d_{FC} , and d_{MC} denote respectively the mean age differences between husband and wife, father and child, and mother and child. If these three averages are known, one can always compute the mean age difference d_{ij} for any pair of reciprocal kintypes as a linear chain of these basic values. Note that $d_{FC} = d_{HW} + d_{MC}$. This is the statistical counterpart of the algebraic equation $f = hm$. For the mean age differences between cross-cousin kintypes, then,

$$d_{FZS\ MBD} = d_{CM} + d_{ZB} + d_{FC} = -d_{MC} + d_{FC} = d_{HW}, \text{ but}$$

$$d_{MBS\ FZD} = d_{CF} + d_{BZ} + d_{MC} = -d_{FC} + d_{MC} = -d_{HW}.$$

Therefore, in general the mean age difference between a man and his matrilateral cross-cousins is not equal to that between a man and his patrilateral cross-cousins. Age differences between all types of cross-cousin are equal if and only if $d_{FC} = d_{MC}$, that is, if and only if $d_{HW} = 0$. In real societies husbands are usually older than their wives. Quantitative research cited earlier has drawn our attention to a number of systems for which d_{HW} is significantly greater than zero. In all such societies the age difference constraints effectively

rule out the possibility of a model with sister exchange and simultaneous bilateral cross-cousin marriage, although the exchange of classificatory sisters or other female relatives may occur. Moreover, the classic models of matrilinear cross-cousin marriage and generalized exchange also become untenable. A value of d_{HW} greater than zero implies the existence of an endless age spiral in which

$$\dots < d_{ZH\ WB} < d_{ZH\ ZH\ WB\ WB} < d_{ZH\ ZH\ ZH\ WB\ WB\ WB} < \dots$$

Age spirals are obviously not compatible with the principle of generational closure, which assumes discrete generations and closed marriage cycles. (A similar argument can be made for systems with d_{HW} less than zero; cf. Hammel 1976.)

A model of MBD-marriage with $d_{HW} = 0$ is sketched in figure 3.1. For situations where $d_{HW} > 0$, my solution replaces the closed marriage cycles with open helices (fig. 3.2) while retaining the algebraic constraint of a commutative group structure. I model only mean age differences, which do not of course, tell the whole story. In Reid's (1974) analysis computation of standard deviations is essential, as is the assumption of a normal (Gaussian) distribution for age differences. However, given the present state of our knowledge, one cannot accept the assumption of normality as a general axiom.¹⁰ The consideration of standard deviations and higher moments of age distributions is necessary but must await the results of further detailed empirical research.

HELICAL MODELS

Analogous to the more complex formulae of exchange developed in the previous chapter, a family of simple metricized helical exchange structures is defined here.

The formal set of objects (interpreted as primary exchange units or sibling groups) is defined as nodes on the surface of a cylinder (cf. Lévi-Strauss 1970:xxxvii) and a basic set of mappings is introduced. Each node is indexed by means of a double subscript. The subscript i indicates the coordinate parallel to the main axis of the cylinder and will be used to map relative age and descent within patrilines; subscript j refers to sibling groups in distinct patriline of the model.

Nodes and mappings.

Let Z be the set of integers and let λ be the index set $\{1, 2, \dots, n\}$. Then N , the basic set of objects or nodes,

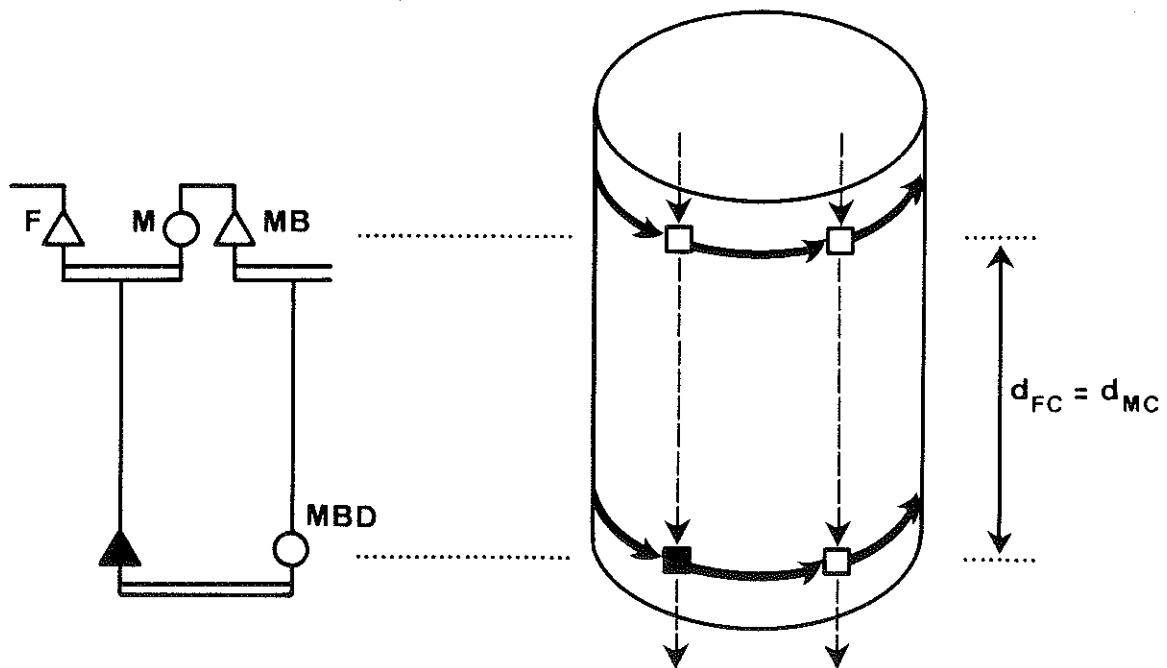


Fig. 3.1. Matrilineal cross-cousin marriage. Closed, cyclical model with $d_{HW} = 0$.

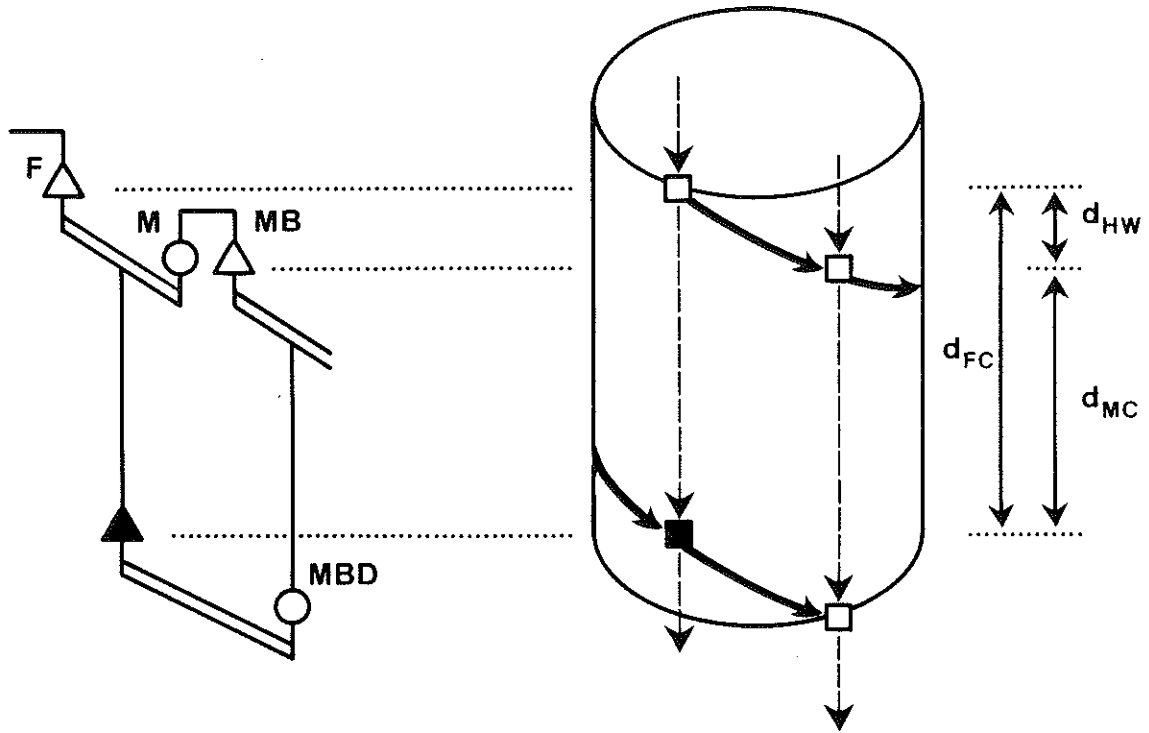


Fig. 3.2. Matrilineal cross-cousin marriage. Open, helical model with $d_{HW} > 0$.

is defined as $\{N_{ij} | i \text{ in } Z \text{ and } j \text{ in } \ell\}$. Let h be the basic helical mapping defined by $(N_{ij})h = N_{i+1 \ j+1}$ with $j+1$ reduced modulo n . For any x in Z the composite mapping h^x is defined as $(N_{ij})h^x = N_{i+x \ j+x}$ with $j+x$ reduced modulo n . The helical mapping h links a man's node, or sibling group, to that of his wife. Now, for any particular number n of patriline, there are only a limited number of possibilities for combining exchange helices and descent mappings along the surface of the cylinder. For any solution, the resulting model will be homogeneous, that is, from a structural point of view

Table 3.1. Values of $|B|$ and elements of the set B for n less than 15.

n	$ B $	B
1	0	-
2	1	2
3	1	3
4	2	2, 4
5	1	5
6	3	2, 3, 6
7	1	7
8	3	2, 4, 8
9	2	3, 9
10	3	2, 5, 10
11	1	11
12	5	2, 3, 4, 6, 12
13	1	13
14	3	2, 7, 14

there are no 'privileged' nodes or sibling groups. The range of possible structures is now defined for any number of patriline (given the basic helical mapping h defined above). For any positive integer n , let B be the set of integers defined by $B = \{b | b \neq 1 \text{ and } b \text{ a divisor of } n\}$. The order of B is denoted as $|B|$. Values of $|B|$ and the elements of B for all $n < 15$ are given in table 3.1.

The patrilineal mapping linking the node of a man to that of his children is now introduced. For any n and b , let s be the translation defined by $(N_{ij})s = N_{i+b \ j}$. For any y in Z , the composite mapping is defined as $(N_{ij})s^y = N_{i+by \ j}$. The basic mappings h and s can be

shown to generate a commutative group, each element $s^y h^x$ denoting one or more kintype relations.

Lemma 1: h and s generate the commutative group $G(h, s)$ of infinite order and $h^n = s^t$, with $bt = n$. $G(h)$, $G(s)$, and $G(h^{-1}s)$ are cyclical subgroups of $G(h, s)$.

Lemma 2: The smallest positive integers x , y , and z such that $h^x = s^y = (h^{-1}s)^z$ are $x = n(b-1)$, $y = t(b-1)$, and $z = n$, with $n = bt$.

These lemmas guarantee that the periodic kinship structures associated with the group $G(h, s)$ are indeed compatible with matrilinear cross-cousin marriage. According to the fundamental kinship theorem mentioned earlier, commutativity is a necessary and sufficient condition for a model with matrilinear cross-cousin marriage.

Helical envelopes and partitions.

The group $G(h, s)$ is now used to partition the basic nodes of the cylinder into subsets corresponding to patriline and matriline and helical exchange cycles. Let N and $G(h, s)$ be defined for some n and b , with $n = bt$. Then for some element N_{pq} of N , let $\text{Hel}(N_{pq})$ be the subset of N defined by $\text{Hel}(N_{pq}) = \{N_{ij} | N_{ij} = (N_{pq})g, \text{ with } g \text{ in } G(h, s)\}$. $\text{Hel}(N_{pq})$ is the *helical envelope*, or the *helical grid*, induced by the group $G(h, s)$ on N , and $N_{pq} = a$ is the *origin* of $\text{Hel}(N_{pq})$. (Note that the origin is defined in a slightly different manner for helical structures than in the previous chapter.) The origin is introduced to make it possible to refer to the exchange nodes and genealogical positions relative to some particular node a .

The set of patriline is introduced as follows. For every j in $\mathbb{Z}$ there exists an integer $x = j - q$ such that

$(N_{pq})h^x = N_{p+j-q} j = N_{rj}$, with N_{rj} in $\text{Hel}(N_{pq})$. Let P_j (patriline j) be the subset of $\text{Hel}(N_{pq})$ defined as

$$P_j = \{N_{ij} | N_{ij} = (N_{pq})h^x s^y \text{ for } s^y \text{ in } G(s) \text{ and } (N_{pq})h^x = N_{rj} \text{ for some } h^x \text{ in } G(h)\}.$$

Let ℓ be the index set defined earlier. Then PLin (the set of patriline) = $\{P_j | j \text{ in } \ell\}$ is a partition of $\text{Hel}(N_{pq})$, that is, a decomposition into disjoint subsets such that every node of $\text{Hel}(N_{pq})$ belongs to some unique P_j . The order of PLin is n (i.e., there are exactly n patriline).

Helical exchange cycles are introduced in a similar fashion. Let H_y be the subset of $\text{Hel}(N_{pq})$ defined as

$$H_y = \{N_{ij} | N_{ij} = (N_{pq})h^x s^y \text{ for } h^x \text{ in } G(h) \text{ and some } s^y \text{ in } G(s)\}.$$

According to lemma 1, $(N_{pq})s^t = (N_{pq})h^n$, with $n = bt$. Hence, $H_t = H_0$ and there are exactly t distinct subsets H_y . Let T be the index set $\{0, 1, \dots, (t-1)\}$. Then $\text{Hex} = \{H_y | y \text{ in } T\}$ is a partition of $\text{Hel}(N_{pq})$ of order t .

Finally, the set of matriline is introduced. Let M_y be the subset of $\text{Hel}(N_{pq})$ defined as

$$M_y = \{N_{ij} | N_{ij} = (N_{pq})g s^y \text{ for } g \text{ in } G(h^{-1}s) \text{ and some } s^y \text{ in } G(s)\}.$$

According to lemma 2, $(N_{pq})s^{t(b-1)} = (N_{pq})(h^{-1}s)^n$, with $n = bt$. Hence, $M_{t(b-1)} = M_0$ and there are exactly $t(b-1) = n-t$ distinct subsets M_y (i.e., $n-t$ matriline). Let U be the index set $\{0, 1, \dots, t(b-1)\}$. Then $\text{MLin} = \{M_y | y \text{ in } U\}$ is a partition of $\text{Hel}(N_{pq})$ of order $t(b-1) = n-t$. Hence $|\text{PLin}| = |\text{Hex}| + |\text{MLin}| = n$. Note that the partitions PLin , Hex , and MLin of $\text{Hel}(N_{pq})$ are defined here without introducing the more efficient coset notation and the concept of a factor group. Also, in the following section exchange structures are defined without making explicit use of the automorphism group of $G(h, s)$.

Simple age-biased exchange structures¹¹

Consider the helical envelope $\text{Hel}(N_{pq})$ with origin $N_{pq} = a$ and partitions PLin , Hex , and MLin . Let T be the index set defined earlier. Then for each H_y in Hex , define the indexed helical mapping h_y by $(N_{ij})_{s^y h_y} = (N_{ij})_{s^y h} = N_{i+by+1, j+1}$ for all N_{ij} in H_0 , with $j+1$ reduced modulo n and the index y of h_y reduced modulo t . Note that the superscript y of s^y is not to be reduced modulo t . Each mapping $h_0, h_1, \dots, h_{t-1}$ is thus defined on the corresponding subset $H_0, H_1, \dots, H_{t-1}$ of the partition Hex . The mapping h_y defines the y -th helical exchange cycle with respect to the origin $N_{pq} = a$. Finally, the simple helical structure of generalized exchange induced by h and s on $\text{Hel}(N_{pq})$ with origin $N_{pq} = a$ is defined by the ordered triplet $\mathcal{H}(a, n, b) = \{h_y \mid y \text{ in } T\}$.

Age metrics

Having derived a commutative exchange structure with openended helical cycles, an age metric d is now defined on the nodes N_{ij} of $\mathcal{H}(a, n, b)$. The objective is to obtain a quantitative scheme for comparing the nodes and thereby the relative age of the sibling groups and kintype relations with which they are associated. This is done by defining d on the first coordinate or subscript index of the nodes on the surface of the cylinder.

For all pairs of nodes N_{ij} and N_{km} of $\text{Hel}(N_{pq})$, define the metric d as $d(N_{ij}, N_{km}) = (k-i)$. It is obvious that d satisfies the following properties:

1. $d(N_{ij}, N_{km}) = 0$ if and only if $i = k$;
2. $d(N_{ij}, N_{km}) = -d(N_{km}, N_{ij})$;
3. $d(N_{ij}, N_{km}) + d(N_{km}, N_{yz}) = d(N_{ij}, N_{yz})$.

It is easy to see that the properties of the metric d

correspond exactly to those of the statistical mean age difference d_{ij} described in the introductory sections to this chapter. The metric d thus will be interpreted as an age metric, that is, as a point estimate at the level of the helical exchange model for the statistical mean age differences measured in actual societies.

A scale of measurement must also be defined. For any positive real number r a new metric d' with the same properties as d can always be defined as $d'(N_{ij}, N_{km}) = r(k-i)$. Hence $d' = rd$ and r is called the *scale* of the metric d' .

The age metric d' with scale r can now be used to obtain the difference between a node N_{ij} and its image $(N_{ij})g$ under any mapping g of the commutative group $G(h, s)$. For example,

$$d'(N_{ij}, (N_{ij})s) = d'(N_{ij}, N_{i+b \ j}) = rb;$$

$$d'(N_{ij}, (N_{ij})h) = d'(N_{ij}, N_{i+1 \ j+1}) = r;$$

$$d'(N_{ij}, (N_{ij})h^{-1}s) = d'(N_{ij}, N_{i+b-1 \ j-1}) = r(b-1);$$

and so on. This is generalized in a simple formula for all elements $g = s^y h^x$ of the commutative group $G(h, s)$ and hence for genealogical relationships or kintypes relative to ego.

Lemma 3: For any element $g = s^y h^x$ of $G(h, s)$ define

$$d'(N_{ij}, (N_{ij})g) = d'(N_{ij}, (N_{ij})s^y h^x) =$$

$$d'(N_{ij}, N_{i+by+x \ j+x}) = r(by+x).$$

This result can also be formulated by defining d'

directly on the elements of the group. Hence

$$d'(e, s^y h^x) = r(by+x). \text{ In particular, } d'(e, e) = 0 \text{ and}$$

the ratio $d'(e, h^{-1}s) / d'(e, s) = (b-1)/b = 1-1/b$ is

independent of the scale r . The earlier notation for the

simple helical exchange structure $H(a, n, b)$ is now

amended so as to include the relevant information

pertaining to the age metric d' with scale r . Let

$H(a, n, b; r, 1-1/b)$ denote the r -metricized simple

helical structure induced by h and s on the helical envelope $\text{Hel}(N_{pq})$ with origin $N_{pq} = a$.

The model of metricized exchange structures is now complete and one can map the system of mathematical relations directly onto the vocabulary of kinship theory. Note that the helical exchange structure is a base model and can also be used to represent relations in domains other than kinship and relative age. Relative status and hierarchical ranking, or systems with a brideprice differential, spring readily to mind (cf. Hammel 1976 and Tjon Sie Fat 1983). For the intended applications, the elements N_{ij} of $\text{Hel}(N_{pq})$ are nodes in a reduced genealogical network. The mappings h , s , and $h^{-1}s$ correspond respectively to the conjugal mapping h , the patrilineal mapping f and the matrilineal mapping m introduced earlier. This particular interpretation of the model is consistent with the work of Courrège (1965) and Lorrain (1975) and with the models developed in other chapters. The partitions $PLin$, $MLin$, and Hex correspond to the n patriline, the $n-t$ matriline, and the t helices of the age-biased kinship structure. Finally, the metric d' with scale r is mapped onto the statistical measure of mean age differences d_{ij} through the following equations:

$$r = d_{HW} \quad rb = d_{FC} \quad r(b-1) = d_{MC}.$$

Hence, the ratio of mean mother-child and father-child age differences, d_{MC}/d_{FC} , is equal to $1-1/b$. All structures $H(a, n, b; r, 1-1/b)$ are indeed compatible with matrilineal cross-cousin marriage since the group $G(h, s)$ is commutative (lemma 1).

For practical reasons, in most of the examples to follow, male ego is situated at the origin $a = N_{pq} = N_{-1n}$ on the helix h_0 . Ego's wife is thereby consistently in node $(N_{-1n})_{h_0} = N_{01}$ on the same helix. The parameters of all simple helical structures $H(a, n, b; r, 1-1/b)$ with fewer than 15 patriline are listed in table 3.2.

Table 3.2. Parameters of simple helical structures with less than 15 patriline

Number of patriline	Number of helices	Number of matriline	Divisors of n	Age ratio d_{MC}/d_{FC}
$n = bt$	t	$n-t$	b	$1-1/b$
2	1	1	2	.500
3	1	2	3	.667
4	2	2	2	.500
4	1	3	4	.750
5	1	4	5	.800
6	3	3	2	.500
6	2	4	3	.667
6	1	5	6	.833
7	1	6	7	.857
8	4	4	2	.500
8	2	6	4	.750
8	1	7	8	.875
9	3	6	3	.667
9	1	8	9	.889
10	5	5	2	.500
10	2	8	5	.800
10	1	9	10	.900
11	1	10	11	.909
12	6	6	2	.500
12	4	8	3	.667
12	3	9	4	.750
12	2	10	6	.833
12	1	11	12	.917
13	1	12	13	.923
14	7	7	2	.500
14	2	12	7	.857
14	1	13	14	.929

Table 3.3. Values of mean age differences d_{FC} for various combinations of d_{MC}/d_{FC} and d_{HW} .

d_{MC}/d_{FC}	d_{HW}										
	2	4	6	8	10	12	14	16	18	20	22
.500	4	8	12	16	20	24	28	32	36	40	44
.667	6	12	18	24	30	36	42	48	54	60	-
.750	8	16	24	32	40	48	56	64	-	-	-
.800	10	20	30	40	50	60	-	-	-	-	-
.833	12	24	36	48	60	-	-	-	-	-	-
.857	14	28	42	56	70	-	-	-	-	-	-
.875	16	32	48	64	-	-	-	-	-	-	-
.889	18	36	54	72	-	-	-	-	-	-	-
.900	20	40	60	-	-	-	-	-	-	-	-
.909	22	44	66	-	-	-	-	-	-	-	-
.917	24	48	72	-	-	-	-	-	-	-	-
.923	26	52	78	-	-	-	-	-	-	-	-
.929	28	56	84	-	-	-	-	-	-	-	-

The scale $r = d_{HW}$ is of course independent of the ratio d_{MC}/d_{FC} so that any helical structure is compatible with an infinite number of possible values of d_{HW} for a given value of d_{MC}/d_{FC} . The results in table 3.2 are perhaps surprising: the family of helical structures is *not* symmetrical or invariant with respect to the mode of descent. For example, there are structures with four patriline and two matriline but no 'inverted' structure with four matriline and two patriline. This important conclusion follows directly from the existence of *positive* average husband-wife age differences, an asymmetric constraint that reflects the nearly universal feature that women marry and get children at a younger age than their spouses.

Table 3.4. Four series of mean age differences (in years) computed according to the formula $r(by+x)$.

Kintype	Kinship mapping	$s^y h^x$ mapping	Mean age differences for			
			b: 2 r: 20	3 14	4 10	6 6
FFZDC	$f^{-2} m^2$	h^{-2}	-40	-28	-20	-12
FFZSC	$f^{-2} mf$	h^{-1}	-20	-14	-10	-6
MFZDC	$m^{-1} f^{-1} m^2$	h^{-1}	-20	-14	-10	-6
FZC	$f^{-1} m$	h^{-1}	-20	-14	-10	-6
MFZSC	$m^{-1} f^{-1} mf$	e	0	0	0	0
Sb	e	e	0	0	0	0
FMBDC	$f^{-1} m^{-1} fm$	e	0	0	0	0
MBC	$m^{-1} f$	h	20	14	10	6
MMBDC	$m^{-2} fm$	h	20	14	10	6
FMBSC	$f^{-1} m^{-1} f^2$	h	20	14	10	6
MMBSC	$m^{-2} f^2$	h^2	40	28	20	12
FFZC	$f^{-2} m$	$s^{-1} h^{-1}$	-60	-56	-50	-42
FSb	f^{-1}	s^{-1}	-40	-42	-40	-36
MFZC	$m^{-1} f^{-1} m$	s^{-1}	-40	-42	-40	-36
FMBC	$f^{-1} m^{-1} f$	$s^{-1} h$	-20	-28	-30	-30
MSb	m^{-1}	$s^{-1} h$	-20	-28	-30	-30
MMBC	$m^{-2} f$	$s^{-1} h^2$	0	-14	-20	-24
FZDC	$f^{-1} m^2$	sh^{-2}	0	14	20	24
ZC	m	sh^{-1}	20	28	30	30
FZSC	$f^{-1} mf$	sh^{-1}	20	28	30	30
MBDC	$m^{-1} fm$	s	40	42	40	36
C	f	s	40	42	40	36
MBSC	$m^{-1} f^2$	sh	60	56	50	42
FFSb	f^{-2}	s^{-2}	-80	-84	-80	-72
FMSb	$f^{-1} m^{-1}$	$s^{-2} h$	-60	-70	-70	-66
MFSb	$m^{-1} f^{-1}$	$s^{-2} h$	-60	-70	-70	-66
MMSb	m^{-2}	$s^{-2} h^2$	-40	-56	-60	-60
ZDC	m^2	$s^2 h^{-2}$	40	56	60	60
DC	fm	$s^2 h^{-1}$	60	70	70	66
ZSC	mf	$s^2 h^{-1}$	60	70	70	66
SC	f^2	s^2	80	84	80	72

In this respect my helical models differ from the class of structures with circulating connubium, MBD-marriage, and double descent developed by the Leiden anthropologists. As demonstrated in Chapter 1, the classic models embody or permit, at the level of the formal structure, an equal number of matri and patridescent lines; marriage is with same-generation kin. There is, however, an intriguing passage in Van Wouden (1968:94) in which he mentions the possibility of intergenerational marriages, but only if the numbers of matri and patriline in the model are unequal!

Table 3.3 presents values of d_{FC} for various combinations of the ratio d_{MC}/d_{FC} and the independent variable d_{HW} ; all values are scaled in years. For obvious biological reasons, only age differences of the order 12 years $< d_{FC} < 60$ years are relevant. (This is a conservative estimate, considering the fact that d_{FC} is the *mean* age difference.) The most interesting combinations of these age constraints are at the lower range of values for d_{MC}/d_{FC} and hence for small b . As d_{MC}/d_{FC} approaches equality, the helical model converges on the conventional model with $d_{HW} = 0$ (cf. figs. 3.1 and 3.2).

One can also use my metricized helical models to analyze the distribution of mean age differences within a genealogical network *without* referring to the traditional criterion of 'generation'. In table 3.4 the composite mappings corresponding to a basic set of 31 kintypes are listed. The Courrège-Lorrain kinship mappings (generated by f and m) are in the second column with the equivalent helical mappings in the third column (substituting s for f and sh^{-1} for m in the formulae, and assuming commutativity). Mean age differences relative to male ego are computed for four different combinations of b and r by means of the formula $r(by+x)$ from lemma 3. Note that r equals d_{HW} . The d_{MC}/d_{FC} ratios

are .500, .667, .750, and .833 (for b equal to 2, 3, 4, and 6).

DISCUSSION OF THE MODELS

For all simple helical models, ego's wife is merged with his MBD, MMBDD, and FMBSD (this is, of course, also true for the standard nonhelical models with matrilinear cross-cousin marriage). These kintypes denote individuals of the 'correct' marriageable age. Significantly, in the first age series of table 3.4, ZD and FZSD are also of marriageable age; in the second series this is the case with FZDD. These observations lead directly to the question of whether there are combinations of d_{HW} and d_{MC}/d_{FC} that allow 'oblique' marriages with kin in addition to MBD, MMBDD, and FMBSD. The answer is summarized in the following lemma.

Lemma 4: Within the range of the 31 basic kintypes of table 3.4 (i.e., for any positive integer b , but with x and y limited to the values -2, -1, 0, 1, 2), there are exactly two helical models with oblique marriage, namely: $H(a, 2, 2; r, .500)$, with two patriline, one matriline, and one exchange helix (wife is merged with ZD and FZSD); and $H(a, 3, 3; r, .667)$, with three patriline, two matriline, and one exchange helix (wife is merged with FZDD).

These oblique structures are examined in greater detail in figures 3.3 and 3.4. The diagrams are obtained by projecting the cylindrical model onto a flat surface parallel to the main axis. Helical exchange cycles are represented as zig-zag lines and patriline are indicated by roman numerals, matriline by capital letters. The 31 basic kintypes of table 3.4 are allocated to the nodes of the models.

Although all helical age-biased structures (as defined in this chapter) are incompatible with symmetric sister

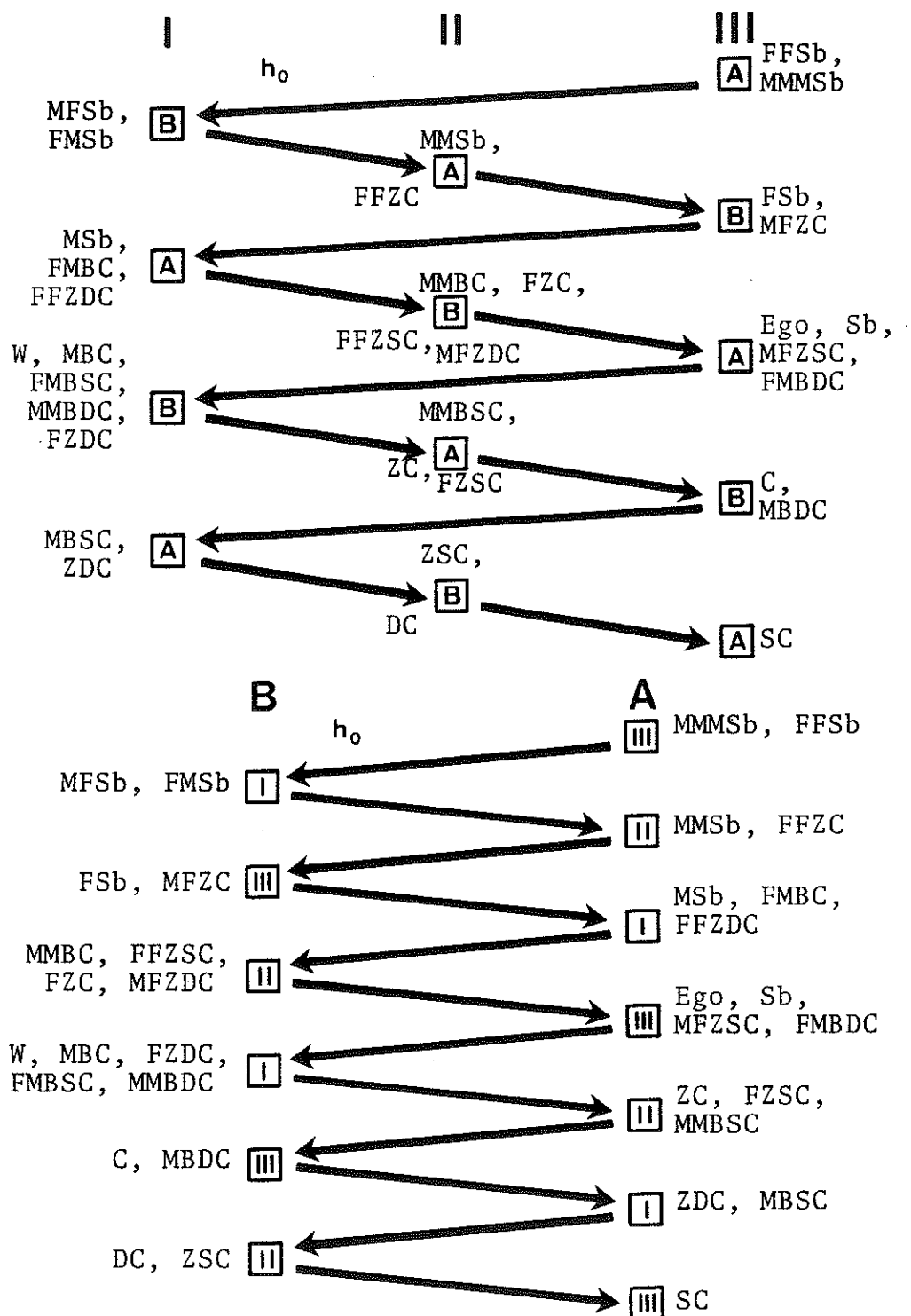


Fig. 3.4. Alternative representations of the model $H(a, 3, 3; r, .667)$ as a three-patriline structure (top) and as a two-matriline structure (bottom). 'Oblique' marriage with FZDD.

direct exchange as 'latent' subsystems or formal possibilities.

The oblique structure of figure 3.3 models a patrimoiety system with unilateral cross-cousin marriage, not sister exchange. It conforms to Lévi-Strauss's (1970: 433) description of a 'still more simple structure of reciprocity than that found between two cross-cousins, viz., that which results from the claim that a man who has given his sister can make on the sister's daughter.' In systems with 'avuncular privilege' (described for South America and southern India), 'ego has a right to his sister's daughter because he has given his sister to be his niece's mother, while he must give away his own daughter because he has received his daughter's mother as his wife' (Lévi-Strauss 1970:145). My model adds one new item to Lévi-Strauss's analysis: ideally, the d_{MC}/d_{FC} ratio should approximate .500.

Although descriptions of marriage with own sister's daughter had appeared prior to 1949 (particularly in discussions of data from southern India; cf. Kirchhoff 1932, Held 1935; Good 1980 lists many of the early sources), Lévi-Strauss's comments generated a spate of reactions. Most of the subsequent discussion focussed on his classification of ZD marriage as an oblique sub-type of patrilateral cross-cousin marriage. Leach (1971:59-60 [1951]) discusses this type of union within the context of MBD marriage; Moore (1963) relates marriage type to descent and terminology. Other authors describe oblique marriage as a secondary union (Shapiro 1966), as direct exchange (Lave 1966), or, following Lévi-Strauss, as a variant of prescriptive patrilateral cross-cousin marriage, that is, 'oblique discontinuous exchange' (Rivière 1966a, 1966b, 1969). Thomas (1979) points to the classification of wife's father as a brother-in-law as a solution for resolving strain in affinal relationships. Good (1980) recently summarized many of these discussions.¹²

My analysis situates ZD marriage firmly within the class of age-constrained matrilineal cross-cousin marriages. It is in fact the minimal or limiting structure in the series. This is also the view expressed by Barnard and Good (1984:98-100). The qualification 'oblique' within the context of helical exchange models is actually incorrect, as it refers to kintypes not in ego's own generation. However, the traditional concept of generation does not really apply to helical structures: marriage is by definition always within the same helix, whatever the kintype.

ZD marriage may in practice occur in free variation with other marriage types. Reid's (1974:271) data on 616 unions among the Telaga-Kapu caste of coastal Andra Pradesh furnish the following percentages for marriage with various types of kin: MBD 12.7, 'distant kin' 6.5, ZD 5.5, FZD 2.8, not related 72.5. Reid mentions corrected mean age differences of $d_{FC} = 36.6$ years and $d_{MC} = 26.9$ years (1974:261-263). Hence the average d_{MC}/d_{FC} ratio for all marriages is .64, somewhat greater than the value of .50 that the model predicts for 'pure' MBD/ZD marriage. Lévi-Strauss's (1970:428-429, 432-437, 447-449) analysis of avuncular privilege among the Sanyasi and the Korava of southern India also points to the coexistence of distinct marriage types or rules 'without contradiction or opposition', a view shared by many of the anthropologists reporting on ZD marriage.¹³ The model of ZD marriage straddles the boundary between symmetry and asymmetry.

The three-patriline, two-matriline kinship structure of figure 3.4 offers a number of fascinating prospects for further research. The marriageable kintypes include FZDD; that is, a man gives his sister in marriage and receives her daughter's daughter as a wife for his son. Exchange of women is asymmetric with respect to the three patrilineages but symmetric from the perspective of the matrilines. Both perspectives are diagrammed in

figure 3.4.

An identical three-patriline model with unilateral marriage with MBD and FZDD is described by McConnel for the Ngamiti, one of the Cape York societies (1950:142, diagram 5). Thus (1950:119):

In the Ngamiti system, a man marries his m.br.d., who is also his f.sis.d.d. That is to say, a man marries back into his mother's line and there is no w.f. ... line as such A woman marries her f.sis.son, who is also her m.m.br.son. Ego and his sister thus marry into different classificatory lines, ... so that the system is completely unilateral. ... There are therefore three classificatory lines: m.f. (w.f.), f.f. and sis.husband.

The exchange rule is that a man marries his m.y.br.d. and gives his sister's d. to his w.br., to whom she is f.y.sis.d.d.; a woman marries her f.o.sis.son and gives her daughter to her m.br.son. Ego's sister's son's child is Ego's d.d. ...

The Yäraidyana, a neighbouring tribe, also have a FZDD marriage rule (proscribing marriage with the MBDD) (McConnel 1950:113):

The general rule is that Ego marries his f.y.sis.d.d. and gives his sister's daughter to his m.o.br.son. Neither Ego nor his sister marry into the mother's father's line. Both marry into a *third* classificatory line, with which f.m.f., m.m.f. and the (taboo) m.f.sis.husband's lines are identified.

These rules are embedded in a more elaborate system. Ideally, ego's 'correct' marriage is not with his FZDD, but with the daughter's daughter's daughter of his father's father's younger brother. Moreover, each of the three 'classificatory lines' is subdivided into three, giving a total of nine patridescent lines for the complete system. And while the system is unilateral, the Yäraidyana acknowledge the possibility of bilateral sister exchange (McConnel 1950:113-119).

A three-patriline, two-matriline model also suggests certain parallels with the 'anomalous' Ambrym system. Ambrym kinship has been described in terms of direct exchange with six sections, intermediate between the Kariera and the Aranda.¹⁴ Lévi-Strauss (1970:125)

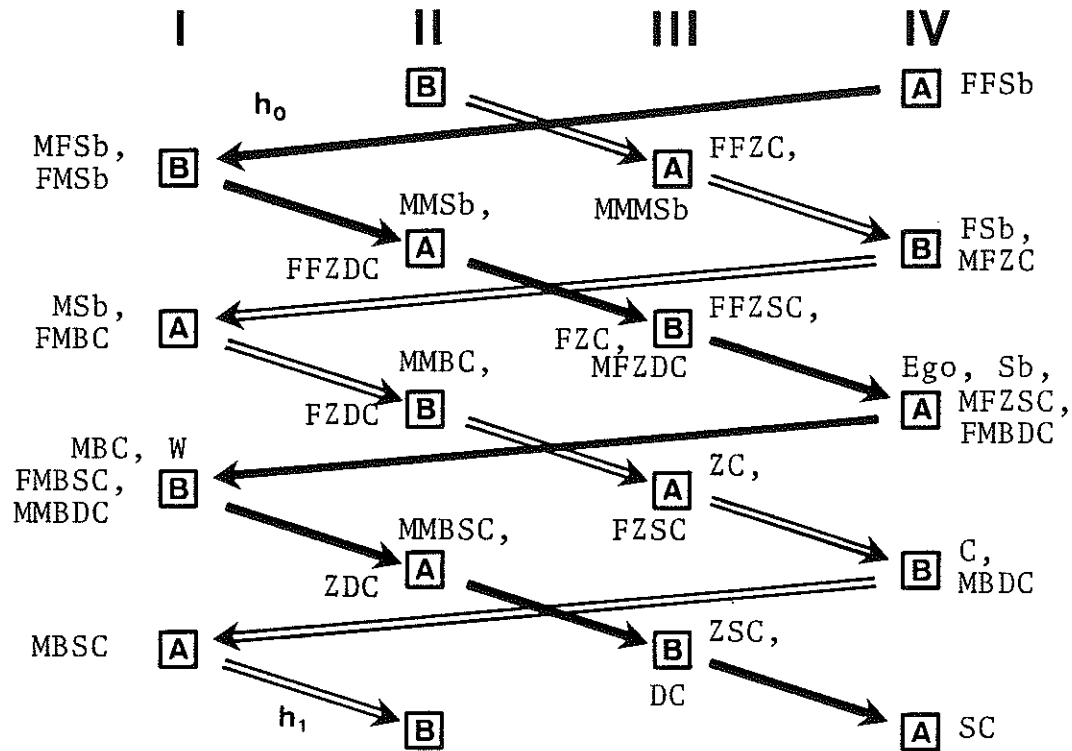


Fig. 3.5. Helical exchange structure $H(a, 4, 2; r, .500)$. Karadjeri-type with direct exchange of ZD and FZSD.

apparently disagrees, referring to the Ambrym as 'one of the most complex systems known'. Furthermore (Lévi-Strauss 1970:465),

... even a superficial examination shows them the Ambrym/Pentacost systems to be the result of the combination of a dualistic and a triadic rule of reciprocity, the intention being to make these coincide... As the combination of the principles of restricted and generalized exchange seems to be at the heart of the so-called Crow-Omaha systems of America ..., we prefer that the eastern area of complex structures should start to the south of Melanesia.

But also see his remarks on the Ambrym in Guilbaud (1970:24-32).

Concern with the relationship between symmetry and asymmetry is a recurrent theme in Ambrym studies, as is

the problem of reconciling the data on 'intergenerational marriages' with an adequate model of the intergroup relations and the kinship terminology. A proper analysis of the Ambrym system lies beyond the scope of this chapter. However, a 9-line helical scheme has been formulated that is compatible with the available data. In addition, the structure of figure 3.4 as well as a number of alternative Ambrym models appear as quotient structures. This model is also a more complete model for the Yäraidyana. (A publication is in preparation.)

Finally, the matrilineal Yombe of Central Africa combine a Crow-type terminology with a rule of preferred marriage with the patrilateral cross-cousin once removed, chosen from the senior paternal matrilineage, i.e., a FZDD (Doutreloux 1967:135-150).¹⁵ However, Doutreloux does not attempt to formulate a model with oblique marriage. His solution is to combine a model with matrilateral cross-cousin marriage and generalized exchange with a model of symmetric sister exchange and bilateral cross-cousin marriage, presenting them as alternative marriage structures for the Yombe (Doutreloux 1967:139, figure 9; 144-147). The matrilineal BaKongo of Lower Zaire, close neighbours of the Yombe, have a similar marriage system. According to MacGaffey (1986: 90-99) there are congruences between the indigenous model of alliance and descent (a complex variant of patrilateral cross-cousin marriage combining elements of both cyclical alliance and direct exchange), and Kongo cosmology, conceived of as a 'reciprocating' or 'spiral' universe in which successive cycles in time generate a helical structure rather than a simple repetition. With the BaKongo emphasis on spiral symbolism clearly evident in all aspects of their culture, further investigations on the occurrence of native models of helical kinship structures may be rewarding.

I provide diagrams of three other helical structures. The model of figure 3.5 comprises four patriline, two

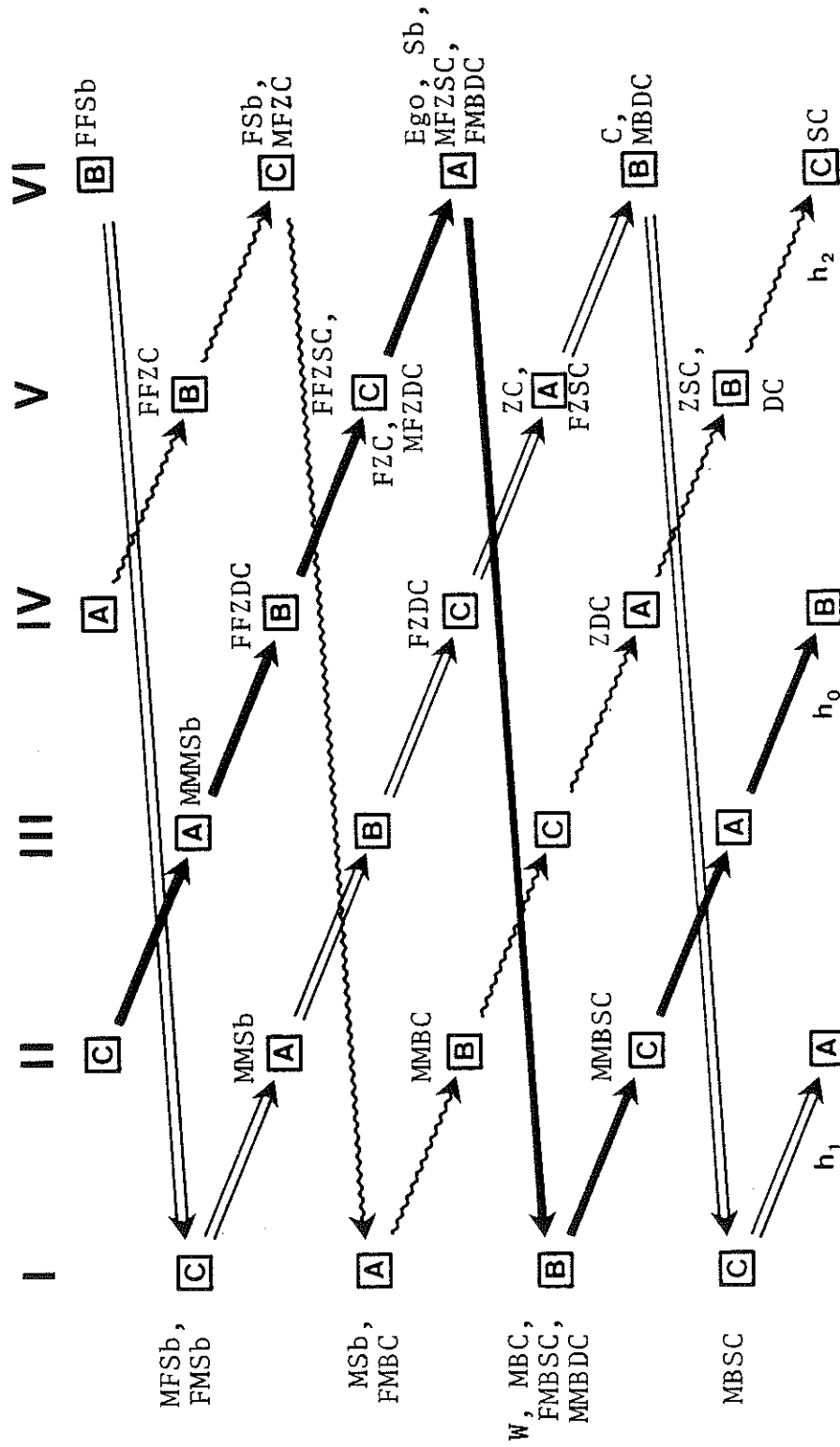


Fig. 3.6. Helical exchange structure $H(a, 6, 2; r, .500)$. Early Wikmunkan type with generalized exchange of ZD and FZSD.

matrilines, and two alternating helical marriage cycles; d_{MC}/d_{FC} equals .500 (for any value of d_{HW}). This model is also symmetric in the sense that men of the same mean age espouse each other's ZD and FZSD. This structure would presumably appear in Karadjeri-type asymmetrical systems with large husband-wife age disparities. It is compatible with a division into either four sections or eight subsections. Indeed, in reduced form it is identical to Lévi-Strauss's (1970:343) figure 66 (right), half of his 'Murngin' model. It is also compatible with J.P.B. de Josselin de Jong's (1952) analysis. However, considering the range of extension of the actual 'Murngin' terminology over seven-odd patriline and five generations (Warner 1964 [1937]:45-49), and Shapiro's data on the symmetric exchange of ZDD (not ZD) (1968:347-351), a different helical model seems more appropriate for the Murngin. (I develop such a model in a future publication.)

The helical structure depicted in figure 3.6 brings us to the heart of the Wikmunkan controversy. This model is based on six patriline, three matriline, and three distinct and intertwined helices; d_{MC}/d_{FC} is .500. The exchange structure can also be glossed as two series of asymmetric connubia, each linking three patriline (II, IV, and VI versus I, III, and V), with men of the same mean age passing on their ZD or FZSD in closed unilateral cycles of exchange. Lévi-Strauss (1970:209) summarizes McConnel's (1940) early analysis as follows:

The Wikmunkan practice a characteristic form of marriage with the mother's younger brother's daughter; the mother's older brother's daughter is prohibited. The structure of alliance and kinship thus does not simply possess the cyclical form of systems of generalized exchange, ... for the cycle takes on the additional appearance of a spiral, a man always marrying into a younger branch, and a woman into an older branch. The adjustment is made by closing the cycle with an absolute displacement of three generations in every six lines ...

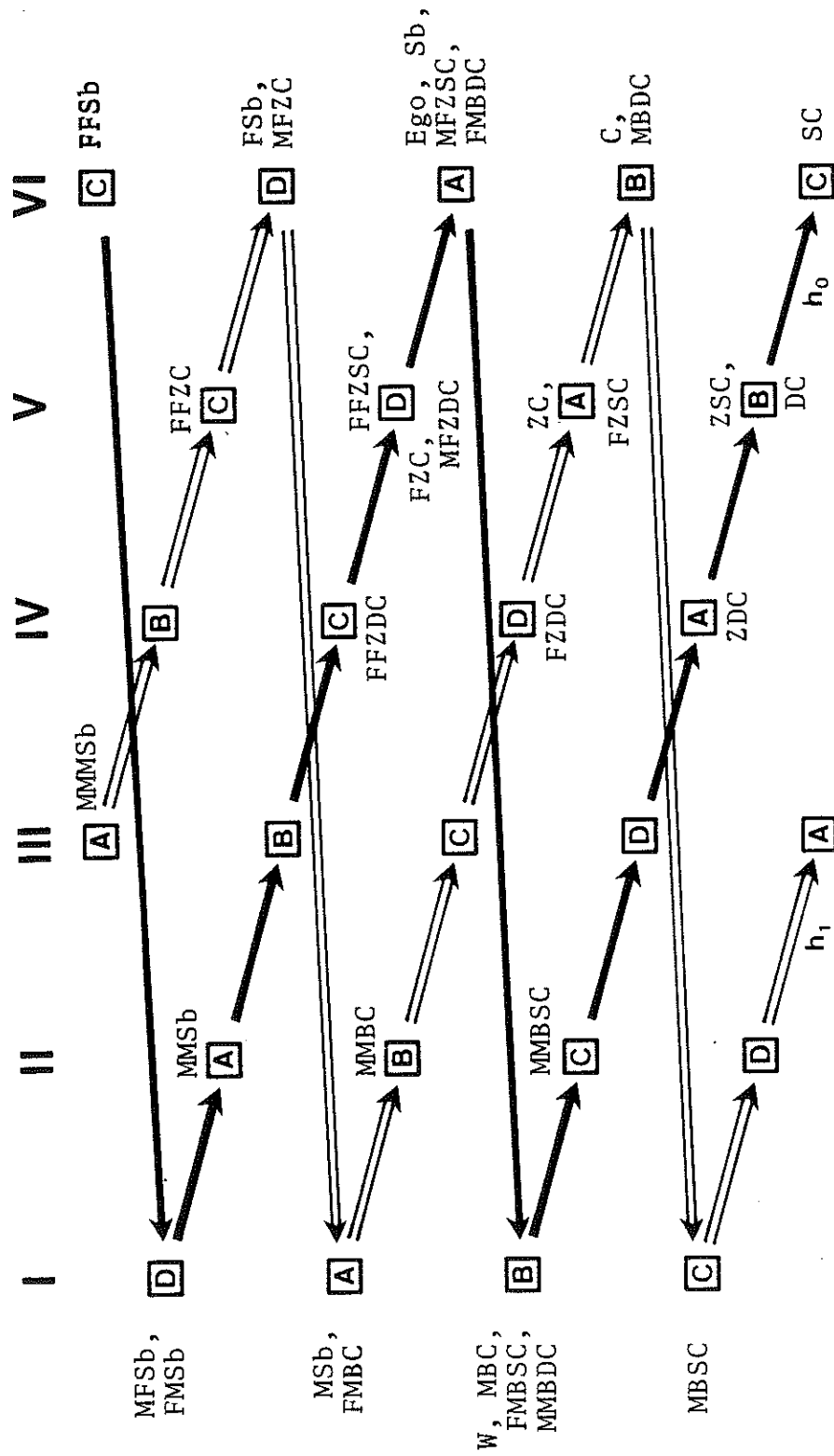


Fig. 3.7. Helical exchange structure $H(a, 6, 3; r, .667)$. Alyawara-Wikmunkan type with direct exchange of FZDD.

This description (see, e.g., figure 3.6) was unfortunately not amended after the 1949 edition of *Elementary Structures* to include McConnell's later (1950, 1951) corrections and additions. Her first analysis is indeed not entirely consistent. Reading from one of her uncorrected kinship charts (McConnell 1940:445, Diagram 3A), the exchange structure is closed with a displacement of *three* generations, MMF(G+3) marrying into the generation of ego's children (G-1). The helical structure in my figure 3.6 is the only homogeneous solution, given the assumption of exactly six patriline. However, problems arise when the Wikmunkan kin terms are mapped onto this exchange structure. Within each patriline the terminology repeats itself after *four* generations, not *three*. These inconsistencies are corrected later in McConnell's (1950, 1951) notes, and her revised Wikmunkan model is described in a new chart (McConnell 1950:121, 145). Here again, a helical model with six patriline is postulated, but now the structure closes with a displacement of only *two* generations, MFB (G+2) marrying into ego's generation (G 0), and the terminology again repeats after *four* generations. McConnell's later analyses are entirely consistent with a helical model embodying six patriline, four matriline, two alternating helices, and d_{MC}/d_{FC} ratio of .667. This corresponds to the scheme developed independently by Denham et al. (1979) for the Alyawara.

Not everyone is satisfied by McConnell's Wikmunkan analysis, however. Needham (1962, 1963, 1971) and McKnight (1971), in direct opposition, dismiss outright all such helical models of generalized exchange. They subscribe to a simple system of moieties with direct exchange regulated by a scheme of 'social classification'. (A moiety structure is indeed one of the model's quotient structures.) The many points of contention cannot be dealt with here;¹⁶ in any case, I do not find the Needham-McKnight Wikmunkan critique very convincing.

The Alyawara structure (fig. 3.7) is indeed one of my

helical models. All age-difference inferences are in perfect agreement with the actual values reported: $d_{MC}/d_{FC} = .667$, hence if d_{HW} is 14 years, then d_{MC} is 28 years and d_{FC} is 42 years. The complete set of mean age differences can be read from table 3.4 (for $b = 3$ and $r = 14$). It is essential to remember that my Alyawara-Wikmunkan model is *not* derived *from* the actual Alyawara and Wikmunkan field data: it belongs to an entire class of formal helical structures developed from a basic set of structural axioms. Hence, the fact that this particular proper model is a perfect fit for the empirical data is not due to a circular argument. The Alyawara structure $H(a, 6, 3; 14, .667)$ can be reduced to a four-section quotient structure, but it is not compatible with a reduction to an eight-subsection Aranda-type kinship structure. This conclusion is also reached by Denham et al. (1979).

As noted by Denham and Atkins (1982), the Alyawara structure permits a form of symmetric exchange, with men of the same mean age espousing each other's FZDDs. This 'latent' property of the model is highly significant in the light of McConnel's (1950) description of the Ngamiti and Yäraidzana marriage systems with three 'classificatory' patriline. It is an interesting possibility that the six-patriline, four-matriline, two-helix Alyawara-Wikmunkan model, with *symmetric* exchange of FZDDs, is a transform of the Ngamiti structure. Under this hypothesis, a three-patriline structure with *asymmetric* exchange of FZDDs is transformed (possibly through fission of the descent lines into 'junior' and 'senior' branches) into a six-patriline structure, with the age difference and age ratio constraints remaining unchanged. A reverse transformation from six lines to three is also possible. Any substantive theory of the transitional configurations of Australian systems (e.g., Turner 1980) will benefit from a systematic investigation of the class of homomorphisms linking kinship models.¹⁷

I have so far not been able to discover clear empirical examples of many of the other structures listed in table 3.2. Age differences have not always been reported in ethnographic studies. In any case, additional constraints on the basic model are called for, as it is hardly likely that kinship structures with large numbers of helices and a large husband-wife age differential are viable descriptions of real societies.

The following extensions to the basic helical scheme should be explored. First, the basic helical mapping generating all models in this chapter has been defined as $(N_{ij})_h = N_{i+1 \ j+1}$ (with $j+1$ reduced modulu n). Under this assumption the minimum d_{MC}/d_{FC} ratio is .500. It has been pointed out to me that models with a minimum d_{MC}/d_{FC} ratio less than .500 cannot be excluded *a priori*.¹⁸

Second, the basic helical model is compatible with matrilateral cross-cousin marriage. As a consequence, the data from Rose's (1960) Wanindiljaugwa analysis, perhaps the best-known description of an age-biased kinship system, does not fit my helical scheme. The model must be extended to include helical structures proscribing first cousin marriage.

Finally, the family of helical models (suitably extended and constrained) offers a unique framework for modelling the structure and dynamics of kinship systems whose alliance structures are not based on the exchange of *sisters* (the standard assumption under the Lévi-Straussian approach).

These extensions are sketched in figure 3.8. The first two examples illustrate the classic models with sister exchange. Ego (indicated by a solid triangle) and some man marry each other's sisters. Exchange is symmetric, repeating the same pattern in consecutive generations (top, left) or in alternating generations (top, right). The simplest full models are of course the 'moiety' structure and the 'Aranda'-type structure. (The symmetries of restricted exchange are discussed in Chapter 4 where the series is extended beyond the classic structures mentioned

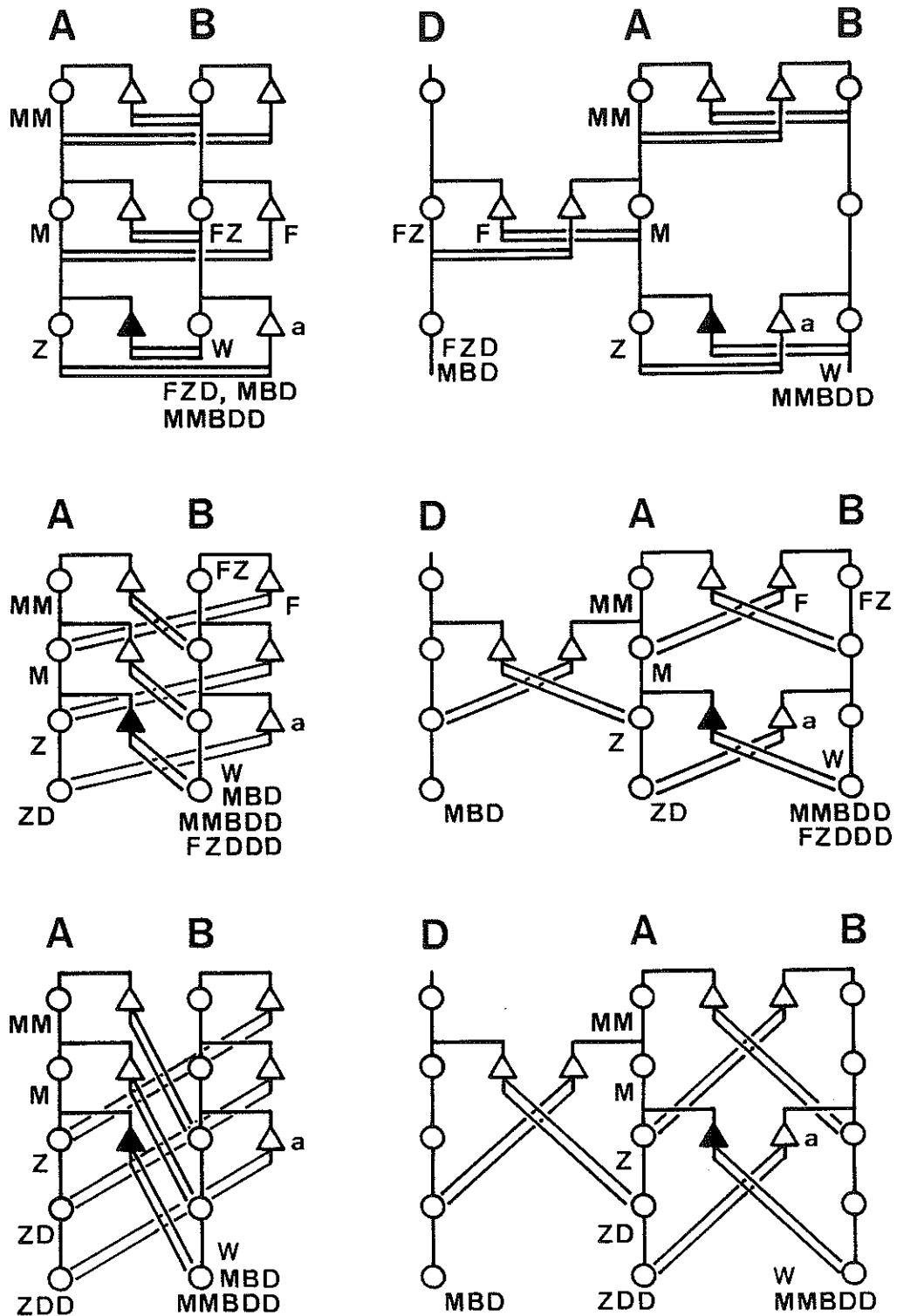


Fig. 3.8. Partial models based on the exchange of sisters (top), sister's daughters (centre), and sister's daughter's daughters (bottom). Consecutive and alternating exchange.

above.) The d_{MC}/d_{FC} ratio is of course equal to unity in all standard models.

The literature on Australian kinship provides numerous examples of sisters being exchanged or bestowed, not as spouses but as mother-in-laws. This is formally equivalent to a rule stating that two men marry each other's sister's daughters, i.e., a rule based on the symmetric exchange of ZDs. A partial model illustrating the exchange of ZDs by men from two matrilineages (with the pattern repeating in consecutive generations) is given in figure 3.6 (centre, left). The simplest proper model is the four-patriline, two-matriline model with two helical cycles $H(a, 4, 2; r, .500)$ pictured above in figure 3.5. Other models with d_{MC}/d_{FC} equal to .500 are also possible (see Table 3.2). Figure 3.8 (centre, right) shows a partial structure based on the exchange of sister's daughters, with the exchange cycle repeating every two generations. In genealogical terms, ego marries his MMBDD who is merged with his FZDDD. In contrast to the previous model, ego's MBD is now allocated to a third matriline. The simplest proper model is a structure with d_{MC}/d_{FC} equal to .500, defined on eight patrilineages, four matrilineages, with four helical exchange cycles. It seems to be fully compatible with the data on the Gidjingali (see Martin and Reddy 1987, Hiatt 1965, 1968). A detailed analysis of this model will be presented shortly.

The final examples (figure 3.8, bottom row) are of models with d_{MC}/d_{FC} equal to .333, i.e., where patrilineal 'generations' are, on average, three times as long as matrilineal 'generations'. In the model with consecutive exchange (figure 3.8, bottom, left), ego and man exchange sister's daughters as mothers-in-law. The simplest proper model is a six-patriline, two-matriline model with four helical exchange cycles. This helical model is indeed compatible with recent analyses of the 'Murngin' (Yolngu) material (see Morphy 1978, Shapiro 1968, 1981, Keen 1982, Maddock 1970, Liu 1986, Heath 1982, Martin and Reddy 1987, and Glowczewski and Pradelles de Latour 1987). Reasons of

space do not permit further discussion of this model (a more detailed presentation is in preparation).

Finally, in the last example (figure 3.8, bottom, right) sister's daughters are exchanged or bestowed as mothers-in-law, with the cycle of exchange repeated in alternating generations of the same matriline, with d_{MC}/d_{FC} again equal to .333. The corresponding proper models must contain at least four matriline.

The last four models illustrate an important feature already stressed in this chapter: *asymmetric* exchange of sisters (with or without the possibility of MBD-marriage) is compatible with a structure generated by the *symmetric* exchange or bestowal of sister's daughters as spouses or mothers-in-law.¹⁹ Hence knowledge of the form of exchange involving *sisters* (symmetric or asymmetric exchange) is not a sufficient criterion for characterizing an entire kinship structure. This conclusion supports the argument put forward long ago by J.P.B. de Josselin de Jong (1952: 46-49) in his discussion of the Lévi-Straussian scheme (see Chapter 1). In fact, it can be demonstrated that the 'Aranda' and 'Gidjingali' models (figure 3.8, top and centre, right), representing distinct kinship structures, are both associated with the same underlying group of transformations.²⁰

APPENDIX

Proof of lemma 1: Under the usual associative composition of mappings, any finite product $g = z_1 z_2 \dots z_r$ with z_i an integral power of h or s belongs to $G(h, s)$. There are an infinite number of such elements g . In particular, $g_1 = h^x s^y$ and $g_2 = s^y h^x$. Then $(N_{ij})h^x s^y = (N_{i+x \ j+y})s^y = N_{i+x+by \ j+y}$ and $(N_{ij})s^y h^x = (N_{i+by \ j})h^x = N_{i+by+x \ j+y}$, with $j+y$ reduced modulo n . Hence, $h^x s^y = s^y h^x$, so that any $g = z_1 z_2 \dots z_r$ can always be rewritten as $g = h^p s^q =$

$s^q h^p$ by a reordering of the factors z_i . The inverse of g is $h^{-p} s^{-q}$, since $gg^{-1} = s^q h^p h^{-p} s^{-q} = s^q h^0 s^{-q} = s^q s^{-q} h^0 = s^0 h^0 = ee = e$. Thus $G(h, s)$ is a commutative group. Now $(N_{ij})h^n = N_{i+n} j+n = N_{i+n} j$ and $(N_{ij})s^t = N_{i+bt} j$. Since by definition $bt = n$, it follows that $h^n = s^t$, and n and t are the smallest positive integers for which this is the case. Finally, under the composition of mappings, any one element g of $G(h, s)$ generates a cyclical subgroup. $G(h)$, $G(s)$, and $G(h^{-1}s)$ are such subgroups, with $h^0 = s^0 = (h^{-1}s)^0 = e$.

Proof of lemma 2: According to lemma 1, $h^n = s^t$, hence $h^{pn} = s^{pt}$ for any p . Now $(N_{ij})h^{pn} = (N_{ij})s^{pt} = N_{i+pn} j$ and $(N_{ij})(h^{-1}s)^z = (N_{ij})h^{-z}s^z = N_{i+z(b-1)} j-z$. Then $p = b-1$ and $z = n$ are the smallest positive integers such that $pn = z(b-1)$, with $j-z$ congruent to j modulo n . Thus $h^{n(b-1)} = s^{t(b-1)} = (h^{-1}s)^n$.

Proof of lemma 4: Ego's wife is denoted by the mapping h and $d_{HW} = r$. According to lemma 3, mean age differences relative to male ego are computed by the formula $r(by+x)$ for any composite mapping $s^y h^x$. The general problem is to discover combinations of b , x , and y for which $r = r(by+x)$ or, equivalently, $by+x = 1$. The basic set of 31 kintypes in table 3.4 is restricted for positive integers b by limiting x and y to the values $-2, -1, 0, 1$, and 2 . By substitution in the equation it is easily seen that the only solutions are:

a. $y = 0$, $x = 1$, and any b ; $s^y h^x = h = \text{MBD, MMBDD, and FMBSD}$.

b. $y = 1$, $x = -1$, $b = 2$; $s^y h^x = sh^{-1} = \text{ZD and FZSD}$;
 $d_{MC}/d_{FC} = .500$.

c. $y = 1$, $x = -2$, $b = 3$; $s^y h^x = sh^{-2} = \text{FZDD}$;
 $d_{MC}/d_{FC} = .667$.

Of course, if one extends the list of basic kintypes, other 'oblique' marriages become a structural possibility.

For example, $FZDDD = s^2 h^{-3}$, and $FFZDDD = sh^{-3}$. These kintypes are of the 'correct' marriageable age for, respectively, structures with $b = 2$ and $b = 4$ (i.e., for d_{MC}/d_{FC} equal to .500 and .750).

NOTES

- 1 This chapter is a much revised and expanded version of a paper with the same title published in *American Ethnologist* (1983a) 10(3):585-604. A number of persons commented on earlier drafts and on the published version of the 1983 paper. I especially wish to thank John Atkins, Paul Jorion, and Woody Denham for their many helpful suggestions. My helical model owes much to the analysis of the Alyawara data presented by Denham, McDaniel, and Atkins (1979).
- 2 Macfarlane's paper is discussed in Needham (1971) and is mentioned in the introduction to Ballonoff (1974a); it is reprinted in Ballonoff (1974b). One of the original discussants was Sir Francis Galton, a cousin of Darwin's and founder of the science of eugenics. Galton's nonstatistical contributions to kinship theory are limited to a few short notes (Galton 1889). According to Needham, Radcliffe-Brown's early attempts at formalising made use of a Macfarlane-type notational system (1971:xxiv-xxviii).
- 3 Greenberg (1986:9) acknowledges an early debt to Kroeber's 1909 paper and to Carnap and the logical positivists. In turn, Carnap later included an entire section on axiom systems for kinship relations in his *Introduction to Symbolic Logic and its Applications* (1958:220-225, 230), first published in German in 1954. Gellner's controversial paper on 'Ideal Knowledge and Kinship Structure' (1957) parallels Greenberg's 1949 proposal in many ways. See also his discussion with Needham and Barnes (Gellner 1960 and 1963).
- 4 André Weil (b. 1906), in the 1930s co-founder (together with J. Dieudonné and H. Cartan) of the group of mathematicians publishing under the collective *nom de plume* of 'Nicolas Bourbaki'. See Chapter 1, note 19.
- 5 See Chapter 1, note 40.
- 6 Similar conclusions have been reached by demographers (Henry 1976:273-284, Cavalli-Sforza et al. 1966, and Casterline et al. 1986). References to early discussions of age bias are found in Needham (1966) and Rivière (1966c).
- 7 Lorrain (1975:127-128) summarizes the first two basic assumptions. I take the term 'generational

closure' from Atkins (1981). This principle is of course also found in the classic models of circulating connubium discussed in Chapter 1 and in the generalized exchange structures of the previous chapter. The use of 'prescription' (as opposed to 'preferential') has evoked one of the most vitriolic debates in recent anthropological history. For Lévi-Strauss (1970:xxxiii), 'a preferential system is prescriptive at the model level; a prescriptive system must be preferential when envisaged at the level of reality'. For a recent discussion see Barnard and Good (1984:100-106).

- 8 All necessary mathematical concepts are summarized in the appendix, together with short proofs of the lemmas. See also the appendix to Chapter 1.
- 9 My definition of d_{ij} differs from that of Reid (1974). I refer directly to paired reciprocal kintypes.
- 10 Cavalli-Sforza et al. (1966:55) suggest that the normal (Gaussian) approximation may be adequate for age differences between first cousins, but less so for relationships across generations.
- 11 'Simple' is not identical to 'elementary'. I use the term 'simple' in the sense of Granger (1983:135); see also Chapter 5. At a more pragmatic level, by simple structures I mean those that, when interpreted, are expressed by means of easily stated rules; e.g., 'One should marry a younger woman from the village, line, blood, etc. of one's mother's brother'. This pragmatic use of 'simple' is suggested by Jorion (1981: personal communication).
- 12 Although he stresses the importance of relative age (as opposed to relative generation), Good describes ZD marriage as reflecting a 'symmetric prescriptive terminology', not a system of exchange. Good's conclusions are phrased in reference to Needham's distinctions between preference and prescription, and rules, categories, and behaviour. See also Shapiro (1968), Brumbaugh (1978), Good (1981), Bossé (1983), Henley (1983-84), Barnard and Good (1984:98-104), and Hornborg (1988). The merging of kintypes through various types of marriage (including ZD-marriage) is also discussed by Kasakoff (1974). See also Yalman (1971).
- 13 The Sanyasi form a caste; the Korava (Lévi-Strauss 1970:425-426) are divided into three sections which are again divided into two exogamous moieties.
- 14 References to the literature on Ambrym can be found in Lane and Lane (1958), P.E. de Josselin de Jong (1966), Scheffler (1970a, 1984). An excerpt from T.T. Barnard's 1924 doctoral dissertation has recently been made available (Barnard 1986). See also Jorion (1986) and the historical overview by Langham (1981). I have now learned that John Atkins (1982: personal

communication) once proposed a helical scheme for the Ambrym identical to the model of figure 3.4 in an unpublished paper. Indeed, many of the helical models presented in this chapter have been obtained independently by Atkins. He is also the inventor of the helical scheme in Denham et al. (1979). Significantly, his metrical analyses (Atkins 1974a, 1974b, 1974c) extend the research pioneered by Macfarlane nearly a century earlier. However, as published, Atkins' helical model is not developed as a metrical structure superimposed on a commutative group.

- 15 This English gloss is taken from MacGaffey's summary of Doutreloux's marriage rules (1986:263, note 3).
- 16 Lévi-Strauss has recently (1983a:93-105) published his comments on McKnight. Thomson (1972; edited by Scheffler) presents additional material on the kin terminology systems in the Cape York area, including the Wikmunkan system.
- 17 See Boyd's (1969) seminal article and De Meur and Jorion (1981).
- 18 Denham 1982: personal communication.
- 19 See Shapiro (1968, 1969, 1971a, 1971b, 1981), Keen (1986), Fox (1969), Maddock (1969, 1972:45-71), and Needham (1986).
- 20 The reduced 'Gidjingali' helical model is based on eight patriline, four matriline and four helical exchange cycles. The corresponding algebraic group is isomorphic to the group described by Thomas and Wood (1980) as 'type 16/9', i.e. the group generated by $(m_G)^4 = (f_G)^2 = (h_G)^4 = e$, with $f_G = h_G m_G$. This group is also isomorphic to the order-16 group introduced by De Meur and Jorion (1981) in a somewhat *ad hoc* fashion, and to the 'expanded Aranda' kinship model with $(m_A)^4 = (f_A)^2 = (h_A)^2 = e$. The 'Gidjingali' and 'Aranda' groups are isomorphic under the following mapping:
 (i) $m_A \leftrightarrow m_G$; (ii) $f_A \leftrightarrow h_G$; (iii) $h_A \leftrightarrow h_G m_G^{-1}$.
 Between them, the lattices of quotient structures contain nearly all of the classic kinship structures. The group contains 7 elements of order 2 and 8 of order 4 (together with the identity).

4. SYMMETRIES OF RESTRICTED EXCHANGE: THE TWOFOLD PATH TO COMPLEXITY

There is a curious description in the closing paragraph of the first volume of the famous *Feynman Lectures on Physics* (Feynman et al. 1963:52-12):

Why is nature so nearly symmetrical? No one has any idea why. The only thing we might suggest is something like this: There is a gate in Japan, a gate in Neiko, which is sometimes called by the Japanese the most beautiful gate in all Japan; it was built in a time when there was great influence from Chinese art. This gate is very elaborate, with lots of gables and beautiful carving and lots of columns and dragon heads and princes carved into the pillars, and so on. But when one looks closely he sees that in the elaborate and complex design along one of the pillars, one of the small design elements is carved upside down; otherwise the thing is completely symmetrical. If one asks why this is, the story is that it was carved upside down so that the gods will not be jealous of the perfection of man. So they purposely put an error in there, so that the gods would not be jealous and get angry with human beings.

Perfect symmetry and absolute harmony may well be the prerogative of the gods. However, the search for symmetry - order *par excellence* - remains eminently fascinating to the human mind. Any notion of symmetry implies invariance: a thing (an object, a structure, a basic law) is symmetrical if one can subject it to a certain operation and it then appears unchanged.

The role of symmetry operations is now paramount in the elucidation of complex phenomena: invariants of physical laws are expressed by a set of transformations that form a mathematical group. For example, the symmetry group of Newtonian physics is the Galilean group, that of special relativity the Lorentz group, and that of general relativity the group of all automorphisms of the space-time manifold. For quantum chromodynamics

(QCD) the symmetry group is SU_3 , the group of unitary transformations in three dimensions.¹ It is now widely believed by physicists that the next step in the quest for a few simple laws underlying the apparent complexity of the universe and uniting Einstein's theory of gravity with the precepts of quantum mechanics might be achieved by a more comprehensive, supersymmetric theory of nature.²

To state the obvious: anthropology is not physics, a point made long ago by Frazer with regard to the study of Australian marriage classes: 'no one, so far as I know, has yet ventured to maintain that society is subject to a physical law, in virtue of which communities, like crystals, tend automatically and unconsciously to integrate or disintegrate, along mathematical lines, into exactly symmetrical units' (cited in Lévi-Strauss 1970: 136). Human societies, as opposed to natural systems, are fundamentally self-reflexive. Hence, under the programme of research set out in *Les Structures élémentaires de la parenté*, structures of marriage exchange arise directly from man's ability to think of biological relations as systems of oppositions: between men who own and women who are owned; between wives who are acquired and sisters and daughters given away; between bonds of alliance and of kinship; between consecutiveness and alternation. The awareness of such oppositions underlies and is used to build up a structure of reciprocity. 'Duality, alternation, opposition and symmetry, whether presented in definite forms or in imprecise forms, are not so much matters to be explained, as basic and immediate data of mental and social reality which should be the starting-point of any attempt at explanation' (Lévi-Strauss 1970:136). To paraphrase Lévi-Strauss, the study of symmetry operations (including duality, alternation and opposition) should be the starting-point of any attempt at explaining the occurrence of particular formulae of exchange. It is at this abstract level of analysis, experimenting on the

rules governing these transformations and operations as a group of invariants that the Lévi-Straussian approach approximates the level of theoretical discourse current in physics.³

In this chapter I investigate the global symmetries of one of the classical modes of reciprocity: the structure of restricted exchange, expressed in the symmetrical exchange of sisters for wives. I carry out this analysis by constructing an entire family of group-theoretic models. Symmetries of kinship structures are represented as automorphisms of the basic structure of direct sister-exchange. My purpose is to extend and to generalize Lévi-Strauss's treatment of the 'pure' forms of Australian class systems with dual organization and sister-exchange based on moieties, sections, or subsections to include recently described systems with 'exclusive straight sister-exchange' (in which first or second cross-cousin marriage is forbidden; cf. Muller 1980). I demonstrate that the family of kinship models developed here provides adequate representations for (1) systems with exclusive straight sister-exchange and lineal marriage prohibitions or a positive rule for the renewal of alliances only after a certain number of generations; (2) the intra- and intersystemic variation of alliance structures with direct exchange. I conclude with a discussion of hybrid forms and the problem of modelling complex marriage systems.

THE ROAD TO EXCLUSIVE STRAIGHT SISTER-EXCHANGE

Lévi-Strauss's elementary structure of restricted exchange, where the gift of a woman is reciprocated directly, is defined as follows (1970:146):

The term 'restricted exchange' includes any system which effectively or functionally divides the group into a certain number of pairs of exchange units so that, for any one pair X-Y there is a reciprocal

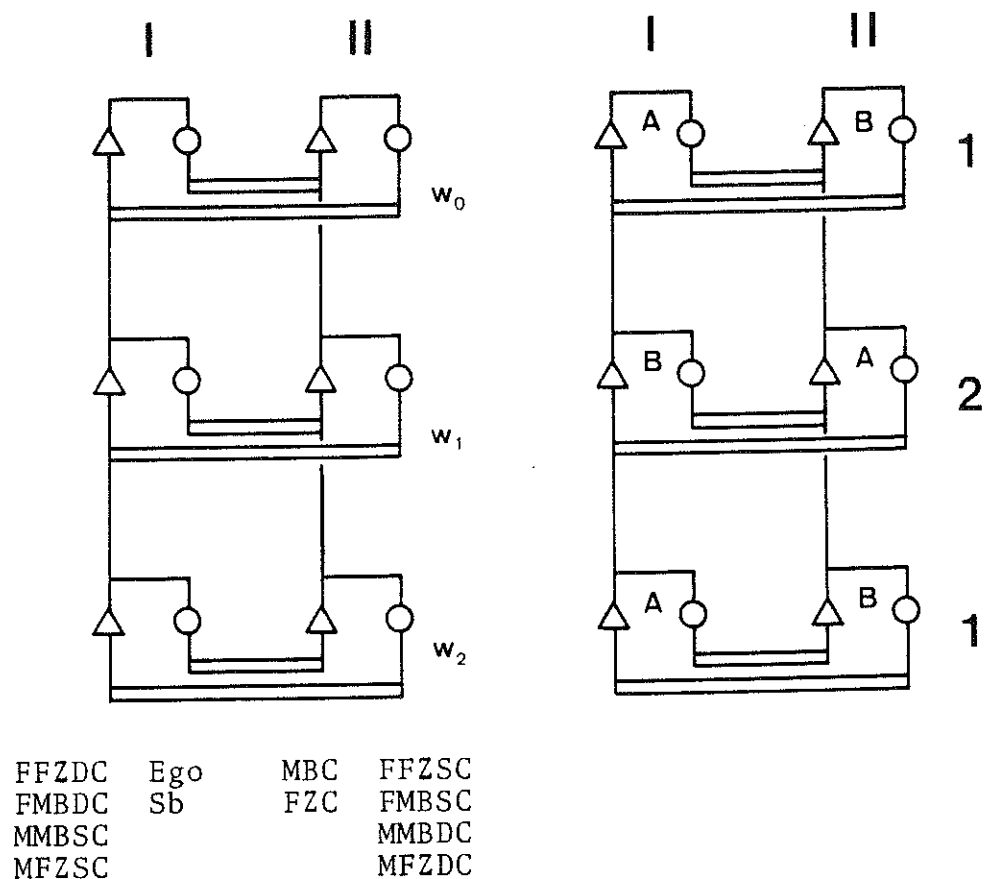


Fig. 4.1. Patrimoiety structure with restricted exchange (left); Kariera-type four-section system (right).

exchange relationship. In other words, where an X man marries a Y woman, a Y man must always be able to marry an X woman.

According to Lévi-Strauss, the 'simplest' form of restricted exchange is encountered in societies with dual organization and patrilineal or matrilineal exogamous moieties. An exogamous moiety division partitions all first cousins into cross and parallel cousins, excluding the latter as possible spouses. In such an elementary kinship structure a law of restricted exchange is directly expressed in straight sister-exchange and in a marriage rule prescribing marriage between

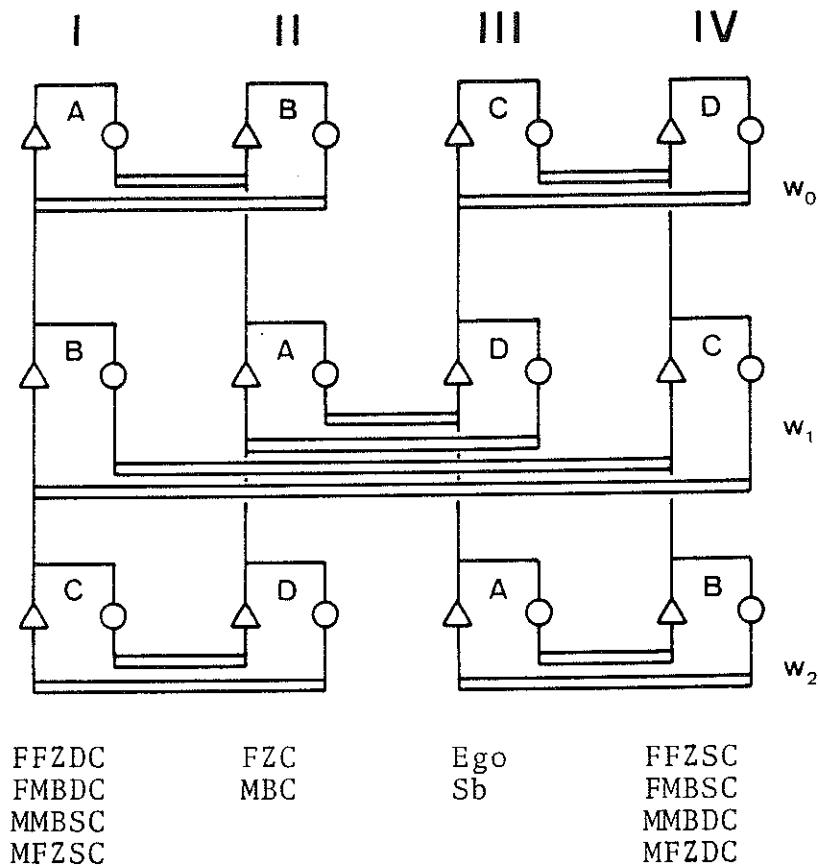


Fig. 4.2. Aranda-type kinship structure. Direct exchange renewable in alternative generations.

bilateral first cross-cousins. A model of a patrimoiety structure is presented in figure 4.1.(left).

The only other kinship systems with restricted exchange considered by Lévi-Strauss in detail in *Les Structures élémentaires* are two classical Australian systems, the Kariera and the Aranda. The Kariera system originally described by Radcliffe-Brown (1913:153-156) recognized two separate lines of patrilineal descent in the second ascending generation, i.e., of FF and MF. Marriage, based on the direct exchange of sisters, was with the MBD, who was also a FZD whenever possible. In addition, a sociocentric division of the entire society into four

named sections or classes (Banaka, Karimera, Burung and Palyeri) was superimposed on the kinship system. Ideally (Lévi-Strauss 1970:156):

Banaka necessarily marries Burung, and Karimera, Palyeri. The rule of descent is that the children of a Banaka man and a Burung woman are Palyeri, while the children of a Burung man and a Banaka woman are Karimera. Likewise, the children of a Karimera man and a Palyeri woman are Burung, and reversing the sexes with the classes remaining the same they are Banaka.

Elevated by Lévi-Strauss to the status of a structural model, the four-section system may be generated by imposing a matrilineal dichotomy (A, B) upon a patrilineal dichotomy (I, II), or vice versa. An equivalent four-section model is obtained by the intersection of alternating generations (1, 2) with either a patrimoiety (I, II) or a matrimoiety (A, B) structure. In any case, the basic structure of restricted exchange remains identical to that of a simple moiety system: bilateral cross-cousin marriage and sister-exchange, renewed in consecutive generations (cf. figure 4.1, left and right. See also the codings in Chapter 1.).

According to Lévi-Strauss, it is convenient to interpret the more complex Aranda-type eight-subsection structure as arising from a further dichotomy imposed on a Kariera-type model. Thus, at the level of the second ascending generation, four patriline, not two, are distinguished, i.e., of the FF, MF, FMB, and MMB. Direct exchange of sisters is practiced, but with the identical exchange now occurring in alternative generations, not consecutively as in the Kariera and moiety systems. First cross-cousins are distinguished from parallel cousins; both categories of relatives are considered unmarriageable. The category of second cousins is now bifurcated: FFZDC, FMBDC, MMBSC, and MFZSC (all unmarriageable), versus FFZSC, FMBSC, MMBDC, and MFZDC (the prescribed spouse category). Finally, the society is divided into eight non-overlapping subsections linked pairwise in direct

exchange. Under this interpretation, the subsection model, ideally in harmony with the kinship system, derives from the intersection of four patriline (I, II, III, IV) with a matrimoiety structure (cf. figure 4.2; A and C versus B and D) (Lévi-Strauss 1970:162-167).

In *Les Structures élémentaires* Lévi-Strauss argues that the transformational or generative potentialities of restricted exchange are realized by imposing a nested series of simple dichotomies upon the basic structure (1970:146):

The simplest form of restricted exchange is found in the division of the group into patrilineal or matrilineal exogamous moieties. If we supposed that a dichotomy based upon one of the two modes of descent is superimposed upon a dichotomy based upon the other the result will be a four-section instead of a two-moiety system. If the same process were repeated, the group would comprise eight sections instead of four. This is a regular progression and embodies nothing faintly resembling a change in principle or a sudden upheaval.

However, as the 'mechanical' development (i.e., the increase in the number of participating units) goes hand in hand with the 'organic' development of restricted exchange (i.e., development in the degree of interaction between units), the process is apparently ultimately functionally sterile (1970:441). Lévi-Strauss appears to question the very possibility of a rule of restricted exchange (as opposed to generalized exchange) being compatible with or able to generate kinship structures of any real complexity (1970:265):

... with generalized exchange the group can live as richly and as completely as its size, structure and density allow, whereas with restricted exchange ... it can never function as a whole both in time and in space. The latter ... is obliged, sometimes from the spatial point of view (local groups), sometimes from the temporal (generations and age classes), and sometimes from both at once, to function as if it were divided into more restricted units, even though these themselves are interconnected by the rule of descent. These rules, however, only succeed in restoring unity by, as it were, spreading it out in time, in other words at the price of a loss, i.e., the loss of time.

A further development of the disharmonic series⁴ towards 'theoretical types' with restricted exchange and $8 \times n$ classes is indicated (1970:216, fig. 44), as is the 'regression' from simple forms of generalized exchange to equally simple forms of restricted exchange (1970:474). However, the crucial transition from elementary to complex structures (through the substitution of bridewealth for the right to the cousin, or by the imposition of Crow-Omaha type marriage prohibitions) can only be realized by renouncing a simple form of *generalized* exchange for a more complex form (Lévi-Strauss 1970:xxxix-xlii, 471, 474). In this developmental sequence all structures with restricted exchange are clearly relegated to a secondary role.

Lévi-Strauss's views on the transition to complex structures have recently been challenged by Muller (1980, 1982). Muller's critique has particular bearing on the important question of how to amend the Lévi-Straussian scheme, with the complex structures of restricted exchange awarded a more prominent position in the sequence of transitions.

According to Muller (1980:518), *exclusive* straight sister-exchange (where sister-exchange is not followed by cross-cousin marriage) is ill-treated in *Les Structures élémentaires*: first, on account of a paucity of adequately described ethnographical cases at the time the book was written; second, by the seeming reluctance of Lévi-Strauss to consider even the possibility of such systems functioning as more than a 'technical form of the institution called "marriage by exchange"', i.e., as a nonstructural form (Lévi-Strauss 1970:143-144). The impression is given that exclusive straight sister-exchange is not considered structurally interesting by Lévi-Strauss on account of its supposed inability to generate or to sustain alliances through time between the same groups. Consequently, restricted exchange seems doomed to create only elementary structures by reverting to the marriage

of cross-cousins in consecutive generations (Muller 1980:520-521).

Muller goes on to demonstrate how Lévi-Strauss's scheme is partly invalidated by the literature on Africa (some of which was already available in the nineteen-thirties). Thus, contrary to Lévi-Strauss's expectations, in a relatively large number of African societies with sister-exchange, cross-cousin marriage is strictly forbidden, i.e., there are numerous examples of societies where exclusive straight sister-exchange constitutes the basic structure of marriage. Moreover, exclusive straight sister-exchange is often seen to coexist with an alternative marriage form, marriage with bridewealth payments. For Lévi-Strauss, bridewealth marriage should only occur in the transition of *generalized* exchange to more complex structures of exchange (Lévi-Strauss 1970: 471). Finally, exclusive straight sister-exchange is not restricted to 'simple', small-scale societies, or to societies having undergone dispersion and regression. This form of marriage structure is described for the Tiv (numbering about 800,000 people) and the Mossi, one of the most complex kingdoms of West Africa (Muller 1980: 518-520). Far from being a minor technical form, marriage structures based on exclusive straight sister-exchange are seen to be fully compatible with highly complex forms of social organization, including marriage with bridewealth transactions and complicated political systems.

On the basis of this evidence, Muller suggests that the Lévi-Straussian scheme is only one of several possibilities for passing from elementary to complex structures. Thus (Muller 1980:523):

I am inclined to think that a good number of societies having marriage with bridewealth do not derive from elementary structures, but have their origin in exclusive straight sister-exchange that has evolved to become bridewealth marriage without passing through the stage of an elementary structure. A modality of complex structure - exclusive straight sister-exchange - has

simply been replaced by another modality, marriage with bridewealth.

Other transitional possibilities are also envisaged (Muller 1980:526):

... the choice of repetition of alliances through cross-cousin marriage, and the opposite choice of diversifying them, is also given from the start. Some exclusive straight sister-exchange systems become elementary structures, whereas others choose to remain within complex structures, or at least within semi-complex structures, either through remaining that way or through the substitutive form of bridewealth marriage. All this is possible without any idea of an evolutionary sequence.

The possibility of treating systems with exclusive straight sister-exchange as semi-complex structures is explored in more detail in Héritier's *L'exercice de la parenté* (1981; see also Muller 1982).⁵ In semi-complex systems an extensive series of lineal marriage prohibitions, together with a privileged choice of spouses (not necessarily defined purely in genealogical terms) ensures that an alliance will, ideally, be renewed only after a specific number of generations. This is the type of model I develop in the following sections, articulating Lévi-Strauss's 'pure' systems with sister exchange (moieties, the Kariera, the Aranda) with the more complex systems of restricted exchange described by Muller and Héritier.

SEMI-COMPLEX STRUCTURES AS AUTOMORPHISM GROUPS

The Bardi and the Ngawbe

The family of algebraic models extending the basic structure of restricted exchange is subject to a fairly stringent set of constraints. First, at the lower level of complexity, all classical models embodying the principle of restricted exchange (i.e., moieties, the

Kariera, the Aranda) must be adequately represented. Second, a simple and unambiguous formal procedure is required for expanding the basic model so as to include, at the more complex end of the spectrum, aspects of the semi-complex structures of alliance described by Muller (1980, 1982) and H  ritier (1981). Third, the formal model should encompass all structures of restricted exchange occurring within the context of a particular kinship system as optional strategies or free variants. (This is only a minimal set of constraints which may be expanded if necessary.)

The Bardi-type kinship system provides constraints of the third type. As described by Elkin (1932), the Bardi system, though similar to the better-known Aranda in many respects, had neither moieties nor sections.⁶ The Bardi (and the Aranda) distinguished four patriline at the level of the second ascending ('grandparent') generation: of the FF, MF, FMB, and MMB. Marriage was based on the direct exchange of sisters. However, the Bardi preference was for a spouse obtained from the patri-group or patriline of one's MMB, not, as with the Aranda or the Kariera, the patriline of the FM, or that of the MB.

Turner has recently published an interesting reanalysis of Elkin's data. He demonstrates that an entirely consistent structure, incorporating the Bardi marriage and exchange preferences as well as the kinship terminology can be formulated (Turner 1980:64-68). The Bardi-type structure (see my figure 4.3, isomorphic with Turner's figure 12) is based on a rule of restricted exchange linking four patriline, with the identical alliance renewed every third generation, not in alternating generations (cf. the Aranda, fig. 4.2) or consecutively (cf. the Kariera and moiety structures, fig. 4.1). At the level of the model, the Bardi impose a series of dichotomies on relatives in ego's generation similar to the Aranda model. Thus, first cross-cousins

are distinguished from parallel cousins; both categories are unmarriageable. The second cousin category is also bifurcated: FFZDC, FMBDC, MMBSC, and MFZSC, versus FFZSC, FMBSC, MMBDC, and MFZDC. However, in direct opposition to the Aranda, the preferred spouse is obtained from the category of the MMBSC, not the FMBSC. As can be seen in figure 4.3, each of the patriline enters into an exchange relationship with each of the remaining three in turn, precluding the formation of an exogamous moiety structure.

As analysed by Turner (1980:67), Elkin's data (1932: 311) suggest that the Bardi recognized as a valid alternative, marriage with the MMBDC, i.e., an Aranda-type marriage preference (cf. fig. 4.3 and fig. 4.2). In sum: three-generation and two-generation forms of restricted exchange occur as alternatives within the context of a four-patriline model.

The phenomenon of a society acknowledging the possibility of two apparently incompatible marriage preferences, both hinging on the direct exchange of sisters, has been documented in even more convincing detail for the Ngawbe or Western Guaymí. As described by Young (1970), the Ngawbe, a Chibchan-speaking people presently inhabiting the three western-most provinces of the Republic of Panama, do not formally recognize any form of unilineal descent reckoning. The primary kin group is cognatic, consisting of all living people born in a given hamlet who are consanguineally related. Marriage, ideally a direct (symmetric) exchange of 'sisters' (or 'sister's daughters'), is conceived of as the basis of a system of alliance between kin groups, not as a simple union of individuals. Ego's generation is characterized by a Hawaiian-type cousin terminology, with terms glossed as 'same-sex-sibling' (*edaba*) and 'opposite-sex-sibling' (*ngwae*) referring to all first cousins, all parallel second-cousins, and some second cross-cousins. No *ngwae* is considered marriageable (Young

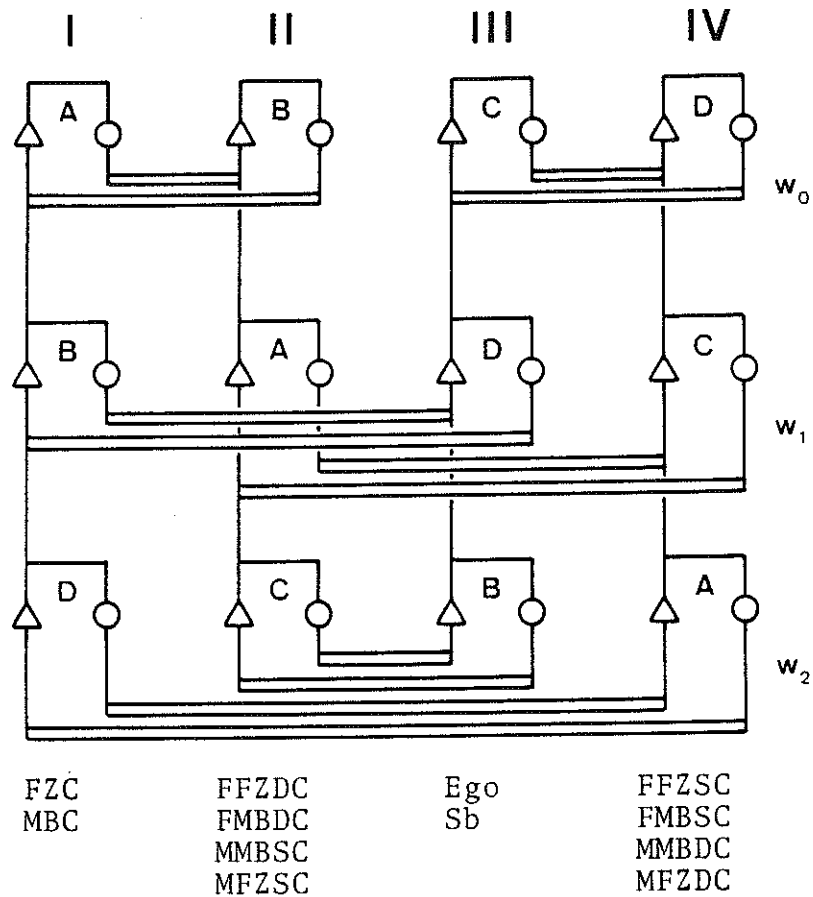


Fig. 4.3. Bardi-type kinship structure. Direct exchange renewable every third generation.

1970:86-87, 90-91).

According to Young (1970:87-88) the same term *uduaw*, 'payment', is used when referring to a woman received in direct exchange and in the context of goods of any kind exchanged. All marriage links function in the context of economic and political mobilisation, with sororal polygyny strengthening existing links, and nonsororal polygyny allowing the formation of new alliances. From the perspective of male ego, same-sex and opposite-sex sibling terms repeat in cyclical fashion every third generation, with FFF and male same-sex sibling equated.

However, from the female point of view, the kinship terminology exhibits a number of alternate generation equivalences.

The Ngawbe prohibit marriage with all types of first cousin. Young reports 'a theoretical preference' for marriage with a man's FFZDD, a patrilateral second cross-cousin (1970:90). His figure 1 (1970:89) incorporates all of the above-mentioned features; as a structure of restricted exchange it is isomorphic with the Bardi-type three-generation model illustrated in my figure 4.3. In addition, the Ngawbe recognize the option of re-establishing an initial alliance after only two generations: in this case marriage is with a matrilateral second cross-cousin (ego's MMBDD).⁷ If practiced systematically, this option will generate the Aranda-type structure of figure 4.2. These marriage strategies are complementary: the Ngawbe forbid marriage with a person who is both FFZDD and MMBDD to male ego (1970:91-93).

I now introduce a set of formal exchange models that is fully compatible with the constraints mentioned above, including the Bardi-Ngawbe type of intra-systemic variation.

Automorphisms of the dihedral group

The family of direct exchange models presented here is developed in complete analogy to the models of generalized exchange introduced earlier.⁸ Thus, the set of formal objects (exchange units) is obtained by mapping a two-dimensional coordinate grid onto the surface of a cylinder. 'Descent lines' (i.e., the trajectories of particular units through time) parallel the cylinder's main axis, with 'generations' located on distinct planes of simultaneity perpendicular to the major (time) axis. After defining a basic structure of restricted exchange as a permutation r of order 2, the exchange configuration in each successive generation is defined recursively as

an automorphism of its immediate predecessor. In algebraic terms, a semi-complex structure of restricted exchange is defined by means of the automorphisms of the dihedral group of which r is one of the two generating permutations.

Let I be the index set $I = \{1, 2, \dots, 2m\}$ and let Z be the set of integers. Obj , the basic set of objects, is defined as $\{O_{ij} | i \text{ in } Z \text{ and } j \text{ in } I\}$. Let $Obj(i)$ be the subset of Obj defined as $\{O_{ij} | j \text{ in } I\}$ for some i in Z . As mentioned in Chapter 2, $Gen = \{Obj(i) | i \text{ in } Z\}$ is a partition of Obj (representing a partition into non-overlapping generations).

Consider any $Obj(i)$ in Gen . Define the permutations r and q of order two as

$$r = (O_{i1}, O_{i2})(O_{i3}, O_{i4}) \dots (O_{i2m-1}, O_{i2m})$$

and

$$q = (O_{i2m}, O_{i1})(O_{i2}, O_{i3}) \dots (O_{i2m-2}, O_{i2m-1}).$$

To simplify the notation, in the following sections r , q and other permutations defined on $Obj(i)$ will be formulated as permutations of the index set I , e.g.,

$$r = (1, 2)(3, 4) \dots (2m-1, 2m) \text{ and}$$

$$q = (2m, 1)(2, 3) \dots (2m-2, 2m-1).$$

By direct computation,

$$rq = (1, 3, 5, \dots, 2m-1)(2m, 2m-2, \dots, 6, 4, 2)$$

and hence $(rq)^m = e$ (the identity permutation).

The group $D_m(r, q; r^2 = q^2 = e, (rq)^m = e)$ generated by r and q is the *dihedral group of order $2m$* .

D_m is one of the classical groups arising in the study of symmetry operations and the invariant properties of finite patterns. For example, it represents the group of rotational symmetries of a regular m -sided prism, a three-dimensional object, or equivalently, the group of all symmetries (including reflections) of a plane (two-dimensional) regular m -sided polygon.⁹ In fact, D_m and C_n (the cyclic group of order n associated with the automorphisms of generalized exchange structures; cf. Chapter 2) are the *only* possible infinite families of

finite symmetry groups in three-dimensional space.¹⁰

In contrast with the cyclical group C_n generated by the one permutation $c = (1, 2, \dots, n)$ of order n , every dihedral group D_m requires two generators. Conversely, if a group is generated by two permutations of order two, it is always dihedral. However, there are alternative ways of defining D_m . For r and q as before, let $a = rq$ be the permutation of order m defined on I . Then $r^2 = a^m = e$, $ra = a^{-1}r$ are optional defining relations for D_m .

I now sketch the strategy to be followed. First, for some index set $I = \{1, 2, \dots, 2m\}$, define the permutations r , q , and $a = rq$ and generate the dihedral group $D_m(r, a; r^2 = a^m = e, ra = a^{-1}r)$. The permutation r represents the basic structure of restricted exchange linking the $2m$ exchange units of $Obj(i)$ in pairs. Second, derive the automorphism group $Aut(D_m)$ of D_m . Among the automorphisms of D_m select the subset A of those which do not leave the permutation r invariant. Then, for any α in A , $\alpha^p = 1$ (the identity automorphism) for some integer p . Finally, the ordered p -tuple $D = (r, (r)\alpha, (r)\alpha^2, \dots, (r)\alpha^{p-1})$, recursively defined, is interpreted as a closed, semi-complex structure of restricted exchange defined on $2m$ descent lines and repeating after p generations.

Structures of restricted exchange

Let $D_m(r, a; r^2 = a^m = e, ra = a^{-1}r)$ be the dihedral group of order $2m$ generated by r and a . Let H and K be the cyclical subgroups of D_m generated by, respectively, r and a . For $m > 2$, K is the only cyclic subgroup of order m , and K is a normal subgroup of D_m .¹¹ Note that $K \cap H = \{e\}$. Then every element g of D_m can be expressed as $g = r^i a^j$, where i is 0 or 1 and $0 \leq j < m$, i.e., $D_m = HK$.

Let $Aut(D_m)$ be the automorphism group of D_m . For α an automorphism and r and a elements of D_m , then let

$$\begin{aligned}(r)\alpha &= ra^i \text{ and} \\ (a)\alpha &= a^j\end{aligned}$$

for some integers i and j , with j relatively prime to m and $0 \leq i < m$. Conversely, for any integers i and j , with j relatively prime to m , the mapping α of D_m into itself defined by $\alpha: r^s a^t \rightarrow (ra^i)^s (a^j)^t$ for s equal to 0 or 1 and $0 \leq t < m$, is an automorphism of D_m ($m > 2$).

All automorphisms of D_m can be systematically derived from the generating permutations r and a of D_m by obtaining mappings for all combinations of integral values of s , t , i and j (as constrained above). The automorphism group $\text{Aut}(D_m)$ can be formulated as $\text{Aut}(D_m) = H_0 K_0$, where H_0 is the set of all automorphisms which leave a invariant, and K_0 is the set of all automorphisms which leave r invariant. Note that $K_0 \cap H_0 = \{1\}$, with 1 the identity automorphism. H_0 is a normal cyclic subgroup of order m , and K_0 is an abelian group of order $\phi(m)$, where ϕ is the Euler function (see Chapter 2, table 2.1). Finally, the following proposition is presented (see the Appendix for a definition of the important construction called a *semidirect product*).^{1,2}

Lemma 1: Consider D_m with subgroups H and K and $\text{Aut}(D_m)$ with subgroups H_0 and K_0 as defined above. Then D_m is isomorphic to the semidirect product of H by K , and $\text{Aut}(D_m)$ is isomorphic to the semidirect product of H_0 by K_0 , i.e., $D_m \cong H \rtimes_f K$, and $\text{Aut}(D_m) \cong H_0 \rtimes_g K_0$.

The group-theoretic apparatus is now applied to the cylindrical structure Obj . For some $Obj(\bar{a})$ in Gen , define the direct exchange permutation w_0 by $O_{\bar{a}j} w_0 = O_{\bar{a}j} r$ for all $O_{\bar{a}j}$ in $Obj(\bar{a})$. Let s , the lineal successor mapping, be the one-to-one translation defined by $(O_{ij})s = O_{i+1j}$ for all O_{ij} . Now, for some α in A (the subset of $\text{Aut}(D_m)$ which does not leave the basic permutation r invariant), define the exchange permutation w_1 of $Obj(\bar{a} + 1)$ by $O_{\bar{a}j} s w_1 = (O_{\bar{a}j} w_0) \alpha s$. Continuing in this

manner, define further exchange mappings w_2, w_3 , etc. of, respectively, $Obj(\bar{a}+2), Obj(\bar{a}+3)$, etc., as $O_{\bar{a}+1} j^{sw_2} = (O_{\bar{a}+1} j^{w_1})\alpha s$, $O_{\bar{a}+2} j^{sw_3} = (O_{\bar{a}+2} j^{w_2})\alpha s$, etc.

In general, for any positive integer x the direct exchange permutation w_x of $Obj(\bar{a}+x)$ may be recursively defined by $O_{\bar{a}+x-1} j^{sw_x} = (O_{\bar{a}+x-1} j^{w_{x-1}})\alpha s$, with $w_0 = r$. An equivalent non-recursive definition is given by

$$O_{\bar{a}j} s^x w_x = (O_{\bar{a}j} w_0) \alpha^x s^x.$$

The structure of restricted exchange with origin $Obj(\bar{a})$, induced on Obj by the permutations r and s for D_m and some α in A , is defined by $D(\bar{a}, 2m, \alpha) = \{w_x | \text{for all integers } x\}$. The period of $D(\bar{a}, 2m, \alpha)$ is given by $p(\alpha)$, the order of α , and is the smallest positive integer x such that $w_x = r = w_0$.

In the following section I apply these results, deriving all possible kinship structures with restricted exchange and up to ten descent lines, i.e., the complete set of exchange models generated by D_1, D_2, D_3, D_4, D_5 and their automorphism groups.

The classical systems: automorphisms of D_1 and D_2

For $I = \{1, 2\}$, i.e., for $m = 1$, define the basic permutation $r = (1, 2)$. Then $D_1(r; r^2 = e)$, the dihedral group of order 2, is the only dihedral group generated by a single permutation. D_1 is isomorphic to C_2 , the cyclic group of order 2, and $\text{Aut}(D_1) \cong E(1)$, the trivial automorphism group consisting of the identity automorphism 1. Interpreted as a kinship structure, D_1 and $\text{Aut}(D_1)$ give rise to a simple exogamous moiety structure with sister exchange and bilateral first cross-cousin marriage. Successive exchange cycles are identical: $w_x = w_0 = (r)1 = r$ for all x .

$D(\bar{a}, 2, 1)$ is identical to $W(\bar{a}, 2, 1)$, the limiting structure obtained by applying the formula of generalized exchange to a two-line system. The reduced structure of

period $p(1) = 1$ associated with $D(\bar{a}, 2, 1)$ is presented in figure 4.4 (top, left). The Kariera four-section model is obtained by doubling the basic period (figure 4.4, top, right).

For $I = \{1, 2, 3, 4\}$, i.e., for $m = 2$, define the basic permutations $r = (1, 2)(3, 4)$ and $a = (1, 3)(4, 2)$. Then $ra = (4, 1)(2, 3)$, and $D_2(r, a; r^2 = a^2 = e, ra = ar)$ is the commutative dihedral group of order 4. Since $m = 2$, the automorphisms of D_2 cannot be obtained by substituting the appropriate values for s, t, i and j in the mapping $r^s a^t \rightarrow (ra^i)^s (a^j)^t$.

By direct computation, $\text{Aut}(D_2)(\beta, \alpha; \beta^2 = \alpha^3 = 1, \beta\alpha = \alpha^2\beta)$ is obtained, with $\beta = (a, ra)$ and $\alpha = (r, ra, a)$. Also, $\text{Aut}(D_2) \cong D_3$, hence $|\text{Aut}(D_2)| = 6$. The remaining four elements are the identity 1, $\alpha^2 = (r, a, ra)$, $\beta\alpha = (r, ra)$, and $\beta\alpha^2 = (r, a)$. The invariant subgroups for a and r are $H_0 = \{1, \beta\alpha\}$ and $K_0 = \{1, \beta\}$. However, as $m < 3$, $H_0 K_0 = \{1, \beta, \beta\alpha, \alpha^2\} \neq \text{Aut}(D_2)$. $A = \text{Aut}(D_2) - K_0 = \{\alpha, \alpha^2, \beta\alpha, \beta\alpha^2\}$; hence there are exactly four exchange structures based on four descent lines: $D(\bar{a}, 4, \alpha)$ and $D(\bar{a}, 4, \alpha^2)$ of period 3, and $D(\bar{a}, 4, \beta\alpha)$ and $D(\bar{a}, 4, \beta\alpha^2)$ of period 2.

By a simple renumbering procedure, both period 2 structures can be shown to represent the same basic kinship structure, the Aranda-type eight-section system. Similarly, both exchange structures of period 3 represent the Bardi or Ngawbe-type kinship structure. The reduced structures $D(\bar{a}, 4, \beta\alpha)$ (the Aranda) and $D(\bar{a}, 4, \alpha^2)$ (the Bardi or Ngawbe) are illustrated in figure 4.4 (below; compare with figures 4.2 and 4.3).

The Bun and the Manga: automorphisms of D_3

For $I = \{1, 2, 3, 4, 5, 6\}$, i.e., for $m = 3$, define $r = (1, 2)(3, 4)(5, 6)$ and $a = (1, 3, 5)(6, 4, 2)$; $D_3(r, a; r^2 = a^3 = e, ra = a^2 r)$. The other four elements are the identity e , $a^2 = (5, 3, 1)(2, 4, 6)$, $ra =$

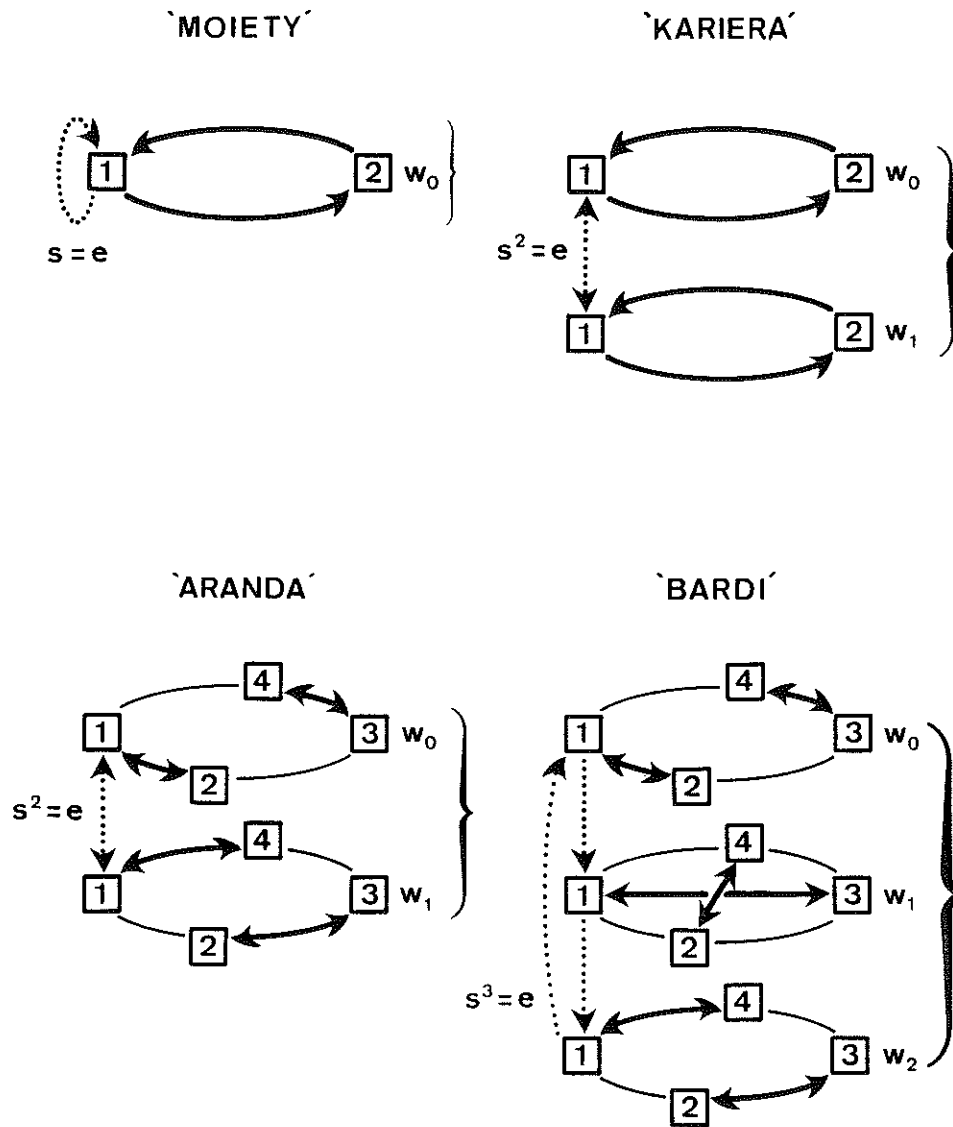


Fig. 4.4. Reduced structures of restricted exchange associated with $D(\bar{a}, 2, 1)$ (top), $D(\bar{a}, 4, \beta\alpha)$ (bottom, left), and $D(\bar{a}, 4, \alpha^2)$ (bottom, right).

$(6, 1)(2, 3)(4, 5)$, and $ra^2 = (1, 4)(2, 5)(3, 6)$. Now, since $m > 2$, the complete set of automorphisms can easily be obtained by substituting the values $s = 0$ or 1 and $0 \leq t < 3$ in $g = r^s a^t$ (giving all elements g of D_3), and then deriving each automorphism γ as a mapping $\gamma: g = r^s a^t \rightarrow (ra^i)^s (a^j)^t$ for combinations of values of i and j . The identity automorphism 1 is obtained for $i = 0$ and $j = 1$ (note that $0 \leq i < 3$, and j is relatively prime to m , i.e., $j = 1$ or 2). In like manner, all of the other automorphisms are derived:

$$\begin{aligned}\alpha &= (r, ra, ra^2) \text{ for } i = 1 \text{ and } j = 1; \\ \alpha^2 &= (r, ra^2, ra) \text{ for } i = 2 \text{ and } j = 1; \\ \beta &= (a, a^2)(ra, ra^2) \text{ for } i = 0 \text{ and } j = 2; \\ \beta\alpha &= (a, a^2)(r, ra) \text{ for } i = 1 \text{ and } j = 2; \text{ and finally,} \\ \beta\alpha^2 &= (a, a^2)(r, ra^2) \text{ for } i = 2 \text{ and } j = 2.\end{aligned}$$

This is indeed the complete set of automorphisms, since $|\text{Aut}(D_3)| = m \times \phi(m) = 3 \times 2 = 6$. Hence the automorphism group is $\text{Aut}(D_3)(\beta, \alpha; \beta^2 = \alpha^3 = 1, \beta\alpha = \alpha^2\beta)$.

The invariant subgroups of a and r are $H_0 = \{1, \alpha, \alpha^2\}$ and $K_0 = \{1, \beta\}$, with $H_0 K_0 = \text{Aut}(D_3)$. Then $A = \text{Aut}(D_3) - K_0 = \{\alpha, \alpha^2, \beta\alpha, \beta\alpha^2\}$ and there are exactly four structures of restricted exchange based on six descent lines: $D(\bar{a}, 6, \alpha)$ and $D(\bar{a}, 6, \alpha^2)$ of period 3, and $D(\bar{a}, 6, \beta\alpha)$ and $D(\bar{a}, 6, \beta\alpha^2)$ of period 2 (see figure 4.5).

One may now derive the complete family of six-line kinship models with restricted exchange. A number of surprising features are revealed. In the first place, there are only two distinct sets of marriage possibilities: male ego's spouse is either (1) his MMBDD and MFZDD, or (2) his FMBSD and FFZSD. Second, MMBDD and MFZDD marriage occurs only in combination with patrilineal descent and a 3-generation rule, or with matrilineal descent and alternating generations. Conversely, FMBSD and FFZSD marriage is associated either with a 3-generation matri structure or with an alternating generation structure based on patrilineal descent. Finally, any

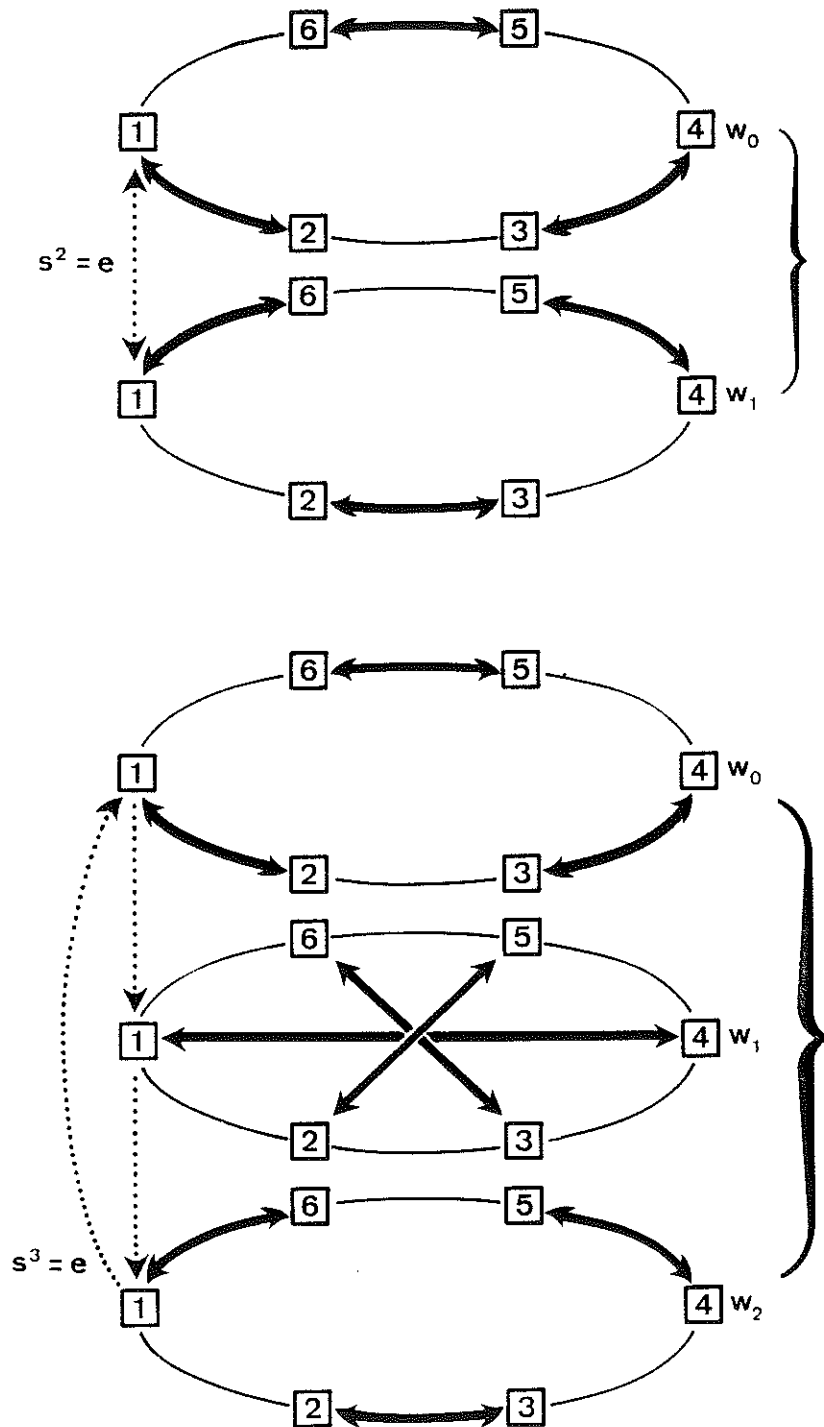


Fig. 4.5. Reduced structures of restricted exchange associated with $D(\bar{a}, 6, \beta\alpha)$ (top) and with $D(\bar{a}, 6, \alpha^2)$ (bottom).

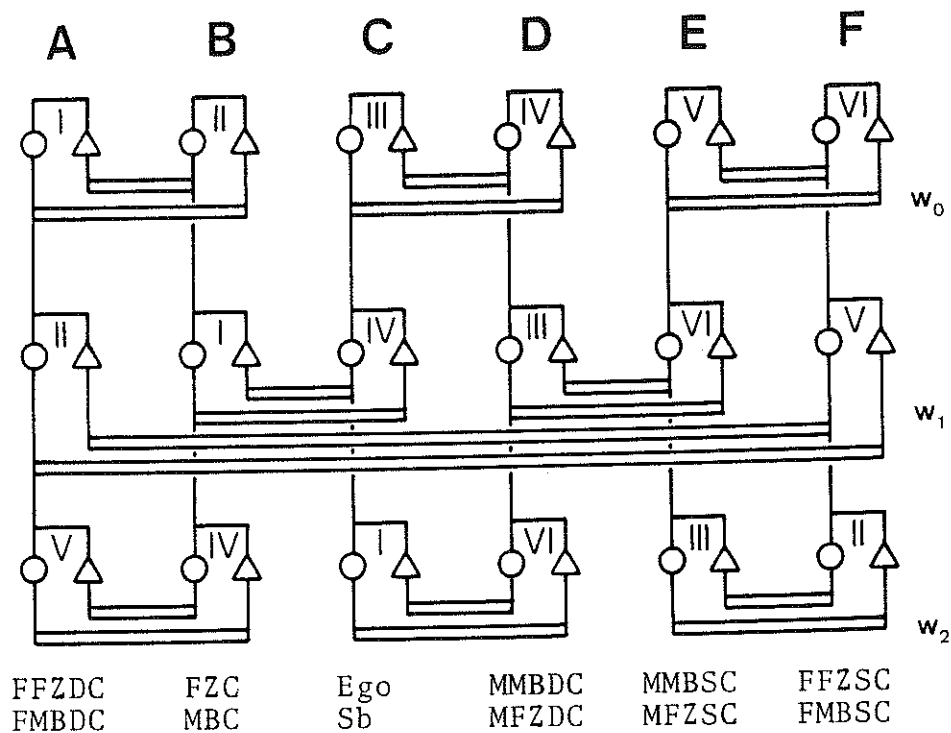


Fig. 4.6. Bun-type kinship structure based on six matrilineal lines and the 2-generation structure $D(\bar{a}, 6, \beta\alpha)$. Latent 3-generation structure based on six patrilineal lines and the exchange structure $D(\bar{a}, 6, \alpha^2)$.

single *kinship* model is seen to incorporate two distinct *exchange* structures with seemingly incompatible periodicities.

These features are highlighted in figure 4.6. The diagram depicts a six matriline structure with restricted exchange. Male ego marries his MMBDD (merged with his MFZDD) and the exchange structure repeats itself in alternating generations. With the six matrilineal lines coded as $A = 1, B = 2, \dots, F = 6$, the associated exchange structure is $D(\bar{a}, 6, \beta\alpha)$. However, if the formal structure of six 'submerged' patrilineal lines (numbered I, II, ..., VI) is traced out in the model, a different structure of restricted exchange is seen to apply. In terms of patrilineal descent, the underlying exchange

structure is $D(\bar{a}, 6, \alpha^2)$, with marriages between the same patriline repeating in three generations, not two. In fact, if one so wishes, these apparent inconsistencies may be resolved by noting that the model of figure 4.6, pictured as a structure of *double-descent*, repeats in exactly six generations (six being of course the least common multiple of two and three).

My model is identical to the six matriline model of sister-exchange marriage presented by McDowell (1977: 180, fig.3) for the Bun, a New Guinea people living along the Yuat river in the East Sepik District. Traditionally the Bun, close neighbours of the Mundugumor, recognized six named, matrilineal clans: Parrot, Rat, Pig, Crocodile, Cassowary, and Great Hornbill (the last two now being extinct). Ideally, clans are exogamous, but intra-clan marriages occur. Clan membership is relevant in two contexts: land tenure and marriage. Since title to clan land can be claimed by anyone whose father belonged to the group, a married couple ideally has rights to tracts of land (and the produce) associated with the following four clans: husband's, wife's, husband's father's, and wife's father's (McDowell 1977:176-177).

Clans do not, however, in any real sense 'regulate' marriage exchanges. 'The people of Bun do not conceptualize exchanges as occurring between clans, but between sibling sets. ... marriage arrangements are always discussed in terms of "roads" ... these relations are not conceptualized as structures or positions but as movements and process' (McDowell 1977:181).

Sister exchange is the ideal (as well as the statistical norm). Marriage is forbidden within one's own and one's father's clan, and with all categories of kin except classificatory cross-cousins. Furthermore, marriages should be arranged so that the structural relations repeat in alternating generations (the couple

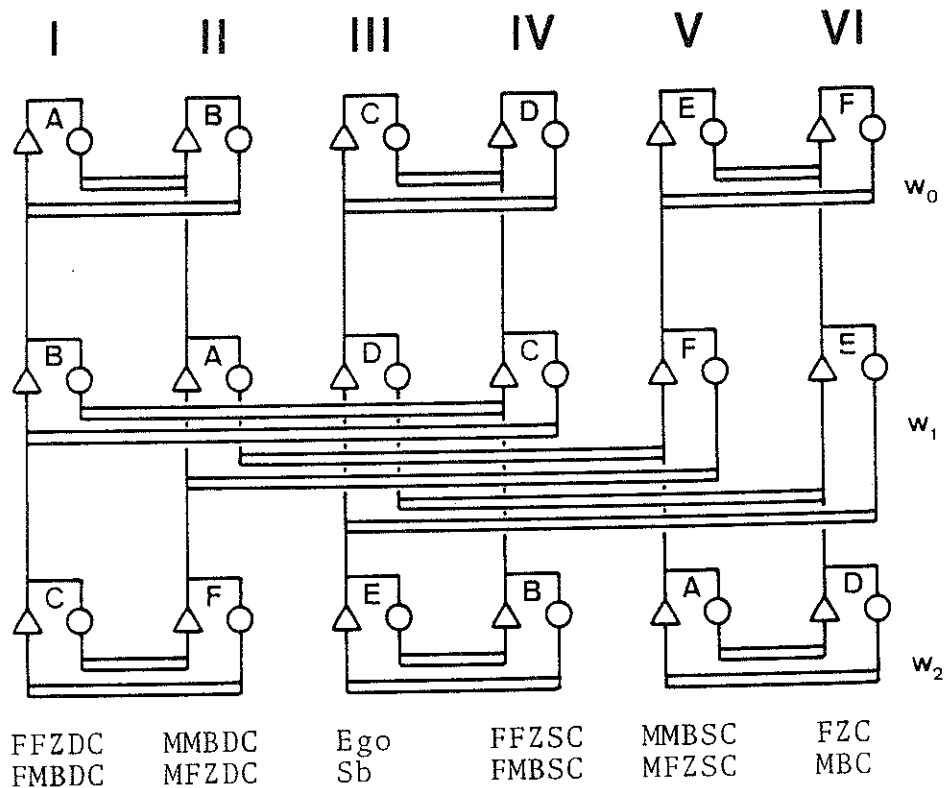


Fig. 4.7. Manga-type kinship structure based on six patrilineal units and the 2-generation structure $D(\bar{a}, 6, \beta\alpha^2)$. Latent 3-generation structure based on six matrilineal units and the exchange structure $D(\bar{a}, 6, \alpha)$.

then re-establishing rights of access to the same lands as their matrilineal grandparents). In the context of marriage and descent, the crucial term 'rope' (*geun*) - associated with a principle of alternating descent - is used metaphorically by the Bun. *Geun* refers to the reduplication of exchanges and expresses the reintegration of Bun society and the ideal of inter-connectedness created and maintained by the structure of exchange (McDowell 1977:178-179, 182-183). McDowell's six line model summarizes the Bun data on kinship and exchange.

The Bun model is a transformational variant of a second six-line model that may be applied to the kinship systems of other New Guinea societies in the Sepik area.

In contrast with the Bun, the Manga, Maring and Kuma recognize exogamous patriclans. As is the case with the Bun, direct sister-exchange (of real or classificatory sisters) is the ideal, with the same marriage exchange renewable in alternating generations. Marriage with any first cousin is forbidden; there is a preference for marriage with a second cousin, specified for the Manga and the Maring as the FFZSD (not the MMBDD or MFZDD) (Rosman and Rubel 1975:115). In native terms, the renewal of marriages in alternating generations is described as 'following one's road', 'forming a pig-woman road', or 'returning the planting material', with the granddaughter marrying back into the clan or patriline of her grandmother (Rosman and Rubel 1975:115; Rubel and Rosman 1978:253).

The kinship structure of figure 4.7 is the simplest proper model for the Manga, Maring and Kuma data. The model (embodying six patriline) is associated with the 2-generation exchange structure $D(\bar{a}, 6, \beta\alpha^2)$. However, if one identifies the exchange structure defined on the six formal matriline $A, B, \dots, F$, this turns out to be the 3-generation structure $D(\bar{a}, 6, \alpha)$. My model is certainly more faithful to the ethnographic data than the Aranda-type model proposed by Cook (described in Rosman and Rubel 1975:115; Rubel and Rosman 1978:256), in which ego's spouse is described as FFZSD and FMBSD, but also MMBDD and MFZDD.¹³

In sum: it can easily be demonstrated that, up to an isomorphism, the kinship models of figures 4.6 and 4.7 are the only distinct models generated by the four six-line structures of restricted exchange. Any other combination of structures (e.g., combining a 3-generation patri-structure based on $D(\bar{a}, 6, \alpha)$ with a 2-generation matri-structure associated with $D(\bar{a}, 6, \beta\alpha^2)$) may, by a simple process of renumbering, be identified with one of the two kinship models described in this section.

Automorphisms of D_4

For $I = \{1, 2, 3, 4, 5, 6, 7, 8\}$, i.e., for $m = 4$, define $r = (1, 2)(3, 4)(5, 6)(7, 8)$ and $a = (1, 3, 5, 7)(8, 6, 4, 2)$. These permutations define the dihedral group $D_4(r, a; r^2 = a^4 = e, ra = a^3r)$ of order 8. The remaining six elements of D_4 are the identity e ,

$$a^2 = (1, 5)(2, 6)(3, 7)(4, 8),$$

$$a^3 = (7, 5, 3, 1)(2, 4, 6, 8),$$

$$ra = (8, 1)(2, 3)(4, 5)(6, 7),$$

$$ra^2 = (1, 6)(2, 5)(3, 8)(4, 7), \text{ and}$$

$$ra^3 = (1, 4)(2, 7)(3, 6)(5, 8).$$

The automorphisms γ of D_4 are obtained by substitution from the mapping $\gamma: r^s a^t \rightarrow (ra^i)^s (a^j)^t$, i.e., for $s = 0$ or 1, $0 \leq t < 4$, $0 \leq i < 4$, and $j = 1$ or 3. They are:

$$1 \text{ (the identity) for } i = 0, j = 1;$$

$$\alpha = (r, ra, ra^2, ra^3) \text{ for } i = 1, j = 1;$$

$$\alpha^2 = (r, ra^2)(ra, ra^3) \text{ for } i = 2, j = 1;$$

$$\alpha^3 = (r, ra^3, ra^2, ra) \text{ for } i = 3, j = 1;$$

$$\beta = (a, a^3)(ra, ra^3) \text{ for } i = 0, j = 3;$$

$$\beta\alpha = (a, a^3)(r, ra)(ra^2, ra^3) \text{ for } i = 1, j = 3;$$

$$\beta\alpha^2 = (a, a^3)(r, ra^2) \text{ for } i = 2, j = 3; \text{ and}$$

$$\beta\alpha^3 = (a, a^3)(r, ra^3)(ra, ra^2) \text{ for } i = 3, j = 3.$$

The automorphism group is $\text{Aut}(D_4)(\beta, \alpha; \beta^2 = \alpha^4 = 1, \beta\alpha = \alpha^3\beta)$. The invariant subgroups of a and r are $H_0 = \{1, \alpha, \alpha^2, \alpha^3\}$ and $K_0 = \{1, \beta\}$. Hence $A = \text{Aut}(D_4) - K_0 = \{\alpha, \alpha^2, \alpha^3, \beta\alpha, \beta\alpha^2, \beta\alpha^3\}$, generating six structures of direct exchange on eight descent lines: $D(\bar{a}, 8, \alpha)$ and $D(\bar{a}, 8, \alpha^3)$ of period 4, and $D(\bar{a}, 8, \alpha^2)$, $D(\bar{a}, 8, \beta\alpha)$, $D(\bar{a}, 8, \beta\alpha^2)$, and $D(\bar{a}, 8, \beta\alpha^3)$, all of period 2.

However, not all kinship models associated with these structures are unique. First, since the automorphisms α^2 and $\beta\alpha^2$ both map the basic permutation r onto ra^2 , the associated kinship models will be identical. In fact, both of these structures may be completely disregarded, as they simply describe two separate Aranda-type

four-line structures, not an integrated kinship model spanning the entire set of eight descent lines. Finally, in analogy to the models on six lines, the remaining four structures can be shown to combine so as to produce (up to an isomorphism) only two distinct kinship models: (1) combinations of a 4-generation matriline structure (e.g., $D(\bar{a}, 8, \alpha)$ or $D(\bar{a}, 8, \alpha^3)$) with a 2-generation patriline structure (e.g., $D(\bar{a}, 8, \beta\alpha)$ or $D(\bar{a}, 8, \beta\alpha^3)$); marriage is with the FFZSD or FMBSD; (2) combinations of a 4-generation patriline structure with a 2-generation matriline structure; marriage is with the MMBDD or MFZDD. The available literature does not provide clear-cut examples of eight-line kinship systems with direct exchange and a rule of second cross-cousin marriage.

Automorphisms of D_5 : modelling the Samo

For $I = \{1, 2, \dots, 10\}$, i.e., for $m = 5$, define $r = (1, 2)(3, 4)(5, 6)(7, 8)(9, 10)$ and $a = (1, 3, 5, 7, 9)(10, 8, 6, 4, 2)$ and hence the dihedral group $D_5(r, a; r^2 = a^5 = e, ra = a^4r)$ of order 10. The remaining eight elements of D_5 are the identity e , together with

$$\begin{aligned} a^2 &= (1, 5, 9, 3, 7)(10, 6, 2, 8, 4), \\ a^3 &= (1, 7, 3, 9, 5)(10, 4, 8, 2, 6), \\ a^4 &= (1, 9, 7, 5, 3)(10, 2, 4, 6, 8), \\ ra &= (10, 1)(2, 3)(4, 5)(6, 7)(8, 9), \\ ra^2 &= (1, 8)(2, 5)(3, 10)(4, 7)(9, 6), \\ ra^3 &= (1, 6)(2, 7)(3, 8)(4, 9)(5, 10), \text{ and} \\ ra^4 &= (1, 4)(2, 9)(3, 6)(5, 8)(7, 10). \end{aligned}$$

According to Lemma 1 the automorphism group is of order $m \times \phi(m) = 5 \times \phi(5) = 20$. As before, all automorphisms γ of D_5 are obtained by substitution from the mapping $\gamma: r^s a^t \mapsto (ra^i)^s (a^j)^t$, i.e., for $s = 0$ or 1 , $0 \leq t < 5$, $0 \leq i < 5$, and $j = 1, 2, 3$ or 4 . They are:

$$\begin{aligned} 1 & \text{ (the identity) for } i = 0, j = 1; \\ \alpha &= (r, ra, ra^2, ra^3, ra^4), i = 1, j = 1; \\ \alpha^2 &= (r, ra^2, ra^4, ra, ra^3), i = 2, j = 1; \end{aligned}$$

$$\begin{aligned}
\alpha^3 &= (r, ra^3, ra, ra^4, ra^2), i = 3, j = 1; \\
\alpha^4 &= (r, ra^4, ra^3, ra^2, ra), i = 4, j = 1; \\
\beta &= (a, a^2, a^4, a^3)(ra, ra^2, ra^4, ra^3), i = 0, j = 2; \\
\beta^2 &= (a, a^4)(a^2, a^3)(ra, ra^4)(ra^2, ra^3), i = 0, j = 4; \\
\beta^3 &= (a, a^3, a^4, a^2)(ra, ra^3, ra^4, ra^2), i = 0, j = 3; \\
\beta\alpha &= (a, a^2, a^4, a^3)(r, ra, ra^3, ra^2), i = 1, j = 2; \\
\beta\alpha^2 &= (a, a^2, a^4, a^3)(r, ra^2, ra, ra^4), i = 2, j = 2; \\
\beta\alpha^3 &= (a, a^2, a^4, a^3)(r, ra^3, ra^4, ra), i = 3, j = 2; \\
\beta\alpha^4 &= (a, a^2, a^4, a^3)(r, ra^4, ra^2, ra^3), i = 4, j = 2; \\
\beta^2\alpha &= (a, a^4)(a^2, a^3)(r, ra)(ra^2, ra^4), i = 1, j = 4; \\
\beta^2\alpha^2 &= (a, a^4)(a^2, a^3)(r, ra^2)(ra^3, ra^4), i = 2, j = 4; \\
\beta^2\alpha^3 &= (a, a^4)(a^2, a^3)(r, ra^3)(ra, ra^2), i = 3, j = 4; \\
\beta^2\alpha^4 &= (a, a^4)(a^2, a^3)(r, ra^4)(ra, ra^3), i = 4, j = 4; \\
\beta^3\alpha &= (a, a^3, a^4, a^2)(r, ra, ra^4, ra^3), i = 1, j = 3; \\
\beta^3\alpha^2 &= (a, a^3, a^4, a^2)(r, ra^2, ra^3, ra), i = 2, j = 3; \\
\beta^3\alpha^3 &= (a, a^3, a^4, a^2)(r, ra^3, ra^2, ra^4), i = 3, j = 3; \\
\beta^3\alpha^4 &= (a, a^3, a^4, a^2)(r, ra^4, ra, ra^2), i = 4, j = 3.
\end{aligned}$$

The automorphism group is $\text{Aut}(D_5)(\beta, \alpha; \beta^4 = \alpha^5 = 1, \beta\alpha^2 = \alpha\beta)$. The invariant subgroups of a and r are $H_0 = \{1, \alpha, \alpha^2, \alpha^3, \alpha^4\}$ and $K_0 = \{1, \beta, \beta^2, \beta^3\}$. Let $A_1 = \{\alpha, \alpha^2, \alpha^3, \alpha^4\}$, $A_2 = \{\beta^2\alpha, \beta^2\alpha^2, \beta^2\alpha^3, \beta^2\alpha^4\}$, $A_3 = \{\beta\alpha, \beta\alpha^2, \beta\alpha^3, \beta\alpha^4\}$, and $A_4 = \{\beta^3\alpha, \beta^3\alpha^2, \beta^3\alpha^3, \beta^3\alpha^4\}$. Then $A = \text{Aut}(D_5) - K_0 = A_1 \cup A_2 \cup A_3 \cup A_4$. Hence there are precisely 16 structures of restricted exchange on ten descent lines.

Again, not all of the kinship models associated with these exchange structures are unique. First, for γ_1 in A_1 , all patrilineal kinship structures generated by $D(\bar{a}, 10, \gamma_1)$ are of period 5. Ego's spouse is equivalent to a second cousin (his MMBDD or MFZDD), and each patri-structure encompasses an alternating generation matrix-structure $D(\bar{a}, 10, \gamma_2)$ such that $\gamma_2\gamma_1 = \beta^2$. Conversely, all patrilineal structures generated by $D(\bar{a}, 10, \gamma_2)$ with γ_2 in A_2 are of period 2, ego's spouse is merged with his FMBSD or FFZSD, and the associated matrix-structure $D(\bar{a}, 10, \gamma_1)$ is of period 5 (with $\gamma_2\gamma_1 = \beta^2$). Second, all structures $D(\bar{a}, 10, \gamma)$ with γ in A_3 or A_4

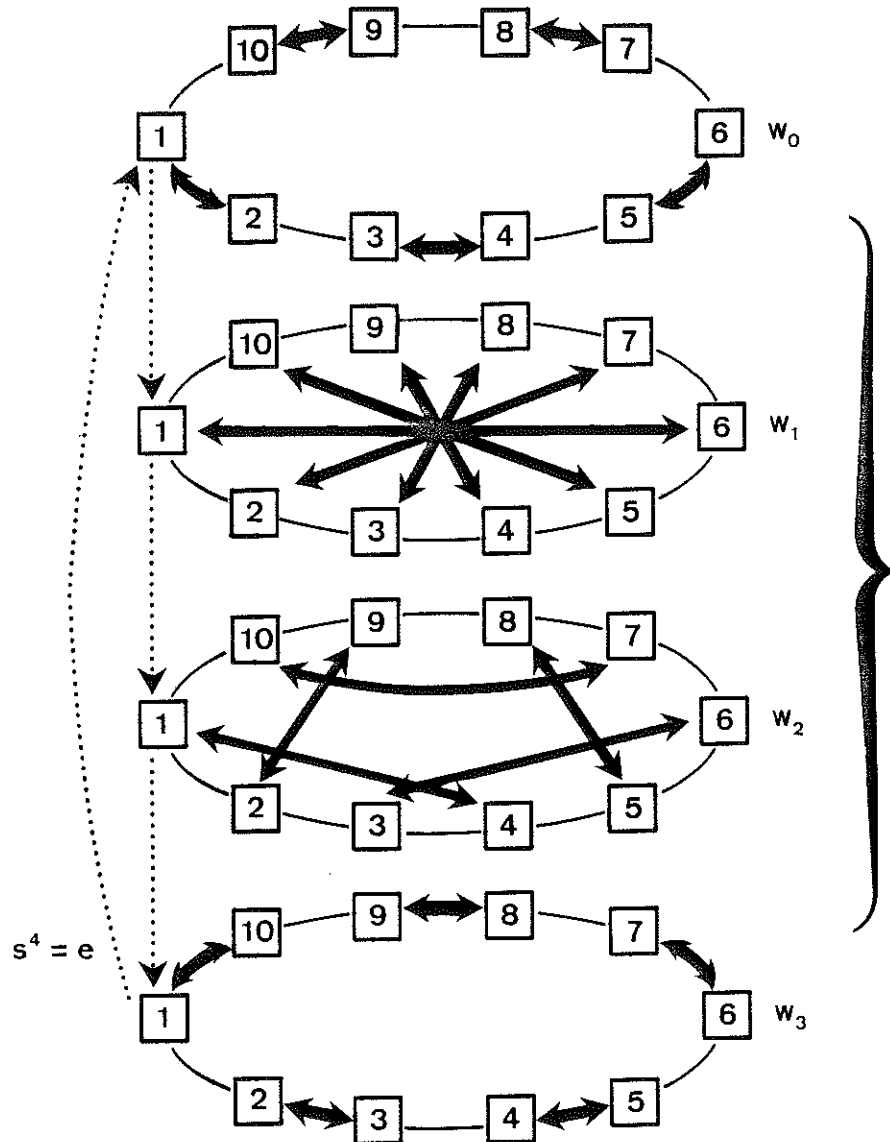


Fig. 4.8. Reduced structure of restricted exchange associated with $D(\bar{\alpha}, 10, \beta\alpha^3)$.

are of period 4 and all first and second cousins are proscribed. For all associated kinship structures, ego marries a third cousin, his FFFZDDD, FFMBDDD, MMMBSSD, or MMFZSSD. Moreover, for patrilineal structures with γ in A_3 (and matrilineal structures with γ in A_4), FMMBDSD and FMFZDSD also belong to the spouse category. Conversely, for matrilineal structures with γ in A_3 (and patrilineal structures with γ in A_4), ego's MFFZSDD and his MFMBSSD are allocated to the spouse category. Hence, up to an isomorphism, the 16 exchange structures generate only four basic kinship models with restricted exchange and ten matri or patrilineal structures.

As an example, I present the reduced structure of period 4 associated with $D(\bar{a}, 10, \beta\alpha^3)$ in figure 4.8. Interpreted as a patrilineal kinship model (i.e., with the formal successor mapping s representing a rule of patrilineal descent), the direct exchange of sisters between two patrilineal structures is renewable only after four generations. In the intervening three generations spouses must be obtained from other patrilineal structures. In genealogical terms, marriage is with a fourth cousin, male ego's FFFFZSSSD (merged with his FFFMBSSSD; see figure 4.9). However, a 4-generation rule does not mean that all marriages with third cousins are proscribed. The model allocates the following third cousin kintypes to the spouse category: MMMBSSD, MMFZSSD, FFFZDDD, FFMBDDD, FMMBDSD, and FMFZDSD (see figure 4.10). This particular patrilineal structure encompasses a 'latent' 4-generation matrilineal structure with the same marriage possibilities, but generated by $D(\bar{a}, 10, \beta^3\alpha^2)$.¹⁴

The family of alliance models with a variety of third and fourth cousin marriage possibilities, and with the structure of sister-exchange renewable after four generations is particularly interesting in the light of Héritier's research on the Samo of Upper Volta. The Samo population of about 120,000 is distributed among a number

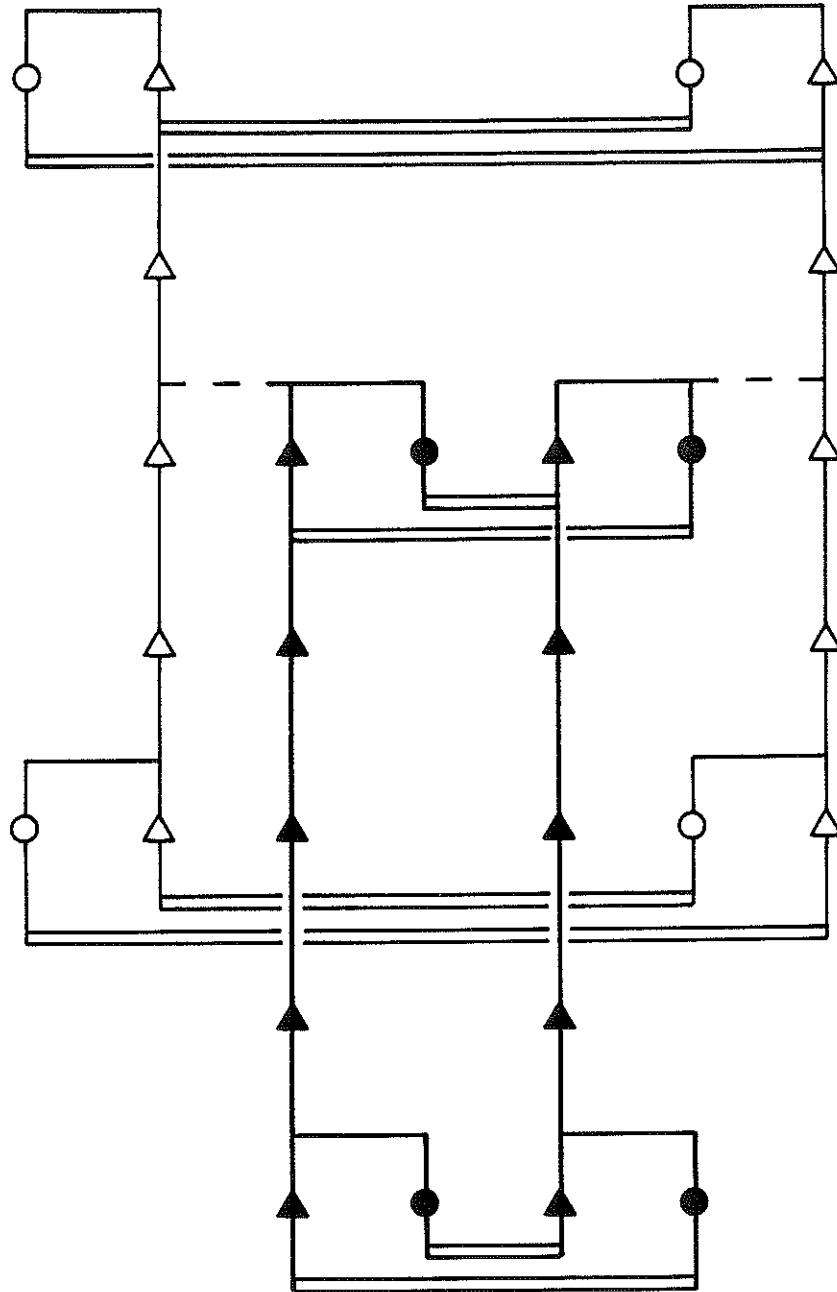


Fig. 4.9. Partial models associated with the 4-generation patrilineal structure $D(\bar{a}, 10, \beta\alpha^3)$. Sister-exchange and marriage with FFFFZSSSD and FFFMBSSSD (cf. Héritier 1981:113, fig. 38).

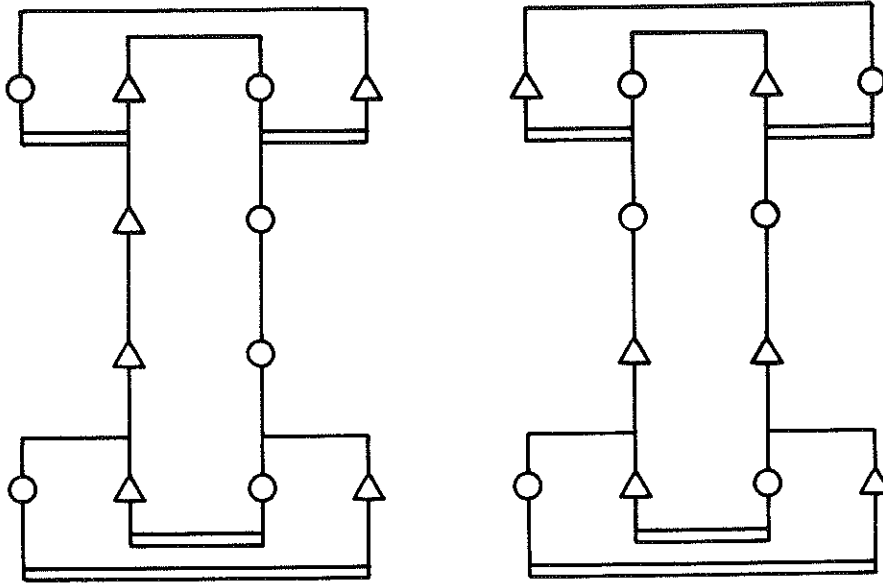


Fig. 4.10. Partial models associated with the 4-generation patrilineal structure $D(\bar{a}, 10, \beta\alpha^3)$. Sister-exchange and marriage with MMMBSSD, MMFZSSD, FFFZDDD, FFMBDDD, FMMBDSO, and FMFZDSO (cf. Héritier 1981:115, fig. 39).

of village clusters, loose associations of three or four villages which constitute an endogamous community. Héritier has recently presented an analysis of the data she collected in one such village cluster (Héritier 1981:73-136). The total population of the three Samo villages Dalo, Gono and Twäre is about 1,500, divided over 26 patrilineal 'lineages' (*sõ*) and 92 discrete 'lines' (*gu1ε*). Lines are genealogically defined segments whose ancestors are identified as classificatory lineage brothers.

The Samo have an extensive series of marriage prohibitions. First, a man may not marry a woman from his own patrilineage, of the patrilineage of his mother, or of the descent lines of his father's mother or

mother's mother. Second, a man may not take a lawful spouse from any lineage into which a male lineage member of his own or his father's generation has already married. (Under special circumstances a secondary marriage is sometimes permitted.) Third, a man may not marry into any of the four 'basic' patrilineages of any of his previous spouses (wife's lineage, wife's mother's lineage, wife's father's mother's lineage, and wife's mother's mother's lineage). Finally, there are a number of other prohibitions on marriage with the affinal kin of one's own affines (Héritier 1981:86).

As analysed by Héritier, the pattern of Samo alliances appears to hinge on a rule or preference for straight sister-exchange, renewable between lineages or lines only after four generations and constrained by a wide-ranging set of prohibitions. However, as an analytic scheme, the classic 'unilineal' prohibitions, even when combined with prohibitions on the replication of previous alliances and with further injunctions on marriage with distant consanguineal and affinal kin, do not appear to constitute a set which is both necessary and sufficient. Héritier's solution is to postulate the existence of an additional series of 'cognatic' prohibitions spanning three generations (1981:100-105). She is then able to derive a regular alliance model for the Samo (1981:113, fig. 38).

The model articulates a basic 4-generation structure of restricted exchange (wife is FFFMBSSSD and FFFFZSSSD) with a supplementary structure of the same type linking different lines of the patrilineages of ego and his spouse. In effect, a man is permitted to replicate the marriage of a classificatory 'father's father' (a FFB), but not the marriage of his actual FF.

Héritier's Samo model is incorporated as a proper substructure of the 4-generation kinship model sketched in figure 4.9. This particular observation is highly significant, as Héritier's alliance model is an

empirical generalization, a structure extracted from the genealogies and marriage histories of a large number of her Samo informants. My 4-generation model is a formal construction deduced as a specific example of straight sister-exchange from a general theory of kinship structures defined as automorphisms of a basic structure of restricted exchange. A 'statistical' model of the Samo data and a 'mechanical' model of the ideal marriage structure lead to equivalent representations of the alliance system.

Héritier's data on the actual pattern of marriage transactions permit further, more detailed testing. Thus, of the 142 marriages in which one of male ego's great-grandfathers is the actual brother of his wife's great-grandmother, in 49 cases (35%) ego's wife's great-grandmother's patriline is identical to that of ego; in 43 cases (30%) to that of ego's mother; in 30 cases (21%) to that of ego's father's mother, and in 20 cases (15%) to that of ego's mother's mother (1981:115, fig. 39).

These lineal identifications may be compared to the series of equivalences predicted by the two types of 4-generation models. Thus, for A_3 and A_4 the subsets of $\text{Aut}(D_5)$ defined earlier, all patrilineal structures $D(\bar{a}, 10, \gamma)$ with γ in A_3 give the following correspondences:

WMMM's patriline = ego's patriline (wife = FFFZDDD);

WMFM's patriline: distinct from the patrilines of ego's great-grandparents;

WFMM's patriline = ego's FM's patriline (wife = FMFZDSD);

WFFM's patriline = ego's MM's patriline (wife = MMFZSSD).

Furthermore, all patrilineal structures $D(\bar{a}, 10, \gamma)$ with γ in A_4 permit the derivation of the following identities:

WMMM's patriline = ego's patriline (wife = FFFZDDD);

WMFM's patriline = ego's M's patriline (wife = MFFZSDD);

WFMM's patriline: distinct from the patriline of ego's eight great-grandparents;

WFFM's patriline = ego's MM's patriline (wife = MMFZSSD).

The conclusion is obvious: if both types of 4-generation structure are allowed, the predictions of the formal model are seen to be fully compatible with Héritier's findings. These results can be formulated more precisely by taking up a suggestion of Courrège's (1974:334).¹⁵

Definition 1: Let A be the subset of $\text{Aut}(D_m)$ defined earlier. By the *spectrum* of the kinship system $K_i(D_m)$ with sister-exchange defined on $2m$ descent lines is meant the collection of kinship structures $\{D(\bar{a}, m, \gamma) \mid \gamma \in C_j(A)\}$, where $C_j(A)$ is a set of constraints imposed on A .

I propose the following hypothesis: the spectrum of the Samo kinship system consists of the collection of kinship structures with sister-exchange defined as $\{D(\bar{a}, 10, \gamma) \mid \gamma \in A_3 \text{ and } A_4\}$. The hypothesis may be tested by referring to two distinct types of data: first, by comparison with additional information on the Samo alliance patterns and actual marriage choices. Second, by comparison with the actors' ideal models and selective representations. (This type of data is not clearly distinguished from the anthropologist's models in Héritier's analysis.) In both cases the empirical claim is that the data, interpreted as intended applications of the formal theory (i.e., as partial structures), may be extended to a set of proper kinship models of the type $D(\bar{a}, 10, \gamma)$.¹⁶

The notion of an entire spectrum of exchange structures is crucial if one wishes to model marriage systems constrained by a wide-ranging series of prohibitions on the replication of previous alliances. Thus, in the case of the Samo, a man may not obtain a

spouse from any lineage into which his lineage brothers have already married, or from the grandparents' lineages of his own previous spouses. At the level of the model this is accomplished by discarding the notion of a single global, sociocentric exchange structure. Instead, a more realistic representation is obtained if one envisages the actual pattern of marriage alliances as resulting from a complex *superposition*¹⁷ of locally defined, interdependent exchange structures of similar type.

Thus, as illustrated in figure 4.9 (see also Héritier 1981:113, fig. 38), if a man's marriage is modelled according to some exchange structure $D(\bar{a}, 10, \beta\alpha^3)$, a lineage brother may obtain a spouse (with no violation of the alliance prohibitions) by marrying in accordance with some other structure $D(\bar{a}, 10, \gamma)$ of the same exchange spectrum. In the case of the Samo, the particular spectrum of exchange structures with sister-exchange and period 4 defined above is consistent with the data on actual marriages published by Héritier (1981:115).

Further development of a realistic mathematical theory of superpositions, in which a probabilistic description of marriage patterns and preferences is derived from the properties of appropriate *ensembles*¹⁸ of alliance structures is essential if one is to measure the extent to which the reported range of Samo marriage choices diverges from chance distribution (cf. Kuper 1982a:158-159). Further analysis should also indicate whether or not evidence for another important prediction from my model - regular alliance circuits encompassing ten lines - may be uncovered. It is perhaps not entirely fortuitous that the kinship system of the Inca, one of the societies whose complex alliance structure appears to be quite similar to that of the Samo (Héritier 1981: 137-146), has recently been analysed in terms of a tenfold division (Zuidema 1986:23-24, 27, 31, 37).

BROKEN SYMMETRIES

The dominant theme in *Les Structures élémentaires*, developed persuasively in the course of more than 500 pages is clear: the sorts of marriage exchange which take place in elementary kinship systems may be categorized as only two main forms, restricted exchange and generalized exchange. Paradoxically, this conclusion is partially undermined in the penultimate chapter in which the transition to complex structures is briefly discussed (Lévi-Strauss 1970: Ch.XXVII, 459-477). The distinction tends to blur. In the final analysis, the two structurally distinct modes of exchange are found only in 'impure', 'corrupted' or 'contaminated' forms: 'Wherever there is restricted exchange there is generalized exchange, and generalized exchange is never free of allogeneous forms' (Lévi-Strauss 1970:464). Even in Australia (a privileged territory for elementary structures), restricted exchange is embedded in a context of matrilinear or patrilinear systems (1970:463). In more abstract terms, the basic symmetry of the simple forms of exchange is broken.

In physics, invariances in symmetry operations imply the existence of conserved qualities. Conversely, the spontaneous appearance of a broken symmetry signals a phase transition - the transformation of a simple system to a more complex state or configuration. A similar process may be discerned in the symmetries of marriage exchange.

The classic view (summarized by Bloch 1978:21) is as follows. Ideally, in systems with sister-exchange, where the gift of a woman is reciprocated directly and both parties exchange identical things, one would expect the relationship between wife-givers and wife-takers to be straightforwardly egalitarian. In a very real sense equality is conserved and any status or prestige differential minimized. Conversely, in systems with

generalized exchange, even where the global structure is conceived of as a closed circle of marriages implying long-term equality, wife-givers and wife-takers exchange different kinds of things. One side will (at least temporarily) gain a woman or a man from the other. In the short term the exchange is unequal; hence the relationship is potentially hierarchical and differentiation of status one way or the other a likely consequence. Under the classic view, any combination of the principles of restricted and generalized exchange is at best problematic (cf. Lévi-Strauss 1970:464-465).¹⁹

In recent years the analysis of more complex matrimonial systems (including systems with a wide variation in bridewealth practices) has led to a shift in perspective. The emphasis is now more on the internal dynamics and the inherent transformational possibilities of a structure. Thus, Taylor (1983) argues that the occurrence of several - apparently contradictory - forms of marriage among the Northern Achuar (a Jivaroan society) are related as variants of a single underlying structure. Moreover, 'It is the play of these variants and their internal contradictions that endows the structure with its flexibility and dynamism' (1983:350). Rosman and Rubel describe the co-occurrence of preferential sister-exchange, indirect exchange, and various marriage forms based on extensive prohibitions in a number of New Guinea societies as alternative patterns 'in complementary distribution' (1975:115-116). Schwimmer suggests a similar approach: 'Je pense qu'on peut obtenir de bien meilleurs résultats en Nouvelle-Guinée si l'on suppose dès le départ que les échanges restreint, généralisé élémentaire et complexe sont simultanément présents et que le système d'alliance est la résultante de l'interaction de ces diverses formes' (1970:33). Finally, Muller (1978), confronted with a combination of two structural types of Rukuba marriage (a 'prescriptive' type with high bridewealth, and marriage by 'random

choice' within the opposite moiety) ventures the following intriguing suggestion: 'This combination is not just a juxtaposition of two systems but forms a model *sui generis* which negates in several ways the structural implications of the two types of marriage taken in isolation' (1978:165).

Muller's brief remark touches upon the core of the problem: the emergence of new levels of organization in complex systems. Complex kinship structures are, perhaps, not really reducible to a juxtaposition of elementary models. The old symmetries may be too simple. The challenge is thus to model complex marriage systems as primary in their own right.²⁰

APPENDIX

The concept of a *semidirect product* is defined in a slightly different manner in the appendix to Chapter 1.

Semidirect product. For H and K any two groups, let $f: K \rightarrow \text{Aut}(H)$ be a homomorphism of K into the automorphism group of H , i.e., $(k_1 k_2)f = ((k_1)f)((k_2)f)$ for all k_i in K and $(k_i)f$ in $\text{Aut}(H)$. Let f_k rather than $(k)f$ denote the value of f at k in K . Since f_k is an element of $\text{Aut}(H)$, f_k is itself a function from H into H . If h is in H , denote the value of f_k at h by $(h)f_k$. Then $H \times K$, the set of ordered pairs (h, k) with h in H and k in K , forms a group when product is defined by $(h, k)(h', k') = (h(h')f_k, kk')$. This group (denoted $H \rtimes_f K$) is called the *semidirect product of H by K* . The identity is (e, e) where e denotes the identity in both H and K , and $(h, k)^{-1}$, the inverse of (h, k) , is defined as $((h^{-1})f_{k^{-1}}, k^{-1})$. Note that when f maps the whole of K onto the identity automorphism 1 of H , the semidirect product becomes the direct product of H and K .

For a proof of *lemma 1*, see Dixon (1973:26, 103) problems 4.1 and 4.2, and Weinstein (1977) section 1.2 and example 4.1. Weinstein also introduces a number of other interesting constructions, including wreath products.

NOTES

- 1 See Friedman (1983) and Elliott and Dawber (1985, 1986). Hargittai (1986) has numerous examples.
- 2 See for example, Haber and Kane (1986).
- 3 There is a clear opposition between Lévi-Strauss's hope for a future awakening of anthropology among the natural sciences (stated in his inaugural address of 1960; see Lévi-Strauss 1978:19) and the position summarized in the final sentence of P.E. de Josselin de Jong's farewell lecture (1987:36).
- 4 Lévi-Strauss (1970:215): 'A harmonic regime is one in which the rule of residence is similar to the rule of descent, and a disharmonic regime is one in which they are opposed.'
- 5 The concept of a semi-complex alliance structure derives from Lévi-Strauss's discussion of Crow-Omaha systems in his famous Huxley Memorial Lecture (published in 1966). Héritier's work is itself a further development of the research programme on kinship set out by Lévi-Strauss in *Les Structures élémentaires* and in *The Future of Kinship Studies*.
- 6 This is probably the reason for Lévi-Strauss not including the Bardi in his discussion of the classic Australian systems. Elkin (1932; cited as 'Elkin 1931') is included in his references, but the Bardi or Bard only figure in his note 1 on page 219 as an example of a system without a clear moiety division (Lévi-Strauss 1970).
- 7 Young does not state his criteria for classifying second cross-cousins; FFZDD is Iroquois-cross, while MMBDD is Dravidian-cross. See Chapter 2, table 2.3.
- 8 See the discussion in the previous chapters and Tjon Sie Fat 1981.
- 9 See Lockwood and Macmillan (1978:11-12), and Budden (1972:187-213, Ch.13).
- 10 The only possible finite symmetry groups in three-dimensional space are: C_n , D_m , A_4 (the tetrahedral group), S_4 (the cubic group), and A_5 (the icosahedral group). If opposite symmetries of three-dimensional objects are permitted, the following direct product groups may be added: $C_n \times C_2$, $D_m \times C_2$, $A_4 \times C_2$,

$S_4 \times C_2$, and $A_5 \times C_2$.

- 11 For a definition of technical terms, see the appendix and the discussion in previous chapters.
- 12 Strictly speaking, *Lemma 1* and the construction are only well-defined for $m > 2$. Then $|H_0| = m$ and $|K_0| = \phi(m)$.
- 13 Many Highland New Guinea societies (including the Manga) apparently recognize the formula of generalized exchange as a valid option. An alternative model based on asymmetrical exchange and marriage with the FFZSD has already been described in Chapter 2. The model of figure 4.7 is also identical to a 6-line model first described by P.E. de Josselin de Jong (1966:72, diag.d).
- 14 Any 4-generation matri or patri-structure $D(\bar{a}, 10, \beta^i \alpha^j)$ encompasses a latent structure of the opposite descent type generated by $D(\bar{a}, 10, \beta^{4-i} \alpha^{5-j})$ (up to an isomorphism) if the origin $\bar{a}$ is situated at generation level G+3 and the basic exchange cycle r on the matriline and patriline is defined as (A, B) (C, D) ... (I, J) as well as (I, II)(III, IV) ... (IX, X).
- 15 I translate the French term *spectre* as 'spectrum', not 'spectre' (cf. Courrège 1974:334, translated from Courrège 1965). This is more in line with standard mathematical usage, e.g., 'the spectral decomposition of linear transformations'.
- 16 See the methodological summary in Chapters 1 and 2.
- 17 As a first approximation, I use the concept of superposition in its classic sense, not as a superposition of macroscopically opposed states (as in later developments of quantum mechanics). Lévi-Strauss's 'regular' and 'alternate' Murngin systems (1970:171, fig.19) may be described as superpositions of an identical exchange structure.
- 18 The standard interpretation of probability in the natural sciences has it that probability deals with relative frequencies within an ensemble.
- 19 According to Lévi-Strauss, the 'contamination' of generalized exchange is one of its *intrinsic* properties. In contrast, the contamination of restricted exchange appears to take an *extrinsic* form (1970:464). In a programmatic statement, the transition to complex kinship structures and Crow-Omaha systems is seen as resulting from the combination of the elementary modes of exchange, resulting in 'new contradictions which are henceforth inherent in these systems' (1970:465). In a paper presented at a seminar conducted by E.R. Leach at King's College, Cambridge, in February 1963, the relationship between symmetrical and asymmetrical marriage systems is summarized by P.E. de Josselin de Jong as follows: 'In sum, the symmetrical and the asymmetrical types prove not to

- be mutually incompatible. There are combinations, borderline cases, and transitions' (1966:66), See also the more recent remarks by Dupré (1981).
 20 Cf. Davies (1987:21-23). This conclusion is also implicit in Muller (1980). The notions of 'elementary' and 'complex' structures (as defined by Lévi-Strauss and elaborated by Héritier) require further discriminations to be made. I provide a number of suggestions for formalizing 'more complex' families of kinship models in the following chapter. The more conservative type of model developed in the preceeding chapters may still be applied with some success to substantial bodies of ethnographic data. For example, Mosko (1985) describes a series of exchange models for the Bush Mekeo. These include superpositions of 'Aranda'-type structures (1985:143, fig.6.11; 146, fig.6.12), and superpositions of a type of structure with a four-generation cycle based on eight descent lines. The exchange cycles are:

$$w_0 = (1, 2, 3, 4)(5, 6, 7, 8);$$

$$w_1 = (1, 5, 3, 7)(2, 6, 4, 8);$$

$$w_2 = (1, 4, 3, 2)(5, 8, 7, 6) = w_0^{-1};$$

$$w_3 = (1, 7, 3, 5)(2, 8, 4, 6) = w_1^{-1};$$

and $w_4 = w_0$ (cf. Mosko 1985:147, fig. 6.13).

Further analysis of Mosko's data, and of the complete set of morphisms linking the family of Bush Mekeo models should prove fruitful. See also Chapter 3, note 20.

5. KINSHIP, COMPLEXITY, AND THE DISCRETE DYNAMICS OF SIMPLE SYSTEMS

Complexity and simplicity, like Yin and Yang, are metaphysical duals; except for a vagrant connection to intuition, it hardly makes a difference what is called which.

David Berlinski,
*The Language of Life.*¹

The understanding of complexity remains a fundamental challenge for kinship theory. Traditionally, much anthropological research has been motivated by what Friedrich (1988:435) has termed a 'rage for order': a preoccupation with predictability and with the extraction of simple rules, patterns or structures from complex social contexts. Making simplifying assumptions is of course a key characteristic of the scientific method: any scientific representation involves the stripping away of unnecessary detail. Scientific models depend on a transformation of common-sensical knowledge, carving significant elements and events out of a wider set of phenomena and reconstituting the relations so extracted in a structural description (cf. Granger 1983).

The trick is to know which details merit consideration. A dairy farmer once wrote to the local university for advice on improving milk production. A multidisciplinary team of scientists was assembled, and after an intensive period of on-site investigation a report was prepared. The farmer received the write-up, and opened it, only to read on the first line: 'Consider a spherical cow ...'.²

Behind this cautionary tale lies an important message: when modelling highly complex phenomena, beware the lure of spurious oversimplification.

Under the conventional approach, systems research has concentrated mainly on the dynamics of 'simple' phenomena: closed, stable systems whose long-term behaviour, described by a few fundamental laws, is assumed to be completely predictable. There should be no surprises. Simple systems, involving only a fairly small number of components, display behavioural patterns that are intuitively well-understood and directly deducible from knowledge of the inputs acting upon the system. Slight changes in the initial conditions or the inputs should only bring about slight changes in the behavioural characteristics. Simple systems should therefore exhibit simple dynamics.

'Complexity' has generally been defined in direct opposition to simplicity. For example, in his classic paper on *The Architecture of Complexity* Herbert Simon has this to say (1969:86):

Roughly, by a complex system I mean one made up of a large number of parts that interact in a nonsimple way. In such systems, the whole is more than the sum of the parts, not in an ultimate, metaphysical sense, but in the important pragmatic sense that, given the properties of the parts and the laws of their interaction, it is not a trivial matter to infer the properties of the whole.

In short, the notion of system complexity is associated with the existence of large numbers of highly connected system components, with unpredictable surprises and highly counter-intuitive behavioural modes. Complex systems behave in complex ways.

Until recently, classical science either ignored complexity or treated it as an embarrassing complication. Where complex results were obtained, one either sought an approximate model or solution to the exact problem, or an exact solution to a much simplified or reduced approximation of the original, intractable system. If

necessary, an element of randomness or indeterminacy could be built into the model by adding on 'noise' or 'error' terms to its equations. By modelling the extensive data obtained from complicated systems with many distinct variables and huge numbers of components as patterns of averaged quantities, the statistical method enabled one to approximate the structure and behaviour of systems so complex that they appeared random.

Thus, according to Stewart (1989:54), by the beginning of the 20th century science had acquired two very distinct paradigms for mathematical modelling:

No longer was order synonymous with law, and disorder with lawlessness. Both order and disorder had laws. But the laws were two distinct codes of behaviour. One law for the ordered, another for the disordered. Two paradigms, two techniques. Two ways to view the world. Two mathematical ideologies, each applying only within its own sphere of influence. Determinism for simple systems with few degrees of freedom, statistics for complicated systems with many degrees of freedom. Either a system was random, or it wasn't. If it was, scientists reached for something stochastic; if not, they polished up their deterministic equations. The two paradigms were equal partners, equally accepted in the scientific world, equally useful, equally important, equally mathematical. Equal. But different. Totally, irreconcilably different. Scientists *knew* they were different, and they knew why: simple systems behave in simple ways, complicated systems behave in complicated ways. Between simplicity and complexity there can be no common ground.

These assumptions would soon be challenged.

ON THE COMPLEXITY OF KINSHIP STRUCTURES

Scientific revolutions, according to Kuhn (1970), involve fundamental changes in world view: transformations of the entire constellation of beliefs determined by the old paradigm. The 'third revolution' - chaos - is now sweeping through the natural and social sciences.³ Under the recent paradigm shift, complexity, not simplicity, now holds

centre stage. The focus is now on the unfolding of organized complexity, not on reductionism or spurious approximation. The new buzzwords are *self-organization, fractals, dissipative structures, bifurcations, strange attractors, downward causation, non-linear dynamics, and chaos*. What is being sought is nothing less than a general theory of organization, with the new approach to complexity cutting across the traditional boundaries of scientific disciplines. *Chaos*, previously defined as 'complete disorder, utter confusion', has taken on the paradoxical meaning of 'stochastic [i.e., random] behaviour occurring in a deterministic system' (Stewart 1989:16-17). Under the old paradigm, the 'simple/complex' and 'deterministic/random' dichotomies coincided: a system was thought to be either deterministic or stochastic. This is no longer the case.

The regions of predictability and chaos are now seen to be closely interwoven, and the types of transformation that mediate between them are of paramount importance. Instead of two opposed polarities, we now study an entire spectrum: a typical system may exhibit a variety of states, some ordered, others chaotic (Stewart 1989:22). As summarized by Stewart (1989:286) the lesson is clear: simple models can have simple solutions - or complex ones. Complex models can have complex solutions - or simple ones. Thus, irregular phenomena do not necessarily require complicated equations; conversely, even simple equations may generate or model the behaviour of complex systems. The provocative generalizations of chaos theory provide, at the very least, an exciting category of new images and scientific metaphors.⁴

These spectacular new developments have yet to make their mark on kinship theory. Thus, as discussed in previous chapters, under the Lévi-Straussian paradigm underpinning the standard view on kinship, 'elementary' kinship structures are associated with deterministic models of alliance. Marital choice is fully determined

by a positive rule prescribing marriage with a certain type of relative (drawn from a particular exchange unit, a terminologically encoded category, or defined genealogically). Elementary systems are bounded or closed, with the cycle of reciprocity generated by the principle of direct (symmetric) exchange or generalized (asymmetric) exchange. The underlying relational properties of elementary kinship systems require only the formulation of simple, 'mechanical' models.

'Complex' kinship structures are situated at the opposite end of the scale. In the Preface to the first (1949) edition of *Les Structures*, the term 'complex structure' is reserved for systems 'which limit themselves to defining the circle of relatives and leave the determination of the spouse to other mechanisms, economic or psychological. ... systems ... which are based on a transfer of wealth or on free choice, would be classified as complex structures' (Lévi-Strauss 1970:xxiii). Fifteen years later, in the 1965 Huxley Memorial Lecture on the future of kinship studies, the elementary - complex distinction is again defined in oppositional terms. In 'elementary' structures the relationship between prospective spouses (whether prescribed or preferred) is formulated in terms of the social structure (i.e., in reference to membership of a particular social group or kinship category). On the other hand, in 'complex' structures marital prescriptions or preferences are never defined in terms pertaining to the social structure (Lévi-Strauss 1966:18). The criterion of social structure is stressed yet again in the Preface to the second edition of *Les Structures* (Lévi-Strauss 1970:xxxiv).

If elementary kinship structures could indeed be adequately represented by simple mechanical models, Lévi-Strauss apparently despaired of ever extending a similar conceptual framework to the study of complex systems (this category would also include 'modern' societies). With simple deterministic models apparently ruled out,

probabilistic methods seemed to offer the only viable solution. Come the revolution, the problems of complex structures would be resolved. As Lévi-Strauss speculated (1966:21):

For such a program to be initiated, the conceptual framework of our studies will have to undergo a transformation whose magnitude is comparable to the one which may be said to exist between Keplerian and quantum mechanics. For the world we should prepare ourselves to enter, will no longer be composed of commutative classes and networks endowed with a periodical structure, but of unpredictable events whose statistical distribution only will show regularities and provide meaningful clues.

With hindsight, Lévi-Strauss's vision of developments in the future of kinship studies, extrapolating from the dominant research strategies of the first half of the 20th century, now seems overly conservative. Phenomena that look complicated might just be governed by a simple - though chaotic - model. Probabilistic models and statistical distributions are not the only techniques for extracting patterns from complexity.⁵

SYSTEMS OF INTERMEDIATE COMPLEXITY

Lévi-Strauss had already posed the problem of quantifying the complexity of kinship systems more than a decade before he presented the 1965 Huxley Lecture. In his celebrated paper on social structure (read at the 1952 Chicago symposium), he suggests a measure based on the number of available marriage choices. This suggestion derives from the mathematical theory of communication (developed by Claude Shannon, Norbert Wiener and others at MIT in the 1940s). Thus (Lévi-Strauss 1953:538):

In the terminology of this theory it is possible to speak of the information of a marriage system by the number of choices at the observer's disposal to define the marriage status of an individual. Thus the information is unity for a dual exogamous system, and, in an Australian kind

of kinship typology, it would increase with the logarithm of the number of matrimonial classes. A theoretical system where everybody could marry everybody would be a system with no redundancy, since each marriage choice would not be determined by previous choices, while the positive content of marriage rules constitutes the redundancy of the system under consideration. By studying the percentage of 'free' choices in a matrimonial population (not absolutely free, but in relation to certain postulated conditions), it would thus become possible to offer numerical estimates of its entropy, both absolute and relative.

As a consequence, it would become possible to translate statistical models into mechanical ones and vice versa, thus bridging the gap still existing between population studies ... and anthropological ones, ... thereby laying a foundation for foresight and action.⁶

Hence a complex kinship system with high entropy is characterized by a large degree of randomness: one needs a great deal of information to specify its state. Conversely, for a simple moiety system with only two possible marriage choices, the entropy (defined as the logarithm of 2 to the base 2) is unity. (For a discussion of information measures and section systems see Buchler and Selby 1968:279-309.)

The matter is actually a bit more complicated than suggested by Lévi-Strauss. In general, not all possible states of a kinship system will be equiprobable. Thus, in order to define the information content of a system, one must consider the ensemble, or collection of all its possible states, and evaluate the information content of each individual state. For a system with an ensemble of n states, each with probability p_i of occurrence, the measure of the information content of the ensemble as a whole is defined as $H = - \sum p_i \log p_i$, not merely $\log n$.

In spite of Lévi-Strauss's initial optimism, the Shannon - Wiener measure of complexity failed to provoke a revolution in kinship theory.⁷ By the mid-1960s the focus of research had shifted. Instead of seeking a further differentiation of the elementary - complex dimension in terms of a quantitative measure, Lévi-Strauss

now concentrated on one particular category of kinship systems of intermediate complexity. If, indeed, the very nature of kinship itself changes in passing from 'simple' societies to 'complex' ones, the problem of identifying the modes of transition might be resolved by studying a type of system which appears to mediate the elementary - complex dichotomy: the so-called 'Crow - Omaha' systems.

Traditionally the terms 'Crow' and 'Omaha' refer to particular types of kinship terminology systems with intergenerational skewing. Thus, an idealized 'Crow' terminology has at least the equations $FZ = FZD = FZDD$. Conversely, in an 'Omaha' terminology $MB = MBS = MBSS$.⁸ Lévi-Straus's usage of the term 'Crow - Omaha' is slightly different. 'Crow - Omaha' systems, according to his view, lack positive marriage rules. Marriage possibilities are defined negatively, by means of an extremely wide-ranging series of lineal prohibitions. Typically a man is forbidden to marry into the lines or clans of his four grandparents (and perhaps other lines too). As these prohibitions operated in societies with relatively small populations and a limited number of lines or clans, the negative rules might produce a statistical pattern of exchange cycles in many respects quite similar to the pattern generated by deterministic marriage rules in elementary kinship structures. As Lévi-Strauss stated (1966:19), 'the generalised definition of a Crow - Omaha system may best be formulated by saying that whenever a descent line is picked up to provide a mate, all individuals belonging to that line are excluded from the range of potential mates for the first lineage, during a period covering several generations', with this process repeating itself with each subsequent marriage. Moreover, while elementary structures with positive, kin-based marriage rules in effect transform kinsmen into affines, 'a Crow - Omaha system takes the opposite stand by turning affines into kinsmen' (1966:19). In Lévi-Strauss's view the 'Crow - Omaha' systems provide the hinge articulating elementary

structures with complex structures, since they encompassed elements of both. Thus (Lévi-Strauss 1966:19):

Crow - Omaha systems still belong to the elementary structures from the point of view of the marriage prohibitions they frame in sociological terms [i.e., in terms of the kinship structure], but they already belong to the complex structures from the point of view of the probabilist alliance network which they produce. In my terminology, they use a negative mechanical model at the level of the norms, to generate a positive, statistical model at the level of the facts.

A great deal of effort has been put into 'solving the Crow - Omaha problem', including a number of attempts to develop Lévi-Strauss's ideas about such systems.⁹ Françoise Héritier's research among the Samo of Upper Volta mentioned in previous chapters is perhaps the most interesting of the recent attempts to pursue the orthodox Lévi-Straussian programme of research. The main thrust of her work centres on the relations between the ideological models representing Samo marriage prohibitions and the statistical pattern of actual marriages, obtained by computer analysis of an extensive body of genealogical material (cf. Héritier 1981). In Héritier's scheme the 'Crow - Omaha' systems (together with certain 'Iroquois' and 'Hawaiian' types) are included as members of a more extensive category, the so-called 'semi-complex systems of alliance'. Héritier has refined Lévi-Strauss's original definition of such systems of intermediary complexity by stressing the following important feature: the existence of prohibitions on the reduplication of marriage patterns by same-sex kin and, conversely, the obligation of opposite-sex kin to repeat the pattern of previous alliances (Héritier 1981:12, 127). (The inverse combination of negative and positive rules is also envisaged; see Héritier 1981:169.) The cumulative result of such prohibitions and preferences will be reflected in the emergence of global cycles of exchange (however partial or incomplete) in the statistical patterning of actually

contracted marriages.

In spite of its undoubted success in elucidating the functioning of the Samo kinship system, Héritier's scheme, as a general research programme, is fundamentally flawed. In the final analysis, the existence of an intermediate category of 'semi-complex' alliance structures is postulated in order to mediate Lévi-Strauss's original elementary - complex dichotomy. As I have sketched above in the introductory section to this chapter, this dichotomy is itself merely a reflection of the fundamental opposition between the two paradigms for mathematical modelling that have hitherto constrained systems research. With the old simplicity - complexity dichotomy (as applied to systems modelling) now transformed, one must now also question the necessity of addressing a separate, mediating category of 'semi-complex' or 'Crow - Omaha' structures.

Lévi-Strauss's information-theoretic approach to systems complexity suffers from another conceptual shortcoming. The complexity of a kinship system is defined exclusively in terms of structural features such as the number of kinship units or possible marriage choices. As Casti has recently argued (1986, 1989:17-19), complexity is also associated with a system's behavioural characteristics such as counterintuitive responses, multiple modes of operation, and irreproducible surprises. Thus (Casti and Karlqvist 1986:xi):

While the structural features are, by and large, objective properties of the system *per se*, the behavioural components of complexity are decidedly subjective: what is counterintuitive, surprising, and so on is as much a property of the system doing the observing as it is a feature of the process under study. Thus, any mathematical formulations of the notion of complexity must respect the subjective, as well as the objective, aspects of the concept.

From this viewpoint complexity, especially in the realm of biological or social systems, is a contingent, rather than intrinsic, property that emerges from the interaction between the system and its observer.

Roughly, given a system N interacting with a system O (i.e., an observer), under the relativistic view of system complexity put forward by Casti, interaction is a two-way relation. For a given mode of interaction, the system N displays a certain level of complexity relative to O ; conversely, O has a level of complexity relative to N (Casti 1986:149). Only in special cases such as classical physics where one of these reciprocal interactions is so much weaker that it can be ignored will the classic approach to complexity as an intrinsic property of the observed system N apply. Let $C_O(N)$ and $C_N(O)$ represent, respectively, the complexity of N as seen by O , and the complexity of O as seen by N . Then, according to Casti (1989:18), it is the reciprocal relation between $C_O(N)$ and $C_N(O)$ that is of crucial importance, not one or the other in isolation. The quantity $C_O(N)$ is defined as the number of *nonequivalent* descriptions of N formed by the observer O . ($C_N(O)$ is defined analogously.)

The idea of nonequivalent descriptions has been formalized and made more explicit (see Casti 1986:151-157, 1989:4-36). Intuitively, if O can interact with or describe a system N in a large number of different ways, then O will regard N as complex; conversely, if O has only a small number of modes of interaction with N or can form only a few descriptions, then N appears simple. Casti uses the following analogy (1986:151):

The pen I used to write this manuscript is a simple system to me. The only mode of interaction with it that I have available is to use it as a writing system; however, if I were, say, a chemical engineer, then many more modes become available. I could analyze the plastic compound of which it is made, the composition of chemicals forming the ink, the design of the writing ball at its tip, and so forth. So, for a chemical engineer my ballpoint pen becomes a far more complex object than it is for me.

In sum: complexity emerges from simplicity when alternative descriptions of a system are not reducible to each other (Casti 1986:157).¹⁰

Casti's proposals need considerable further elaboration before they can constitute a viable research programme for identifying and managing the complexity of interacting systems. As he acknowledges (Casti 1986:166-168), it will be necessary to develop a more general theory of models applicable to the behavioural sciences as well as to the natural sciences. A sufficiently rich theoretical framework is required for the formal modelling of the processes and emergent properties encountered in the social, behavioural and cultural environments, with particular emphasis on features associated with adaptability, hierarchy, and shifts from one behavioural or structural mode to another (bifurcations). Casti's recent book, *Alternate Realities: Mathematical Models of Nature and Man* (1989) is a refreshing and highly accessible introduction to some of these matters. These investigations provide a starting point for the development of simple mathematical models that have greater explanatory power and predictive capability than the traditional 'spherical-cow' approach.

The brief review of recent developments in these introductory sections have clear implications for the Lévi-Straussian programme. First, as the advances in chaos theory have demonstrated, the traditional approach to the problem of systems complexity (associated with randomness or uncertainty) by means of probability theory (e.g., Lévi-Strauss's statistical models and information-theoretical measures) has come under increasing attack. A variety of alternatives to probability theory have now been advanced for representing random-like properties of systems in a nonprobabilistic fashion. Second, complexity itself is associated with a system's behavioural characteristics as well as its structural features. It cannot be measured solely in terms of intrinsic properties of the observed system: complexity emerges from the interaction between the system and the variety of models provided by the observer.

Under the Lévi-Straussian scheme, 'semi-complex systems of alliance'¹¹ (including the 'Crow - Omaha' systems) constitute a uniform category intermediate between the 'elementary' systems based on positive marriage rules, and the 'complex' kinship systems with extensive prohibitions or probabilistic rules. This category is intermediate in two respects. First, it straddles the developmental sequence: 'elementary' structures (seen by Lévi-Strauss as the basic, logical, primeval or 'natural' structures; cf. Muller 1980:527) pass over to 'complex' structures, and the transition is effected through the category of 'semi-complex' structures. Second, the *models* necessary for the formal characterisation of 'semi-complex' structures (and about whose development Lévi-Strauss, writing in the 1960s, is fairly pessimistic) must be able to mediate between determinism and randomness. The first point has occasioned the spilling of much ink.¹² Whether or not a single intermediary category of 'Crow - Omaha' or 'semi-complex' systems actually exists, and whether or not attempts to develop a truly general theory of kinship raise 'tremendous difficulties which are the province, not of the social anthropologist, but of the mathematician' (Lévi-Strauss 1970:xxxvi) are still open questions. Under the new paradigm for complexity - deterministic chaos - and with a growing number of detailed empirical studies of individual societies becoming available, we are now beginning to develop a deeper understanding of the real difficulties involved in modelling kinship systems. In the remaining sections of this chapter I introduce one of the exciting new classes of simple dynamical models - cellular automata. I then consider how this example, elementary as it is, may provide a useful analogy for modelling the discrete dynamics of kinship systems. Semigroups are introduced as a means of extending the basic models. I conclude with a discussion of structures with extended sibling groups, taking as an example the Beliyan marriage system.

CELLULAR AUTOMATA AND DISCRETE DYNAMICAL SYSTEMS

Consider a one-dimensional array of cells (i.e., lattice points or nodes arranged along a line), each of which can only take on a value 0 or 1. If we have a finite array of N cells, the total number of possible configurations or states that the system can assume equals 2^N . Consider one specific configuration (the initial state of the system). Assume that the value of each cell is updated according to a rule of transition involving only the values of its two nearest neighbours. For example, the rule of transition could define the value of a cell at a particular instant to be the sum (under addition modulo 2) of the values of its two nearest neighbours at the previous time period. The system should evolve synchronously in discrete time steps, with the same transition rule applying to each cell. Given a rule of evolution or state-transition and some initial state, we can ask the following fundamental question (cf. Casti 1989:46): What happens to the system as time goes on?

The model described above is an example from the class of *one-dimensional cellular automata*, discrete dynamical systems of simple construction but which may display complex and varied behaviour. Cellular automata were introduced in the 1940s by John von Neumann, following a suggestion by the mathematician Stanislaw Ulam, a colleague at Los Alamos. Von Neumann was interested in the construction of formal models of automata capable of reproducing themselves in a nontrivial way. At the time, Ulam was busy inventing pattern games for the first generation of computers developed for the Manhattan Project. Ulam's games were played on huge grids of square cells, essentially limitless chessboards (triangular or hexagonal tilings were also tried). The computer would be given a set of recursively defined rules for generating the patterns and would then print out a complete

sequence of diagrams documenting the growth and change of the original pattern. All changes took place in discrete jumps, with the fate of each cell only determined by the values of neighbouring cells. This is the framework adopted by Von Neumann.¹³

He presented his theory in a lecture given in September 1948 entitled 'The General and Logical Theory of Automata' (reprinted in Von Neumann 1961-1963:288-328). Von Neumann's cellular automaton model (he used the term 'cellular spaces'; his array employed 29 different states, including the empty state, for each cell) proved that self-reproduction - a logical form abstracted from the self-reproduction of natural systems - could indeed be achieved by *machines*. The growth of crystals is an example of trivial replication. In order to model reproduction more akin to the characteristic behaviour of living organisms, Von Neumann's abstract machine contained a complete description of its own organization. This information is used in two ways: first, as a set of instructions which the original automaton then executes to make a copy of itself. Second, as passive data to be duplicated and attached to the offspring, providing it with the self-description necessary for it, in turn, to reproduce. Von Neumann's insights were brilliantly confirmed with the discovery of DNA's structure and the genetic code. His theory of automata succeeded in formulating the simplest truly general model of self-reproduction, not only for machines, but for living organisms themselves.

Theoretical research since Von Neumann has taken several directions. The recent volume *Artificial Life* edited by Christopher Langton (1989) presents a fascinating account of the general research themes as well as the diversity of ideas now being pursued. Many of these important themes may be illustrated using the simple model of one-dimensional cellular automata introduced above. Many of the recent developments may provide valuable insights into the dynamics

of social structures.

Local rules and emergent properties

For all examples of one-dimensional cellular automata considered below, let $a_i^{(t)}$ denote the value (0 or 1) of cell i at time t . All N cell values are updated at the same time. Updating occurs in a sequence of discrete time steps, with the new value of each cell only depending on its previous value and the previous values of its two immediate neighbours. All cell values of the automaton are updated according to the same transition rule F , which can be expressed as follows:

$$a_i^{(t+1)} = (a_{i-1}^{(t)}, a_i^{(t)}, a_{i+1}^{(t)})F, \text{ with } i = 1, 2, 3, \dots, N \text{ and } t = 0, 1, 2, \dots$$

For example, under the modulo 2 rule described earlier, F is defined as:

$$a_i^{(t+1)} = (a_{i-1}^{(t)} + a_{i+1}^{(t)}) \text{ modulo } 2 \text{ for all } i \text{ and } t. \text{ Under the convention introduced in Wolfram (1983:604), } F \text{ can be defined explicitly. There are exactly eight possible states of triplets of cells at time } t:$$

$$\begin{array}{rcccccccc} t & : & \underline{111} & \underline{110} & \underline{101} & \underline{100} & \underline{011} & \underline{010} & \underline{001} & \underline{000} & : & \downarrow F_{90} \\ t+1 & : & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & \end{array}$$

The lower line then shows the values of the central cell at time $t+1$ (under the modulo 2 rule). Hence any transition rule F may be specified by a sequence of eight binary digits. The modulo 2 rule defined above is thus rule 01011010, or in decimal notation, rule 90. There are precisely $2^8 = 256$ possible transition rules for the class of cellular automata discussed here. F_4 , F_{50} and F_{22} are other transition rules defined as:

$$\begin{array}{rcccccccc} t & : & \underline{111} & \underline{110} & \underline{101} & \underline{100} & \underline{011} & \underline{010} & \underline{001} & \underline{000} & : & \downarrow F_4, \\ t+1 & : & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & \end{array}$$

$$\begin{array}{rcccccccc} t & : & \underline{111} & \underline{110} & \underline{101} & \underline{100} & \underline{011} & \underline{010} & \underline{001} & \underline{000} & : & \downarrow F_{50} \text{ and} \\ t+1 & : & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & \end{array}$$

t :	<u>111</u>	<u>110</u>	<u>101</u>	<u>100</u>	<u>011</u>	<u>010</u>	<u>001</u>	<u>000</u>	: $\downarrow F_{22}$
$t+1$:	0	0	0	1	0	1	1	0	

The transition rules are *deterministic*: they are Boolean functions taking the values 0 and 1 and combining them by operations of conjunction, disjunction, and negation. (A complete list of all rules with their Boolean formulation is provided in Table 1 of the Appendix to Wolfram (1986).) The rules are applied *recursively*: values at $t+1$ are not obtained by direct substitution in a formula but from values at time step t . Finally, the rules are spatially and temporally *local*: each new value of a cell depends only on the values of a close neighbourhood of surrounding cells (not necessarily only the two adjoining cells), for a fixed number of preceding time steps (usually just one step).

The three rules F_4 , F_{50} , and F_{22} are now compared by applying them in turn to the same randomly generated initial configuration of 40 cells. The rules are applied under the assumption that the cells lie on a circle, i.e., with cells 1 and 40 adjoining.

Rule F_4 yields the following sequence:

	40	1	2	3	4	5	6	7	8	9		40	1	2	3	
$t = 0$	01111010010000110111011000101101000001110111															$\downarrow F_4$
1	000000100100000000000000001000010000000000000															
2	000000100100000000000000001000010000000000000															
.																

The configuration generated at time step 1 is maintained unchanged at all following steps: the automaton has achieved a *steady-state*.

The evolutionary sequence obtained by applying rule F_{50} to the same initial pattern is quite different:

	40	1	2	3	4	5	6	7	8	9		40	1	2	3	
$t = 0$	01111010010000110111011000101101000001110111															$\downarrow F_{50}$
1	10000101101001001000100101010010100010001000															
2	010010100101101010101010101010101010101010															
3	101101011010010010101001010100101010101010															
4	010010100101101010101010101010101010101010															
5	101101011010010010101001010100101010101010															
.																

From step 2 on the automaton exhibits a global pattern of cyclic behaviour, with the same configuration repeating in *alternating* time steps. (I.e., the patterns at step 2, 4, 6, 8, ... are identical, as are the patterns at step 3, 5, 7, 9,)

Finally, after 38 time steps, the automaton governed by rule F_{22} continues to generate new patterns (although it may eventually give rise to a cyclic pattern with periodicity greater than 38):

[illegible]

These examples of the 'evolution' of cellular automata from a random initial state illustrate three qualitatively distinct classes of behaviour: evolution to a steady-state (rule F_4), to a periodic structure (rule F_{50}), or to a chaotic or aperiodic sequence of patterns (rule F_{22}). Simple transition rules yield a variety of behavioral patterns, 'simple' as well as 'complex'. Perhaps the most remarkable fact is that the independence of the cell values in the initial, random configuration is destroyed: the rules generate correlations between the values of cells separated both spatially and temporally. For example, rule F_{22} causes the initial random state to evolve to a sequence of states containing long-range correlations and nonlocal structure. In the space-time diagram above, this feature is exemplified by the distinctive pattern of downward-pointing 'triangular' structures made up of zeros. This is the phenomenon of *self-organization* (Wolfram 1983: 607, 1984; Casti 1989:52-53). A system governed exclusively by a set of *local* rules may, in time, exhibit higher-level, *global* structures and dynamics. Complex behaviour and global structures are thus *emergent* phenomena, implicit in the interaction of simple, local rules. This surprising insight leads directly to the exciting possibility that much of the complex behaviour exhibited by kinship systems may be the result of local interactions governed by simple rules. Lévi-Strauss's programme might still be salvaged.

Exchange structures as discrete dynamical systems

The cellular automata introduced above serve as examples of a more general category of mathematical models: *dynamical systems*. In the following sections I summarize some of the general features of dynamical systems analysis as developed by Casti (1986, 1989) and Louie (1985).

Intuitively, a *system* is a particular subset of the real world considered as the object of study. A *state* is a specification of the system at a particular time - the

assumption being that the system can exist in a set of physically distinct states. Note that under the 'relativistic' view of system complexity supported by Casti (1986, 1989:4), what counts as a 'physically distinct state' is not an intrinsic property of the system. A given system will usually have many sets of abstract states. Which set is used depends crucially upon the observer's choice of 'measuring apparatus' and the means at his disposal for probing the structure and behaviour of the system. An *observable* of a system is some characteristic of the system that can, at least in principle, be measured. More precisely, a system S is composed of an abstract set of states $\Omega = \{\omega_1, \omega_2, \omega_3, \dots\}$ (note that Ω may be finite or infinite). Then an observable is a map $f: \Omega \rightarrow R$, i.e., a rule associating a real number with each state ω_i of Ω . Any observable f induces an equivalence relation R_f on Ω defined as follows: (ω_1, ω_2) is in R_f if and only if $(\omega_1)f = (\omega_2)f$. As summarized by Louie (1985:83-84), an observable f will in general only convey limited information about its domain Ω , because it cannot distinguish between states lying in the same equivalence class - effectively, the states of the system are reduced to the quotient set Ω/R_f . This is why 'alternate' or 'nonequivalent' descriptions of a system are crucial: the more observables we have, the more information we have on the workings of the system. This observation ties in with Casti's definition of the complexity of S as seen by the observer: $C_\theta(S)$ is the number of *nonequivalent* descriptions of S formed by θ . 'The more we can "see" ("know") of a system, the finer discriminations we can make about its structure and behaviour, and consequently, the more complex it will appear to us' (Casti 1989:18).

Finally, a *dynamic* on an abstract state-space Ω is a one-parameter family of transformations $T_t: \Omega \rightarrow \Omega$, with t in R . An observable f on Ω is said to be *compatible* with T_t if any two states ω_1 and ω_2 that are equivalent under f (i.e., $(\omega_1)f = (\omega_2)f$), are such that $(\omega_1)T_t = (\omega_2)T_t$ for

all t in R . Hence f and T_t are compatible if T_t preserves the equivalence classes of R_f (Casti 1989:35).

The one-parameter family of transformations T_t (with time being the parameter t) defines a group of *automorphisms* on the set of abstract states (see Louie 1985:100). A *discrete dynamics* is obtained by considering time to be composed of a succession of 'elementary steps'. The parameter t is thus not defined on the continuum of real numbers R but on the integers $Z = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$. Then, following Louie (1985:116), a discrete dynamics on Ω is a mapping T from $\Omega \times Z$ to Ω , i.e., for each t in Z , each $(\omega, t)T$ is a bijection from Ω to Ω such that (i) $(\omega, 0)T = \omega$ for all ω in Ω , and

$$(ii) ((\omega, t_1)T, t_2)T = (\omega, t_1 + t_2)T \text{ for all } \omega \text{ in } \Omega$$

and all t_1, t_2 in Z . Furthermore, $(\omega, n)T = ((\omega, 1)T)^n$, hence a discrete dynamics is a *cyclic subgroup* of the automorphism group $\text{Aut}(\Omega)$ generated by T .

The essence of the idea of a discrete dynamical system has already been illustrated in the examples of the one-dimensional cellular automata provided above. The transition rule F (defined locally) corresponds to the mapping T , and the abstract states are the patterns of ones and zeros at each time step. I now sketch how the exchange structures introduced in earlier chapters may be reformulated as discrete dynamical systems.

In Chapter 2 the permutation w_x of $Obj(a+x)$ was defined recursively as

$$0_{a+x} j^{w_x} = 0_{a+x-1} j^{(w_{x-1})^k} s, \text{ with } w_0 \text{ the permutation of the origin } Obj(a) \text{ defined as } w_0 = c = (1, 2, 3, \dots, n)$$

and k some integer coprime to n . Roughly, a structure of generalized exchange on n lines with origin $Obj(a)$ then consists of a finite sequence of permutations (exchange mappings) of order n , repeating after p steps. Each permutation is obtained by taking the k th power of the permutation at the previous step. Let C_n be the cyclic group generated by the permutation c of order n . Let

$\text{Aut}(C_n)$ denote the automorphism group of C_n . Both C_n and $\text{Aut}(C_n)$ are Abelian, with the order of $\text{Aut}(C_n)$ equal to $\phi(n)$, the value of the Euler function for n (see Table 2.1 in Chapter 2). Then for some element g of $\text{Aut}(C_n)$, $(c)g = c^k$, $((c)g)g = (c^k)^k$, etc. Hence each element of $\text{Aut}(C_n)$ defines a discrete dynamics as formulated by Louie (1985:116), with transition rule defined in terms of k . (The structures of symmetric exchange formulated in Chapter 4 may be redefined as dynamical structures in a similar manner.)

There are, however, a number of differences between the cellular automaton model and the above reformulation of exchange structures as discrete dynamical systems. First, the transition rule for exchange structures is defined *globally*, not *locally*. The entire exchange circuit at time step $i+1$ is defined recursively in terms of the exchange circuit at time step i (under the 'standard' interpretation the time steps are interpreted as discrete 'generations'). However, an equivalent 'local' definition has already been formulated. For example, the exchange structure $W(a, 7, 2)$ (i.e., the seven-line structure with origin at generation level a and with k equal to 2; see figure 2.9 in Chapter 2 and figure 5.1 opposite) is also obtained if one specifies that a man should acquire a spouse from the same line into which his mother's brother married in the previous generation (assuming patrilineal descent). The local rule (stated recursively) is thus: $wg(i+1) = wg^2(i)$. If males in all seven descent lines apply this rule they will be replicating the marriages of their MB, MMB, MMMB (and their FFF), and the result will be a global structure which repeats with a period of three generations ('states').

There are other possibilities. For example, the global structure of $W(a, 7, 2)$ will also be generated if every male marries his FFZDD. Alternatively (see Lemma 5 in Chapter 2), this is equivalent to stating that two patrilineal lines renew an alliance only after 'marrying out' for two consecutive generations (in genealogical terms, a man

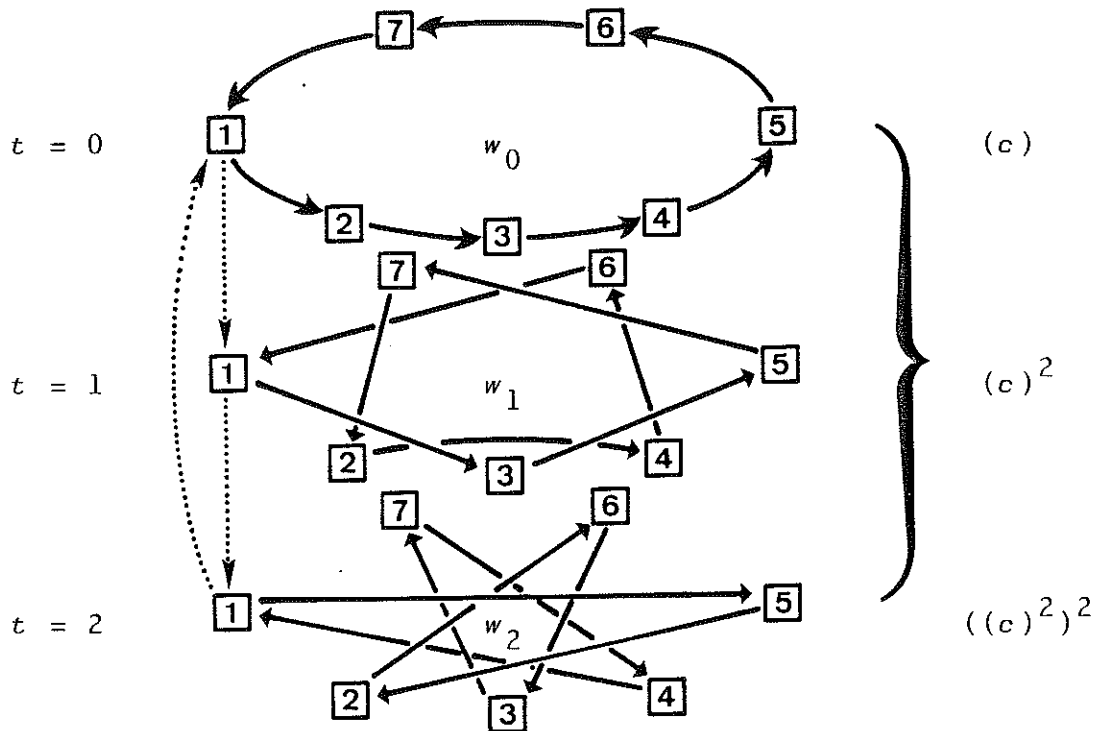


Fig. 5.1. Kinship structure $W(a, 7, 2)$. Discrete dynamical system with a period-3 cycle, i.e., for $t = 3$, $((c)^2)^2)^2 = c^8 = c$ (cf. Fig. 2.9).

marries his FFMBSSD). Such local rules may indeed be expressed or encoded by the participants in a number of ways: in terms of their fundamental social categories, by means of the kinship terminology system, or in the society's key metaphors (for examples, see Chapter 2 and Fox (1980)).

The reformulation of kinship structures as dynamic systems is compatible with the more general methodological framework (the non-statement or structuralist view) introduced earlier. Roughly (see Fig. 5.2 below), 'local rules' may be associated with a special subclass of a theory's 'partial potential models'; the 'global' structures

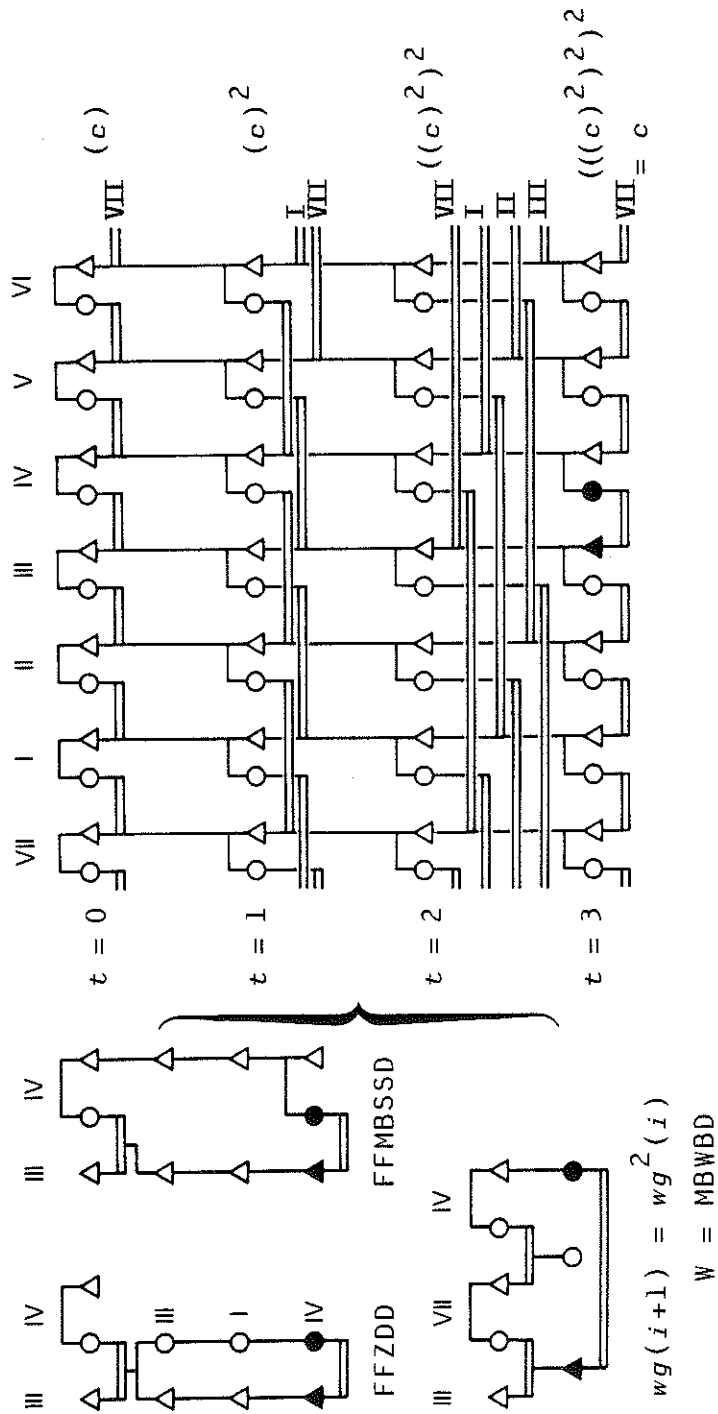


Fig. 5.2. 'Local' rules (partial potential models) generating the 'global' exchange structure $W(a, 7, 2)$ with seven lines and period-3 cycle. Identical genealogical positions are indicated by a solid triangle and circle. (See also Fig. 2.9 and Fig. 5.1.)

generated by their dynamics (with a suitable set of necessary and sufficient constraints on the local rules) are members of the class of proper models. Louie's (1985) extension of system theory, using the framework of the mathematical theory of categories and functors, provides the natural bridge between the Stegmüller - Sneed approach and Casti's (1986, 1989) discrete dynamical systems.

The second difference with the cellular automaton model relates to the definition of the initial state or pattern of exchanges. The structure of generalized exchange (cf. Chapter 2) is defined on the initial permutation $c = (1, 2, 3, \dots, n)$ of order n . The analogous definition of the mathematical structure of restricted exchange (Chapter 4) is based on the permutation $r = (1, 2)(3, 4) \dots (2m-1, 2m)$ of order 2. The full range of exchange patterns (i.e., states) of the structures $W(a, n, k)$ and $D(\bar{a}, 2m, \alpha)$ is then obtained from the automorphism groups $\text{Aut}(C_n)$ and $\text{Aut}(D_m)$ of, respectively, the cyclic group C_n generated by c and the dihedral group D_m generated by r and the permutation $a = (1, 3, 5, \dots, 2m-1)(2m, 2m-2, \dots, 6, 4, 2)$ of order m . In both cases, the initial exchange patterns are chosen so as to conform to the classic anthropological theory of elementary kinship structures. These initial states are thus decidedly non-random. (Cellular automata can of course also be defined on equally specific initial patterns.)

There is at present no general theory available for the dynamics of kinship systems in which some nontrivial initial exchange configuration other than the permutations c and r yields complicated patterns under the application of a variety of local rules. Figure 5.3 gives some insight into the types of evolutionary behaviour that may emerge from such a general approach. In the first example (left) the dynamics of the standard model $W(a, 8, 3)$ with generalized exchange on eight lines is summarized. Under the genealogical interpretation given earlier (see Fig. 2.6 in Chapter 2), this structure represents a kinship model

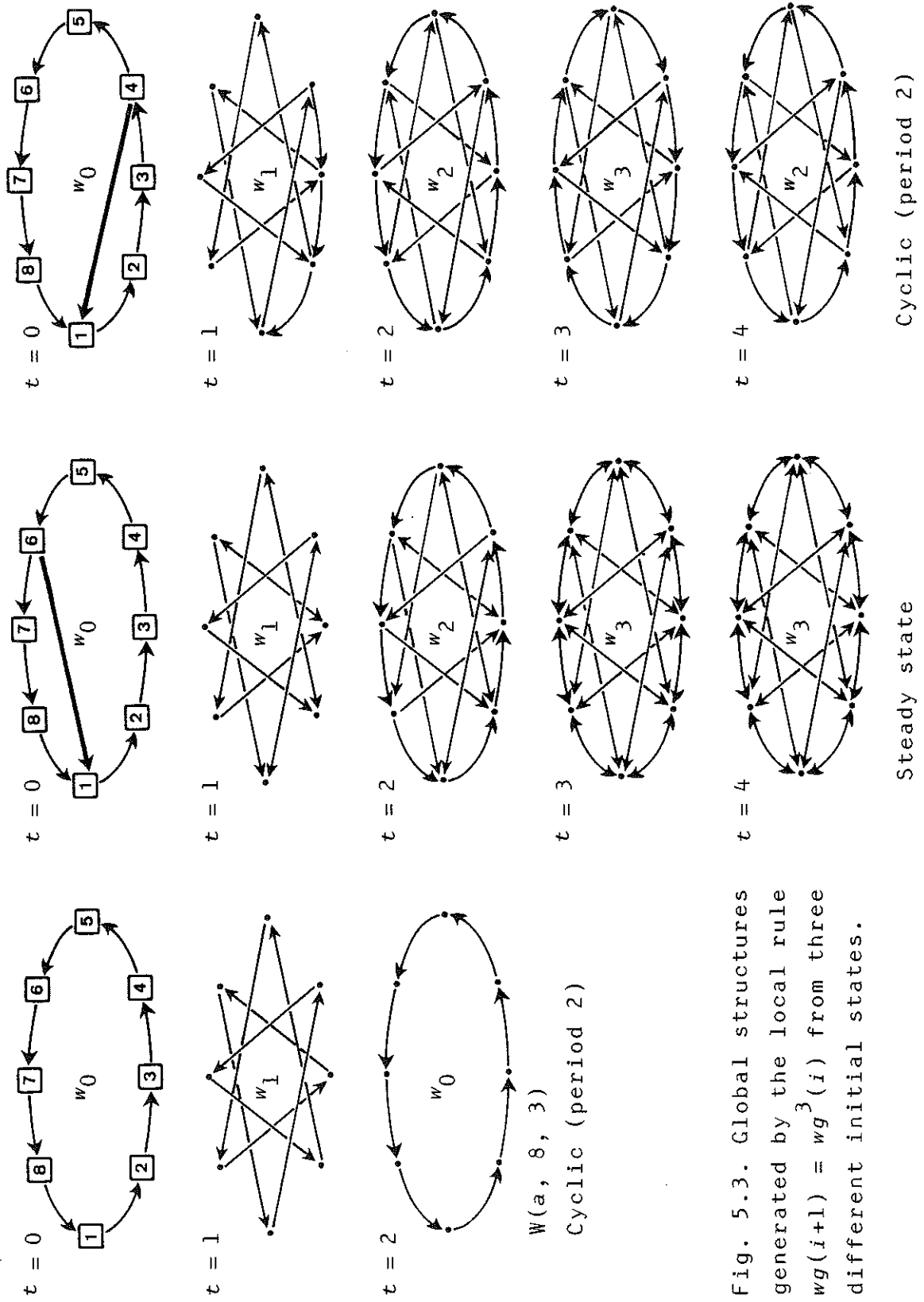


Fig. 5.3. Global structures generated by the local rule $wg(i+1) = wg^3(i)$ from three different initial states.

in which male ego marries his MMBDD or FMBSD and the exchange structure repeats itself in alternating generations (i.e., with a period-2 cycle). Alternatively, given a system based on eight descent lines or 'exchange units', the global pattern is generated by assuming that all men marry according to the local rule $wg(i+1) = wg^3(i)$.

In the second example (Fig. 5.3, centre) the same local rule $wg(i+1) = wg^3(i)$ is applied to a slightly different initial configuration, again defined on eight lines. Here line 6 marries into line 1 as well as into line 7, thus taking advantage of the marriage possibilities defined for successive generations under the previous example. Surprisingly, after only two time intervals (generations), the system has attained a new global pattern: from $t = 3$ on, instead of an exchange structure with generalized exchange, the exchange structure is now based on direct (symmetric) exchange, with the same pattern (w_3) maintained unchanged from then on. The model has evolved to a steady state under the local rule $wg(i+1) = wg^3(i)$. And since exchange is now symmetrical, this rule may itself be replaced or formulated more elegantly as: $wg(i+1) = wg(i)$.

In the third example (Fig. 5.3, right) men in line 4 now marry into two other lines: line 1 and line 5. Under the rule $wg(i+1) = wg^3(i)$ the system then yields a global structure of period 2, but with the pattern of marriage exchanges reversing itself in each generation. Instead of a second-cousin marriage rule, the kinship structure may now be generated by the rule $wg(i+1) = wt(i)$, i.e. one's wife-givers are defined as the wife-takers of the previous generation, and the marriage pattern is compatible with a rule of patrilineal cross-cousin marriage.

These examples of a system's dynamics giving rise to a variety of exchange structures have not, of course, been extracted from kinship data or ethnographic descriptions of actual societies. The models illustrate the following possibility: real kinship networks contain any number of 'short circuits' or alternative ways of tracing genealogical

relationship. The application of a simple, locally defined rule for the allocation of spouses may, if consistently applied, result in a far-reaching transformation of the original system, either at the level of the empirical network of kinship relations or in terms of the society's conception of the ideal structure of exchange. Here again, simple rules may give rise to unexpected dynamics.

PROHIBITIONS, MULTIPLE EXCHANGES AND SEMIGROUPS

The introduction of multiple exchanges and alternate marriages in the initial state of the model $W(a, 8, 3)$ (see Fig. 5.3) requires an extension to the basic kinship model. Under the standard Courrège-Lorrain model introduced in Chapter 1 and applied throughout this book, an *elementary kinship structure* $X = (S, h, m, f)$ with the associated permutation group (S, G) is only defined if h , m , and f are one-to-one mappings of S onto S , i.e., under the assumption that they are permutations of S such that $f = hm$ (under the usual composition of mappings). Under these assumptions the set of permutations on S generated by h , m , and f forms a *group*: the composition of permutations is associative, there is a unique identity permutation e , and every permutation x has an inverse x^{-1} such that $xx^{-1} = x^{-1}x = e$ (see also the appendix to Chapter 1).

In figure 5.3 (top, centre) the initial configuration w_0 is not based on a one-to-one mapping: element 6 is mapped onto element 7 as well as onto element 1. In other words, descent line 6 has two wife-giving lines: multiple exchanges and alternate marriages cannot be represented by permutations, and the kinship structure will not be associated with a group. A more general approach is needed.

One obvious solution is to loosen the group constraints, i.e., drop either one or both of the stipulations for an

identity element and inverse mappings. The simplest and most general mathematical structure fulfilling these conditions is a *semigroup*.

Semigroups

A *semigroup* is a set provided with an associative binary operation that is closed. Semigroups are of wide significance: under the usual composition of relations, any set of binary relations generates a semigroup. For example, let R and S be binary relations on some set Obj . Then the *composition* of R with S is the binary relation RS on Obj defined as $RS = \{(x, z) \mid (x, y) \text{ in } R \text{ and } (y, z) \text{ in } S\}$. Recall that the *converse* of any binary relation R is defined as $\{(y, x) \mid (x, y) \text{ in } R\}$. The converse of R (also called the *inverse* of R) is denoted by R^{-1} .

An element R of a semigroup S_G is *regular* if and only if there is an X in S_G such that $RXR = R$. A semigroup is regular if all of its elements are regular. An important class of semigroups is the class of *inverse semigroups*. A semigroup S_G is called an *inverse semigroup* if for every element R in S_G there exists an element X in S_G such that $RXR = R$ and $XXR = X$ (cf. Boyd et al. 1972:40-41). R and X are called *generalized inverses* and it can be shown that regularity implies the existence of generalized inverses (Kim and Roush 1983:54). Semigroups may contain a *zero element* 0 : an element with the property that, for every element in S_G , $0R = R0 = 0$. Finally, I is called an *identity element* of S_G if $IR = RI = R$ holds for all R in S_G . If zero elements and identities exist, they can be shown to be unique. A semigroup with an identity element is called a *monoid*. Therefore a *group* is a monoid in which every element has an inverse.

There is a simple way to relate the theory of semigroups to the examples of dynamical systems with multiple exchanges illustrated in figure 5.3 (centre and right). First, for binary relations defined on a finite set Obj , it is often

convenient to consider the *Boolean matrix* R associated with each binary relation R . The rows and columns of R are indexed by Obj , with $R[i, j] = 1$ if and only if (i, j) is in R , i.e., if iRj , and $R[i, j] = 0$ if this is not the case. This gives a one-to-one correspondence between the binary relations on Obj and $n \times n$ Boolean matrices. For any binary relation R , the converse relation R^{-1} then corresponds to the *transpose* R^T of the matrix R , i.e., the Boolean matrix obtained by interchanging the rows and columns of R . That is, $R^T[i, j] = R[j, i]$. Composition of relations (the associative semigroup operation) then corresponds to the *Boolean product* of matrices. This product is the same as the product of ordinary matrices, except that addition and multiplication are Boolean.

Hence $RS[i, j] = \sum_k R[i, k]S[k, j]$, with

$$0 + 0 = 0 \quad . \quad 1 = 1 \quad . \quad 0 = 0 \quad . \quad 0 = 0 \quad \text{and}$$

$$1 \quad . \quad 1 = 1 \quad + \quad 0 = 0 \quad + \quad 1 = 1 \quad + \quad 1 = 1.$$

Let w_0 be the Boolean matrix corresponding to the initial pattern of exchange relations w_0 in figure 5.3 (top, centre). Then, under the exchange rule defined as $wg(i+1) = wg^3(i)$,

$$w_0 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad w_1 = (w_0)^3 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$w_2 = (w_1)^3 = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \text{ etc.}$$

$$\text{Hence } W_4 = W_3 = (W_2)^3 = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{pmatrix} \text{ and the system}$$

evolves to a steady state. Note that $(W_3)^T = W_3$, corresponding to the fact that the exchange relation w_t is symmetric for all $t \geq 3$.

This type of exchange system is generated by relations which are not necessarily one-to-one mappings or permutations. The model can be formalized as a kinship structure in analogy to the procedure sketched in Chapter 2.

Let the basic set *Obj* (consisting of the kinship units defined on n descent lines) be partitioned into *generation levels*, with each G_t corresponding to the state of the system at time interval t . Define the patrilineal and matrilineal relations f and m on the product $G_t \times G_{t+1}$ and the marriage relation h on $G_t \times G_t$. Then, in analogy to the Courrège-Lorrain model, (Obj, h, m, f) with $f = hm$ defines a kinship structure associated with the semigroup generated by h , m , and f (with compound relations defined on the relevant products of generation levels). All relations may of course be expressed as $n \times n$ Boolean matrices.

However, this extension of the model still assumes that the basic elements of the set *Obj* are defined in accordance with the two rules introduced in Chapter 1: (a) the unity of the sibling group (merging same-sex siblings and parallel cousins), and (b) marriage prescription (i.e., spouses of structurally equivalent persons are considered to be structurally equivalent; see figure 1.4 in Chapter 1). Under these assumptions the elements of *Obj* correspond to the reduced sibling groups represented (under the standard genealogical notation) by a circle and a triangle.

Hence, under the model (Obj, h, m, f) , with h, m , and f relations such that $f = hm$, the sibling relation is assumed to be equal to the identity relation e . A refinement of the model may be obtained if one introduces one or more types of sibling relation not necessarily equal to the identity.

Boyd, Haehl and Sailer (1972) have demonstrated that a model based on inverse semigroups exists which is able to produce a range of structural reductions similar to that obtained from a Lounsbury-type model of an Omaha system (cf. Lounsbury 1964; inverse semigroups can also be applied to any of the other well known types of kinship system). Their model is based on the following assumptions. First, there is a set $S = \{\delta, \varphi, -, +, *\}$ (the *basis* or *alphabet*)¹⁴ the elements of which may be combined in *strings*. However, not all strings of basic symbols are permitted. Well-defined strings are introduced by adjoining a zero element 0 and the following equations:

- (1) $u0 = 0u = 00 = 0$,
- (2) $xy = 0$,
- (3) $aa = a$, and $\delta\varphi = \varphi\delta = 0$,
- (4) $+a- = *$,

for all u in $\{\delta, \varphi, +, -, *\}$, x, y in $\{+, -\}$, and a in $\{\delta, \varphi\}$. In addition, an identity element 1 is defined by $1x = x1 = x$ for all strings x . The identity element is interpreted as the *empty string*, i.e. the string consisting of no symbols.

Under these constraints, basic symbols and strings may be combined. The concatenation of strings is clearly an associative binary operation, i.e. $x(yz) = (xy)z$ and the composition of strings can be written unambiguously without parentheses as xyz . Well-defined strings are interpreted as kinship relations (the notation is equivalent to the scheme introduced by Romney and D'Andrade (1964)). Boyd et al. introduce one additional assumption: for every string x there is a unique reciprocal or inverse x^{-1} (the converse kinship relation) such that:

$$(5) \quad xx^{-1}x = x \text{ and } x^{-1}xx^{-1} = x^{-1}.$$

The resulting mathematical structure is an inverse semigroup called the *free inverse semigroup for kinship* and is abbreviated *FISK*. As interpreted by Boyd et al. (1972: 43-45) *FISK* is a general structure. Every other kinship system can then be considered as a *congruence* on *FISK*, i.e. as a further reduction obtained by defining a suitable set of equivalence relations on the strings of *FISK*. Note that an equivalence relation C on a semigroup is a congruence if C is compatible with the semigroup binary operation. In the case of *FISK*, C is a congruence if and only if aCb and cCd imply $(ab)C(cd)$ for all a, b, c , and d in *FISK*. Furthermore, if C is a congruence on *FISK*, then *FISK*/ C is a semigroup called the *factor semigroup* or *quotient semigroup* of *FISK* over C and the mapping $\psi: FISK \rightarrow FISK/C$ defined as $(a)\psi = [a]$ (the equivalence class of a) is called the *canonical homomorphism*.

The main focus of Boyd et al. is on the use of semigroup theory as a means of generating and comparing different systems of kin terminology. A *kinship terminology* is defined as a function f from the free inverse semigroup *FISK* onto the set of kinship terms (augmented by a special term 'undefined'). f induces an equivalence relation on *FISK*, namely, ff^{-1} , which is in general not necessarily a congruence. If T is a terminological equivalence on *FISK*, then, according to Boyd et al. (1972:44-45), the problem is to find the 'largest' or 'coarsest' congruence C_T that is contained in T such that if C is any other congruence contained in T , then C is also contained in C_T . C_T is defined as $C_T = \{(x, y) \mid (sxt, syt) \text{ in } T \text{ for all } s, t \text{ in } FISK\}$.¹⁵

Thus, for the so-called Omaha I kinship terminology systems (see Lounsbury 1964), the maximal congruence contained in the terminology appears to be induced by the following set of three equations:

$$(6) \quad +\delta- = +\varphi- = * \text{ (Half-sibling rule),}$$

$$(7) \quad a*a = a \text{ for } a \delta \text{ or } \varphi \text{ (Same-sex merging rule),}$$

$$(8) +\varphi*\delta- = +\varphi* \text{ (Skewing rule I).}$$

Omaha II terminology systems are obtained by substituting the following equation for Skewing rule I:

$$(9) \varphi*\delta- = \varphi* \text{ (Skewing rule II).}$$

A third variety of 'Omaha' systems (Omaha III) is obtained by the following rule:

$$(10) \varphi*\delta = \varphi+\delta \text{ (Skewing rule III).}$$

Lounsbury's Omaha IV systems are generated by applying Skewing rules II and III (together with the half-sibling and merging rules (6) and (7)). Finally, Boyd et al. generate a fifth variety, the so-called Omaha Lineage systems, by adding the following equation to the Omaha I rules:

$$(11) a-b+a = a \text{ for } a \text{ and } b \text{ in } \{\delta, \varphi\}.^{16} \text{ (Unique Parenthood axiom; 1972: 43-44, 47.)}$$

As demonstrated by Boyd et al. (1972:46-49), the congruences associated with the Omaha I, II, III, IV and 'lineage' terminologies are related by inclusion mappings. For example, the congruence associated with the Omaha II equations is a *reduction* of the congruence given by the Omaha I rules (cf. Chapter 1) (conversely, Omaha I is a *refinement* of Omaha II) since the equivalence classes generated by Omaha I are all contained in classes given by the congruence under the Omaha II rules: equivalence of strings in Omaha I implies Omaha II equivalence. More precisely, the quotient semigroup associated with the Omaha II congruence is a homomorphic image of the quotient semigroup associated with the Omaha I congruence on *FISK* under the 'inclusion' mapping φ . That is, if X and Y are, respectively, equivalence classes associated with Omaha I and Omaha II systems, then $(X)\varphi = Y$ if and only if X is a subset of Y (Boyd et al. 1972:46).

Thus, the Omaha Lineage system is a reduction of Omaha IV; Omaha IV is a reduction of Omaha III as well as Omaha II, and both Omaha II and Omaha III are reductions of Omaha I.

Finally, an important class of isomorphisms between

FISK congruences is generated by the 'sex change' mapping σ that interchanges the 'male' and 'female' symbols δ and φ in certain rules or equations. For example, applying σ to the Omaha Skewing rule I gives: $(+\varphi*\delta- = +\varphi*)\sigma = (+\delta*\varphi- = +\delta*)$ which is equivalent to Lounsbury's Crow Skewing rule I.¹⁷ The other Crow equations are easily obtained by applying σ to the remaining Omaha rules.

The Boyd, Haehl, and Sailer application of semigroup theory has recently been introduced as an example in a textbook on applied abstract algebra for graduate students in mathematics (Lidl and Pilz 1984:397-408). Within anthropology there is now a small but growing corpus of analyses representing aspects of kinship systems in terms of semigroup structures. Recent work by Liu (1986) and, in particular, by Read (1984) provides a most welcome demonstration of how kin terminology structures may be related to the algebraic framework of semigroup theory.¹⁸

The algebraic approach need not, in fact, be formulated in terms of genealogical assumptions and is logically independent of the 'extensionist' position defended by Lounsbury (1965) and Scheffler and Lounsbury (1971).¹⁹

The use of semigroup theory as the preferred framework for representing and analysing relational complexes discernible at distinct levels of kinship phenomena (e.g., genealogical space, kin terminology, rules for applying kin terms, the marriage system and models of alliance and exchange), as well as for the comparison of different kinship systems has an obvious advantage. The properties of disparate structures may be precisely defined and compared by exploring the possibility of homomorphic mappings and homologies. Furthermore, the algebraic theory of formal languages and automata makes extensive use of semigroups (e.g., Pin 1986, Le Chenadec 1986).²⁰ The further development of semigroup models promises a new synthesis in kinship studies and the prospect of addressing the fundamental problem of how a system's constituent parts and subdomains act together to produce the complexity

of the whole. However, any extension to the classic exchange paradigm must first deal with the problems posed by the existence of negative marriage rules and prohibitions.

Prohibitions on the reduplication of relations

A number of prohibitions governing marriage and the structure of alliance relations occur in both 'elementary' and 'complex' kinship systems. For example, according to Van Wouden (1968:9-10), the system of social organisation described by P. Drabbe for Jamdena, the central island of the Tanimbar group in eastern Indonesia, is characterised by a triple classification: one's own patrilineal descent group (*mirwan'awai*, literally 'men brothers'); one's wife-givers (*nduwe*, 'lord', 'master'); and one's wife-takers (*uranak*, 'sister's child') - the group to which one gives one's daughters and which is regarded as inferior. The marriage structure is asymmetrical and exclusive matrilateral cross-cousin marriage is the designated means by which the *nduwe-uranak* relationship is maintained. One may either marry a daughter of one's *nduwe* (*bate nduwe*) or an unrelated woman (*bat-waljete*). Marriage with an *uranak*-woman is proscribed 'because in that case we should be swallowing our own blood' (1968:10).

Ideally, one son (the eldest or the youngest) should obtain a spouse from the *nduwe* group (the patrigroup of his mother's brother). However, although leviratic unions occur, two brothers may not marry two sisters - the first married would consider this as 'a blow in the face', or even as 'making use of his wife' (Van Wouden 1968:10). The elementary structure of matrilateral cross-cousin marriage is thus constrained by a rule prohibiting the replication of *nduwe-uranak* relations by two brothers. A similar restriction is reported for the Kei islands: an eldest son is obliged to marry the eldest daughter of his mother's brother - but once this marriage has been concluded, all other sons are prohibited from marrying

'in this degree of consanguinity' (1968:12).

The Nuer rules of exogamy and incest analyzed by Evans-Pritchard in his contribution to the 1949 *Festschrift* in honour of Radcliffe-Brown provide another example of a clear prohibition on the reduplication of relations, from a kinship system with no 'elementary' marriage prescriptions. Thus (Evans-Pritchard 1949:85-86), the limits of patriclan exogamy extend to persons able to trace common descent through male links from an ancestor as far back as ten generations. In addition, a man may not marry a close cognate if relationship can be traced through either male or female links as far back as six generations. Marriage is not permitted between close kinfolk by adoption, between close affines, or between persons classified as 'fathers' and 'daughters' in the age-set system.

However, a man may not take his wife's sister or any near kinswoman of his wife as a second spouse, nor can two brothers marry sisters or 'close cousins' (1949:88). The marriage prohibitions are complemented by an extensive series of incest (*rual*) rules proscribing sexual relations with close kin or affines. Evans-Pritchard analyzes the entire set of rules as instances of a more general Nuer regulation which forbids, as *rual*, two close kinsmen from having relations with the same woman (and, conversely, two closely related women from having relations with the same man). For Evans-Pritchard, the Nuer rules of exogamy and incest function so as to prevent confusion between different kinship categories and the patterns of behaviour in which the relationships are expressed, thereby maintaining the consistency of the kinship system and the payment and distribution of bridewealth (1949:101).²¹

The Nuer example cited above, together with similar material on incest prohibitions, illicit relationships and exogamy from a large number of other societies (the Gusii, the Baule, the Ashanti, the Kaguru, the Mkako, the Mossi, and of course, the Samo, among others) provides the ethnographic background to Françoise Héritier's

analysis of the symbolics of incest (1982).²² While she accepts the Lévi-Straussian postulate on the necessity of exchange as the foundation of any society, Héritier focusses on conceptions of incest and incest prohibitions as 'total ensembles of representations concerning the person, the world, social organization, and the multiple interrelations among these three universes' (1982:153). The relations linking such diverse orders of representations are based on an 'elementary symbolics' of *identity* and *difference* (1982:158). In particular, representations of the constituents of the person, of the relationship between the sexes, and of the vertical and horizontal flows and exchanges that operate through channels of descent or contagation and which become manifest in rules forbidding certain sexual relationships and alliances are construed as instances of a fundamental 'principle of nonduplication of relationships'. This principle underlies the extensive series of prohibitions that characterize the Samo kinship system, Héritier's paradigmatic example of a 'semi-complex system of alliance' (1982:158-160, 162-166).

These arguments receive further elaboration in Héritier's monograph *L'exercice de la parenté* (1981) where criteria based on the reduplication of relationships are employed in her classification of elementary, semi-complex, and complex alliance systems. Héritier distinguishes four possibilities (1981:169):

(A) Repetition of a previous alliance by same-sex consanguines is permitted.

(a) Repetition of a previous alliance by same-sex consanguines is forbidden.

(B) Repetition of a previous alliance by opposite-sex consanguines is permitted.

(b) Repetition of a previous alliance by opposite-sex consanguines is forbidden.

Hence an elementary kinship system with asymmetric exchange and matrilinear cross-cousin marriage is based on the combination Ab; semi-complex systems on aB. The combination

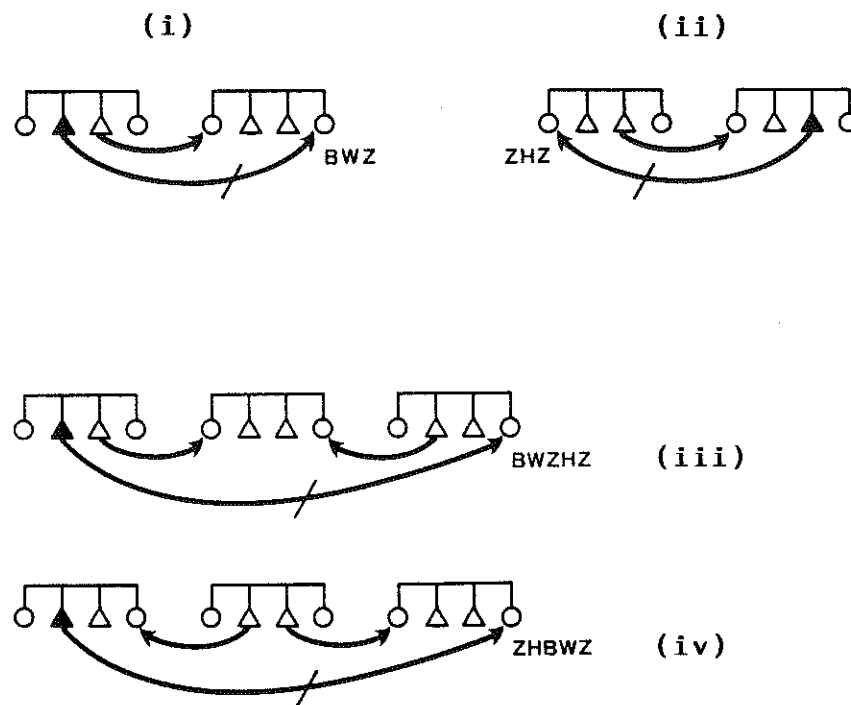


Fig. 5.4. Prohibitions on the repetition of a previous alliance by: (i) a same-sex sibling; (ii) an opposite-sex sibling; (iii) an opposite-sex affine; (iv) a same-sex affine. Adapted from Etienne (1975:9-10).

AB is compatible with either an elementary or a semi-complex structure with symmetric exchange. The final combination, ab, has been described by Pierre Etienne for the Baule (1975). (Indeed, the term 'non-reduplication of matrimonial bonds' was first suggested by Etienne.) Figure 5.4 illustrates some of the basic prohibitions described by Etienne for kin situated at the same generational level. In Héretier's classificatory scheme the combination ab defines a semi-complex system.

Unfortunately, although the fourfold classification described above does possess a certain heuristic value, it proves inadequate when confronted with the full range

of marriage possibilities from actual societies. Thus, in the case of the eastern Indonesian examples cited above, only one son is permitted or required to repeat the alliance previously made by his father and marry his matrilateral cross-cousin. All other sons are prohibited from marrying a sister of their brother's wife. Hence repetition of an alliance contracted in the *previous* generation by a same-sex consanguine is permitted, but the repetition of alliances by same-sex consanguines in the *same* generation is forbidden, giving the combination Aa - a possibility not considered by H  ritier. At the very least, H  ritier's four possibilities (A, a, B, and b) must be contextualized by stating the generational level at which the 'previous' alliance has occurred.

H  ritier has not, I think, provided a definitive solution to the problem of modelling semi-complex alliance systems. However, many of her arguments are highly original and will undoubtedly provoke a reassessment of the traditional studies of Crow-Omaha systems. In any case, her claim that actual marriage patterns are reflections of specific combinations of rules and restrictions on the repetition of previous alliances between close kin (same-sex or opposite-sex relatives) has clear implications for the algebraic models developed in the present work.

First, by defining alliance possibilities recursively, i.e., in terms of previous marriages and alliances, H  ritier's principle of 'nonduplication' of exchange provides the link between the analysis of marriage prohibitions and the recursively defined models of marriage and exchange developed in Chapters 2 and 4 for systems with positive or 'preferential' rules. This suggests the possibility of extending the family of mathematical structures to the analysis of H  ritier-type semi-complex systems. Second, H  ritier's work provides additional justification for finally abandoning, as overly restrictive, two of the classic principles

underlying all attempts at modelling alliance structures as genealogical networks: (a) the unity of the sibling group (interpreted as the structural equivalence of same-sex siblings and parallel cousins), and (b) marriage prescription (essentially the assumption that the spouses of same-sex siblings and parallel cousins are structurally equivalent). The possibility of modifying these assumptions under a semigroup framework has already been discussed above. Héritier's research (her case study of the Samo in particular) offers compelling new evidence for their repudiation.

In the final section of this chapter the arguments introduced above are made more concrete by exploring the formulation of models designed to explicate the structural complexity of ethnographic data from West Africa.

OF BROTHERS AND SISTERS

The basic model (S, h, m, f) introduced in Chapter 1 and elaborated in subsequent chapters defines an algebraic structure on the elements of the set S , i.e., on the set of nodes of a reduced kinship network. Under the standard interpretation elements of S correspond to reduced sibling groups consisting of a brother-sister dyad. Thus, while the basic model is compatible with the cross-sibling (B/Z) distinction, it must be expanded if parallel-sibling relationships (i.e., B/B and Z/Z) are to be distinguished.

A fairly obvious solution is to define a model in which each sibling group is expanded to include two brothers and two sisters. Thus $\{\overline{\Delta} \circ\}$ is expanded to $\{\overline{\Delta_1 \circ_3 \circ_4 \Delta_2}\}$.

With the siblings numbered as above, the following relations are defined within each expanded sibling group:

- (i) $b = \{(1, 2), (2, 1)\}$, the same-sex relation linking

a man to his brother;

(ii) $z = \{(3, 4), (4, 3)\}$, the same-sex relation linking a woman to her sister;

(iii) $x = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$, the opposite-sex relation linking a man to his sisters;

(iv) the converse relation $x^{-1} = \{(3, 1), (4, 1), (3, 2), (4, 2)\}$, i.e. the opposite-sex relation linking a woman to her brothers.

It is easy to show that (under the usual composition of relations), b , z , and x generate an inverse semigroup S_{sb} (the inverse semigroup with relations defined on the expanded four-element sibling unit) with nine distinct elements b , z , x , x^{-1} , b^2 , z^2 , xx^{-1} , $x^{-1}x$, and o .

o is the zero element consisting of the empty set of relations and the remaining elements of S_{sb} are defined as: $b^2 = \{(1, 1), (2, 2)\}$; $z^2 = \{(3, 3), (4, 4)\}$; $xx^{-1} = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$; and $x^{-1}x = \{(3, 3), (3, 4), (4, 3), (4, 4)\}$.

The inverse semigroup S_{sb} is embedded in a generalized kinship structure as follows.

X is a *specialization* of an elementary kinship structure (SEKS) if and only if there exist Obj , h , m , f , b , z and x such that

(1) $X = (Obj, h, m, f, b, z, x)$;

(2) Obj is a non-empty set such that

(i) $Obj = \bigcup S_i$ with $S_i = \{o_{i1}, o_{i2}, o_{i3}, o_{i4}\}$ and $S_i \cap S_j = \emptyset$ for all i and j ;

(ii) $S_i = S_{i\delta} \cup S_{i\varphi}$ with $S_{i\delta} = \{o_{i1}, o_{i2}\}$, $S_{i\varphi} = \{o_{i3}, o_{i4}\}$, and $S_{i\delta} \cap S_{i\varphi} = \emptyset$.

Let $S_\delta = \bigcup S_{i\delta}$ and $S_\varphi = \bigcup S_{i\varphi}$.

(3) h , m , f , b , z and x are mappings such that

(i) $h: S_\delta \rightarrow S_\varphi$ is a bijection (one-to-one and onto);

(ii) $m: S_\varphi \rightarrow Obj$;

(iii) $f: S_\delta \rightarrow Obj$;

(iv) $b: S_\delta \rightarrow S_\delta$ is a bijection, with

- $b = \{(o_{i1}, o_{i2}), (o_{i2}, o_{i1})\}$ for all i ;
 (v) $z: S_{\varphi} \rightarrow S_{\varphi}$ is a bijection, with $z = \{(o_{i3}, o_{i4}), (o_{i4}, o_{i3})\}$ for all i ;
 (vi) $x: S_{\delta} \rightarrow S_{\varphi}$ with $x = \{(o_{i1}, o_{i3}), (o_{i1}, o_{i4}), (o_{i2}, o_{i3}), (o_{i2}, o_{i4})\}$ for all i ;
 (4) $(o_{ij})^f = ((o_{ij})^h)^m$ for all o_{ij} in S_{δ} .

Briefly, a specialization of an elementary kinship structure (SEKS) is formalized by defining a set-theoretical predicate. Under the methodological framework introduced previously, proper models of the expanded kinship theory are defined as set-theoretical entities which satisfy the predicate, and onto which the partial or incomplete structures encountered in data from actual kinship systems are mapped.²³

Each SEKS is associated with a semigroup. In effect, the family of sets S_i (representing expanded four-element sibling groups) constitutes a partition of Obj generated by the equivalence relation $f^{-1}f$ (or $m^{-1}m$). Taking b and z as equivalence relations introduces a further division of each S_i and thus a finer partition on Obj . The elementary kinship structures (EKS) defined in previous chapters are easily recoverable as quotient structures of SEKS (specializations of elementary kinship structures).

Although further sophistication can (and should) be introduced, this definition of the class of models of SEKS is able to cope with most of the relevant detail specified for Héritier's category of semi-complex structures. For example, the four prohibitions illustrated in figure 5.4 are characterized as:

- (i) $W \neq BWZ \rightarrow h \neq bhz$
 (ii) $W \neq ZHZ \rightarrow h \neq xh^{-1}x$
 (iii) $W \neq BWZHZ \rightarrow h \neq bhz h^{-1}x$
 (iv) $W \neq ZHBWZ \rightarrow h \neq xh^{-1}bhz.$

Also, one may now distinguish both types of parallel cousin from each other (i.e., $FBC = f^{-1}bf$ and $MZC = m^{-1}zm$) and from siblings, an operation not feasible under the

earlier formalization of elementary kinship structures. Finally, if one wishes to specify a class of exchange models as discrete dynamical systems, additional criteria would need to be added to the set-theoretical definition of SEKS. These would include an appropriate set of states $\Omega = \{\omega_1, \omega_2, \omega_3, \dots\}$ and a mapping (a discrete dynamics) T from $\Omega \times \mathbb{Z}$ onto Ω (see above).

Further specification of this new class of models is the subject of future research. I now provide examples from theoretical discussions and partial data from actual kinship systems to illustrate something of the range of structural phenomena to be taken into account (as 'intended applications') by any such formalization.²⁴

As a first example I present a hypothesis formulated by H  ritier for 'Omaha' systems. According to H  ritier (1981:91-92), 'Omaha'-type systems of alliance (a sub-category of her 'semi-complex' alliance systems) arise from the intersection of two distinct classes of prohibitions. On the one hand, prohibitions that define the limits of exogamy: certain consanguineals (lineals as well as collaterals), specified with respect to some common ascendant, will be proscribed as marriage partners for ego and his or her full siblings. On the other hand, prohibitions on relations of affinity: injunctions on obtaining a spouse from the line or kin group of one's mother, one's grandmother, etc. I.e., prohibitions on the reduplication of previous alliances, to be observed for a specified number of generations.

H  ritier links the existence of such extensive classes of prohibitions to a more specific hypothesis. Thus, in 'Omaha' systems (generally associated with a rule of patrilineal descent) consanguinity must be well-defined and expressed unambiguously: all relationships between members of the same descent line (*lignage*) are traced through a genealogical chain whose intermediate links are only connected through males. Distinct principles of unilineal descent are not to be combined: one's agnates

are never, simultaneously, one's cognates (Héritier 1981:92). Héritier gives the following example (1981:92, fig. 26):

Or, si mon père épouse une femme apparentée à la mère d'un de ses frères, la fille du frère de cette femme par exemple, alors ce frère de mon père devient également pour moi un parent par les femmes. Nous nous retrouvons tous deux dans le statut mutuel de deux neveux utérins dans le même lignage maternel. Le non-redoublement de l'alliance prévient cette situation.

Héritier's summary diagram (her figure 26) gains considerable clarity and precision of presentation when extended sibling groups are employed and the marriages replicated in consecutive generations are emphasized. As represented below (figure 5.5 (i)), if a man repeats the alliance made by his father's brother and marries his FBWBD, then *a* and *e* are related as FFBS/FBSS as well as MFZS/MBDS. In other words, not only do *a* and *e* belong to the same patriline — so do their mothers. Hence *a* and *e* are agnates who are also related through their mothers. According to Héritier this state of affairs is precluded in 'Omaha' systems by means of a prohibition on the repetition of previous alliances by same-sex consanguines.

A similar contamination of kinship relationships does not occur if a man repeats the previous alliance of his father's sister (i.e., an opposite-sex consanguine) and marries his FZHBD (see figure 5.5 (iii) which is an adaptation of Héritier's figure 27 (1981:93)). In genealogical terms, a man marries his FZHBD and *a* and *e* are related as FFZS/MBSS and as MFBS/FBDS. In this case *a* and *e* do not belong to the same patriline (or matriline); neither do their mothers: in fact, *a* is a member of *e*'s mother's patriline and vice versa. Héritier considers this type of exchange the 'inverse' of the previous (partial) structure, with the implication that it is fully compatible with 'Omaha' prohibitions encountered in actual kinship systems.

Although not formulated by Héritier, the analogous

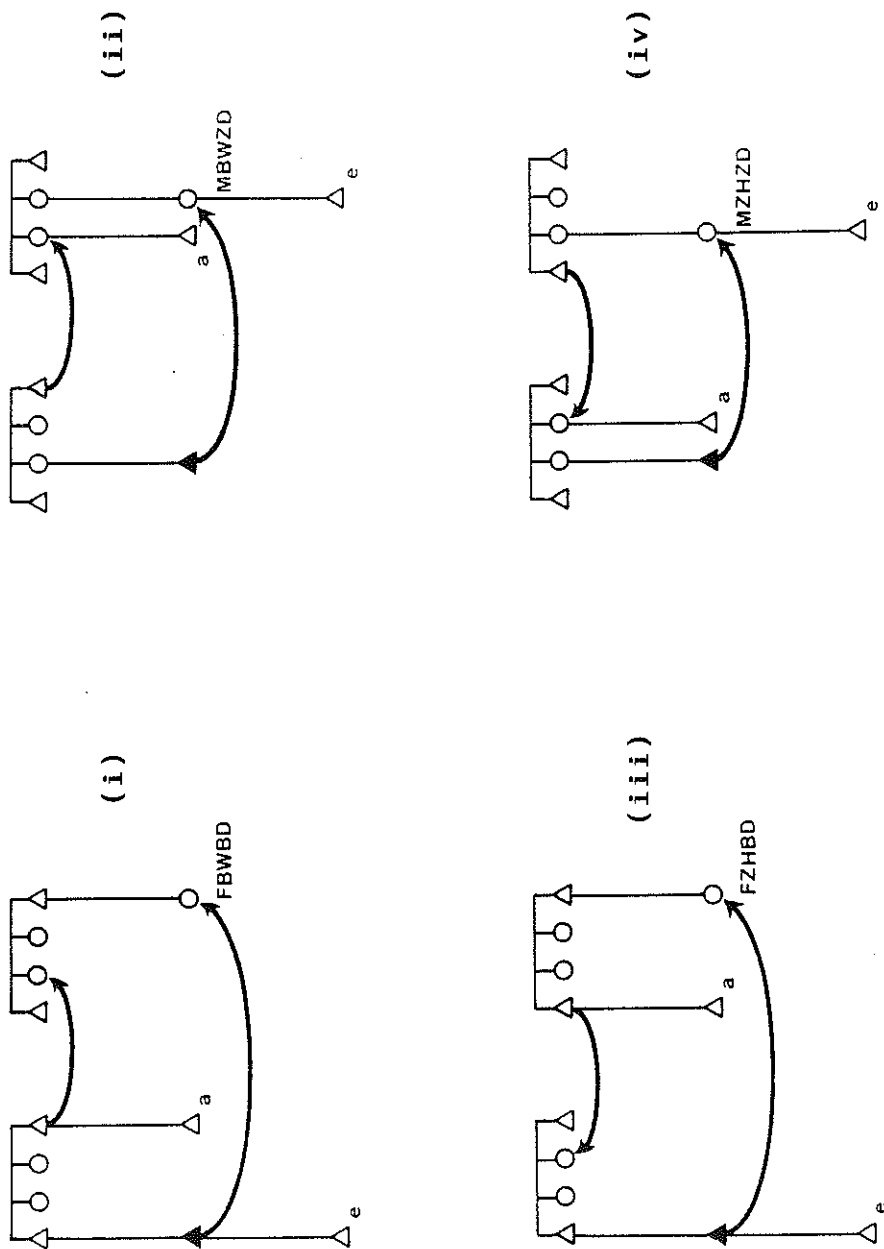


Fig. 5.5. Partial structures representing the repetition of a previous alliance by a same-sex consanguine (i), (ii), and by an opposite-sex consanguine (iii), (iv).

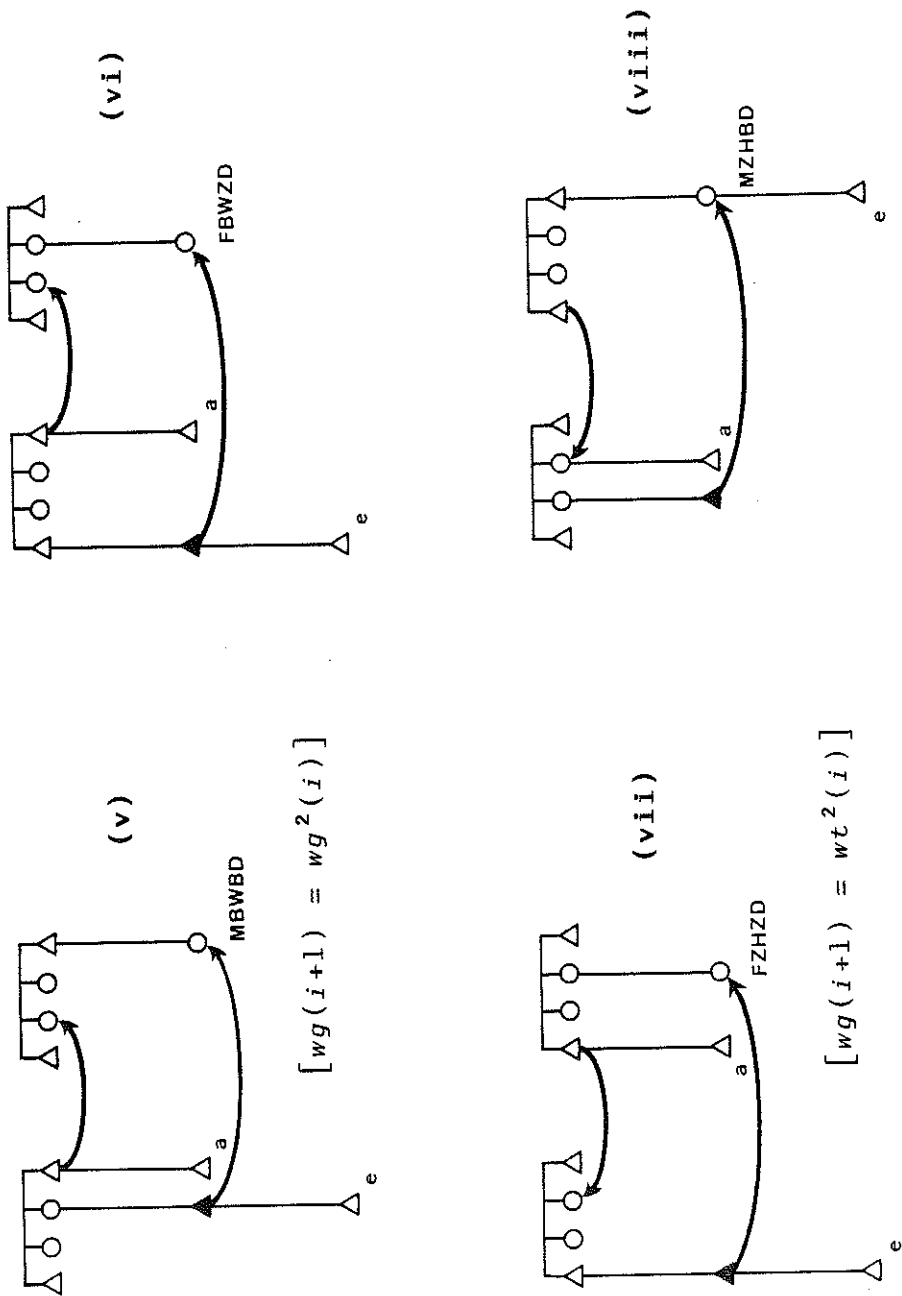


Fig. 5.5 (continued). Partial structures representing the repetition of a previous alliance by a same-sex consanguine (v), (vi), and by an opposite-sex consanguine (vii), (viii).

hypothesis for matrilineal 'Crow' systems is easily obtained by interchanging the 'male' and 'female' symbols (the triangles and circles) in figure 5.5 (i). Thus, if a man repeats the previous marriage of his mother's brother (a same-sex matrilineal relative) and marries his MBWZD (see figure 5.5 (ii)), *a* and *e* (the offspring from the two marriages) are not only related as MMZS/MZDS but also as FMBS/FZSS. Hence *a* and *e* are matrilineal kin, and so are their fathers. (Strictly speaking, under the 'sex change' mapping *a* and *e* should represent women, and from a female ego's point of view a woman marries her MZHZS, replicating the marriage of her mother's sister. The structural implications are, of course, identical.)

Conversely, and again in analogy to Héritier's 'Omaha' hypothesis, in matrilineal 'Crow' systems a man should be able to repeat the marriage of his mother's sister (an opposite-sex matrilineal relative) and marry his MZHZD. If this occurs (see figure 5.5 (iv)), there is no intersection or cross-cutting of kinship categories: *a* belongs to *e*'s father's matriline, and vice versa.

These four structures do not, however, exhaust all of the possibilities for replicating a previous marriage. There are exactly eight distinct possible realizations of a rule either prescribing or forbidding marriage with one's parent's sibling's spouse's sibling's offspring (PSbSpSbC). These are (for a male ego): (i) $W = FBWBD$; (ii) $W = MBWZD$; (iii) $W = FZHBD$; (iv) $W = MZHZD$; (v) $W = MBWBD$; (vi) $W = FBWZD$; (vii) $W = FZHZD$; and (viii) $W = MZHBD$. The complete range of structures is summarized in figure 5.5.

Types (v) and (vii) have been introduced previously: they belong to the family of models formulated in Chapter 2 (see Table 2.4) and defined on *unexpanded* (i.e., two-member) sibling groups. The proper model $W(a, 7, 2)$ is the paradigmatic example, a seven-line model described for the Swazi, the Pawnee terminology, and various American kinship systems (see the discussion in Chapter 2,

figure 2.9, and figures 5.1 and 5.2 above). The global structure of exchange is generated by the recursive formula $wg(i+1) = wg^2(i)$, i.e., one's wife-givers are the wife-givers to the wife-givers of the previous generation, with male ego marrying into his MBW's line (his father's WBW's line). In genealogical terms, marriage is with a FFZDD, merged with the FFMBSSD. Kinship categories intersect at the generational level of the grandparents. Thus, in figure 5.5 (v), e's mother (father) belongs to a's mother's patriline (father's matriline); conversely, a's mother (father) belongs to e's mother's patriline (father's matriline).

Type (vii) is associated with a global structure of exchange generated by the formula $wg(i+1) = wt^2(i)$, i.e., one's wife-givers are the wife-takers to the wife-takers of the previous generation. In figure 5.5 (vii) e's mother (father) belongs to a's father's matriline (mother's patriline), and a's mother (father) is a member of e's father's patriline (mother's matriline). Marriage with the FZHSD is described by Jonckers for the Minyanka in Mali (1983:81, fig. 2). The patrilineal Minyanka have a system of marital alliances based upon the exchange of 'sisters' as well as on an extended set of 'Omaha'-type prohibitions. The terminology exhibits 'Omaha'-type skewing. According to Jonckers' analysis, a man may not marry his MBD or his FZD, but marriage with a FZHBD (type (iii)) or a FZHSD (type (vii)) is permitted (amongst other possibilities), and a global structure is repeated in three generations (1983:80-86, 92-94). Although further analysis is required, the Minyanka data tend to support Héritier's 'Omaha' hypothesis on the replication of opposite-sex alliances.

In type (vi) structures, ego marries his FBWSD, repeating the alliance made by his father's brother. However (see figure 5.5 (vi)), e and a both belong to the same patriline and to the same matriline — hence, under Héritier's hypothesis this type of alliance structure should not occur under either an 'Omaha' or a 'Crow'

regime.

In the final structure (figure 5.5, type (viii)), ego repeats the alliance of his mother's sister and marries his MZHBD. *e*'s mother (father) belongs to *a*'s father's patriline (mother's matriline), and *a*'s mother (father) belongs to *e*'s father's matriline (mother's patriline).

The development of models based on extended sibling groups is essential if H  ritier's 'Omaha' hypothesis is to be refined and articulated with data and relational complexes from other cultural domains. The partial exchange structures of figure 5.5 are already a considerable improvement on the classic assumptions. For example, under the supposition that same-sex siblings are structurally equivalent (or the analogous terminological assumption exemplified by, say, Lounsbury's (1964) same-sex sibling merging rule), a FBWBD (i) or a MBWZD (ii) are equated with a MBD, a FZHBD (iii) or a MZHZD (iv) with a FZD, and a FBWZD (vi) or a MZHBD (viii) with a Z. These are all distinct types under the extended framework.

Future research should also focus on the class of kinship structures embodying what may be considered the reverse of H  ritier's 'Omaha' hypothesis (and its 'Crow' analogue): systems that embrace or specify the exact combination of marriage alliances prohibited under 'Omaha' (or 'Crow') modes of exchange. Kolenda's insightful comparison of wedding rituals and the summary images of the bride in two Indian subcultures provides a clear example of such a structure (Kolenda 1984). Thus, the Rajputs (the dominant landowning group in the north Indian village of Khalapur) practice village and *gotra* (patriclan) exogamy. They also avoid marriages within the geographical subsection *thamba* (a unit including the caste chapters in a cluster of villages) of the mother's *gotra*. Exchange is asymmetrical: marriage is hypergamous, with the permanent subordination of the bride's family to the groom's. Under such a hypergamous regime, no sister-

exchange is possible: men from the bride's patrilineage (the lower ranking group) cannot, of course, obtain wives from the groom's patrilineage. The Rajput rules of *gotra* exogamy prevent the marriage of cross-cousins: a man cannot marry his MBD (as she belongs to the *gotra-thamba* of his mother), or a girl her MBS (who, conversely, is a member of her mother's *gotra-thamba*). However, enduring ties may exist between villages or minor lineages, and a groom may ask for his bride's sister as a spouse for his brother. Possible spouses are a man's BWZ, his FBSWZ or his FBWBD, emphasizing the replication of previous alliances (Kolenda 1984:101-102, 111-114). Kolenda characterizes the Rajput exchange system as follows (1984:102):

The Rajput system is one of *deflected alliance* ... In such a system; one cannot marry a cross cousin but a man can 'repeat' his brother's or his male parallel cousin's marriage by marrying that man's WZ, or he can marry a male parallel cousin's female cross cousin.

Kolenda considers the Rajput marriage system as a modified form of generalized exchange (1984:101). In Hérítier's terminology the alliance structure is based on the intersection of the (lineal) *gotra-thamba* rules of exogamy with a set of rules which (1) forbid the repetition of a previous alliance by an opposite-sex sibling; (2) permit the repetition of a previous alliance by a same-sex sibling or parallel cousin; (3) permit the repetition of a previous alliance by a man's father's brother, i.e. marriage with a FBWBD (figure 5.5, type (i)).

The focus of Kolenda's analysis is actually on the comparative symbology of ritual performance, as codified and validated in weddings of the north Indian Khalapur Rajputs and the south Indian Nattati Nadars. However, her attempt at providing cultural content for Lévi-Strauss's supreme category of the gift — woman — is important to any extension of the classic exchange models. Different summary images of the bride in Indian culture (Rajput:

woman as tribute; Nadar: woman as flower) go together with different conceptualizations of fertility and of the valuation of exchange relations. In the final analysis, any realistic framework for the comparison of kinship systems must be able to deal with the articulation of cultural content with structural form.

To summarize some of the themes introduced above: in the first place, I think it is safe to say that the fundamental opposition between 'elementary' and 'complex' kinship systems introduced by Lévi-Strauss has only been partially successful. Arguments based on the investigation of real-world systems, together with a critique of the dichotomy as a determining factor in the specification of possible models (either 'mechanical' or 'statistical') point to certain fundamental limitations. The field of kinship theory awaits an appropriate, natural measure of 'complexity' (as do many other fields).

In the second place, a number of constraints evident in standard representations of kinship structures are overly restrictive and impose severe limitations on the development of a more realistic theory. For example, the presupposition concerning the equivalence of same-sex siblings with respect to their marriage possibilities must be reconsidered. As a first step, I introduce a family of models (to be formalized as semi-group structures) based on extended (four-element) sibling groups. (See Chapter 3 for a discussion of a different extension to the standard models, e.g., age-constrained, helical structures of exchange.)

Finally, exciting new developments in cellular automata theory and dynamical systems analysis highlight the importance of studying structures as emergent phenomena: complex, global dynamics and developmental processes may emerge from the strictly local interactions of components or entities governed by a limited set of simple rules. Interpreted as a programme of research for kinship and the dynamics of exchange systems, this approach focusses

on the problem of identifying the appropriate set of local rules (some of which may be defined recursively). Such a programme of research appears especially well suited to modelling Héritier's hypotheses on the (non) replication of previous alliances. There is an important advantage. One may dispose of the problem of attempting to define or classify real kinship systems as 'simple', 'semi-complex' or 'complex' in terms of the constituent components of the social structure (e.g., the number of exchange units), the underlying mechanism of exchange (e.g., kinship, the transfer of wealth, or free choice), the type of marital rule (prescription, preference, or prohibition), or the kind of model ('mechanical'-deterministic or 'statistical'-probabilistic). Complexity is an emergent property. The recursive specification of exchange as a set of local rules yields a family of models with the potential for describing either a periodic structure of alliance or a more chaotic behavioural complex.²⁵

Dynamic behaviour in exchange models depends on the specification of local rules which determine the interaction of the system's basic units (extended sibling groups or some other kinship unit), and thus the transformation of its states at discrete time steps. The models developed here are all constrained by wrapping the basic exchange grid or kinship network around a cylindrical structure, with descent lines oriented along the main (time) axis. Under the classic assumptions, alliance cycles are situated at discrete generation levels corresponding to the system's states. However, as demonstrated in Chapter 3, further elaboration of such structures to represent the possibility of large average husband-wife age differences (or, equivalently, the exchange of sister's daughters or other close kin other than sisters) leads directly to a series of models with helical structures of exchange. Alliance networks are neither closed nor confined to separate generation levels but wind across the

surface of the cylinder in a series of interconnected spirals. Articulated as a family of discrete dynamical systems, this implies that closed circuits or exchange trajectories are not allocated to separate states of the system.

The development of age-constrained models incorporating extended sibling groups is more than just a novel technical challenge. Such extensions to the classic models of alliance theory are required if recent material on various forms of West African marriage systems is to be treated adequately. My final example outlines a few steps in the direction of such a class of extensions. The model I discuss accommodates data from the Beliyen or Bassari, one of the Tenda peoples.²⁶

The Tenda constitute a group of related societies with structural similarities (presumably reflecting a common origin) as well as differences: the Coniagui (numbering approximately 15,000), the Beliyen (about 12,000), the Boin (approximately 1,000), the Badyaranke (nearly 6,000), the Bedik (about 1,500), and the Tenda Mayo (less than 1,000). The Tenda inhabit an area lying across the frontier of south-eastern Senegal and the Republic of Guinée; Tenda populations are also found in Guinée Bissau and the Gambia (Gessain and de Lestrang 1980).

The Coniagui and Beliyen are matrilineal with Crow-type terminologies; the Badyaranke are apparently bilineal, while the Boin and the Bedik are both patrilineal (there is little information on the descent system of the Tenda Mayo). The Boin are evidently a Beliyen group which converted to Islam in the 19th century; a similar process is now taking place among the Badyaranke (Gessain and de Lestrang 1980; Simmons 1980).

Beliyen society is essentially a gerontocracy, with power and authority vested in the elders. There is a highly structured system of male and female age-classes, with recruitment occurring at about the age of seven. Promotion and class progression take place at six-year

intervals, with the ritual of promotion intimately associated with rights and duties related to the system of agriculture and communal labour (Nolan 1975; Gessain: 1971; Dupré 1965). According to Gessain (1971:159, 175) the relationship terms applied within the age-class system are linked to the kin terminology and the 'real' kinship system. Thus, in the case of consecutive classes, members of the senior age-class are the 'fathers' (or the 'mothers') of members of the succeeding class (their 'sons' or 'daughters'); the relationship is one of strict authority. Similarly, members of alternating classes refer to each other as 'grandparents' and 'grandchildren' (*tyatya*) in what may be regarded as a reciprocal joking relationship. The age-class idiom of joking and respect may be extended to the actual children of age-class members.

Beliyan marriage possibilities are constrained by the following set of rules (cf. Gessain 1963, 1982; Ferry and Guignard 1984):

(1) Exogamy of the matrilineage: persons with the same lineage name should not intermarry.

(2) The necessity of reciprocal exchanges: when a man marries, his matriline should, in turn, provide a young girl as a marriage partner for a man of his wife's matriline (Gessain 1963:145). However, sisters are not exchanged symmetrically, nor are two brothers permitted to marry two sisters (Ferry and Guignard 1984:54-56).

(3) Marriage between first cousins is forbidden. However, the following kin-type marriages are allowed (Ferry and Guignard 1984:54-56): (i) $W = ZHBD$; equivalently a man obtains his brother's daughter as a spouse for his wife's brother; (ii) $W = FZSD$; i.e., a man gives his ZSD (a classificatory daughter under a Crow terminology) to his son as a wife. Alternatively, since sons of men of the same matrilineage (for example, the reciprocal pair $FZSS/FMBS$) are *ruwis* — 'offshoots of the same yam stalk' (Ferry and Guignard 1984:36, 48-49), a man marries the

sister of his *ruwis*-'brother'; (iii) W = MZHZSD, an extension of (ii), i.e., a man marries the FZSD of his MZS, a lineage brother.

(4) Around 1930 the average husband-wife age difference d_{HW} was approximately 6 years (the span of one age-class), with most women marrying and giving birth to their first child at the age of 18-23 years. Earlier, d_{HW} may have been greater; it has since decreased, together with the average age at marriage for both men and women (Gessain 1982:636-647). However, even as recently as 1970 (if Gessain's data on Etyolo village can be generalized) Beliyen society exhibited 'gerontocratic' tendencies, with the older men claiming a disproportionate number of spouses. Thus, all 27 men aged 42 or older were married; the average number of spouses was 2.4 with a range of 1-5 women per man. In contrast, the 59 men younger than 42 were married to 73 women for an average number of spouses equal to 1.2; 14 men remained single and the maximum number of spouses for this age group was only 3 (adapted from Gessain 1982:642, Table VI). (The relative imbalance may be even greater, as a number of unmarried young men had left Etyolo. See Nolan 1975.)

(5) Finally, the Beliyen marriage system is fairly closed, with over 85% of the marriages taking place within the village or a small group of villages already linked through previous alliances (Gessain 1963:172, 1982:637).

The mixed set of Beliyen rules involves lineal prohibitions, prohibitions on the repetition of marriages by siblings, as well as directives on the importance of reciprocal exchanges. Moreover, the occurrence of relative age constraints on marriage (and thus of different average generation lengths for males and females; see Chapter 3) leads to the possibility of 'oblique' unions: marriage with the ZHBD, FZSD, or MZHZSD, all situated at the generation level below male ego.

Perhaps surprisingly, this miscellaneous set of rules and constraints can be accommodated by a fairly simple

exchange model based on extended sibling groups. I present the main characteristics of such a model as a partial structure in figure 5.6. More precisely, I claim that much of the information provided for the Beliyen marriage system can be reformulated as a set of partial structures which may then be extended to proper models of the type described earlier as specializations of elementary kinship structures.

The partial model of figure 5.6 comprises five distinct clusters of matriline, each cluster representing some named 'matrilineage'. Though not explicitly diagrammed, collateral lines are superimposed. Hence cluster 3 includes ego's matriline as well as the matriline of his MZ, his MMZ, etc. Note that while matri-parallel kin (e.g., Sb and MZC, MSb and MMZC, etc.) are allocated to the same cluster, patri-parallel kin are not, since brothers are required to marry into different matrilineages. There are a number of ways in which a global structure of alliance is generated by a rule of exchange. For example, by assuming the symmetrical exchange of sister's daughters, not sisters: two men in different matrilineages marry each other's sister's daughter (WMB is thus equivalent with ZDH), and the cycle of exchange repeats in alternating generations, not consecutively. This particular formulation of a generating rule is compatible with a number of terminological equations. For example, the Beliyen kin term *ayú* is applied self-reciprocally. The class of its denotata includes the following kintypes: MB/ δ ZC, MMZS/ δ MZDC, WSb/ZH, WMZC/*MZDH*, δ ZHZ/*qBWB*, δ ZHZS/*MBWB*, HMB/ δ ZSW (Ferry and Guignard 1984: 51-52; Ferry 1974:624-627. Kintypes in italics are not given explicitly but are implied by the principle of self-reciprocity.). As can be seen from figure 5.6, *ayú* is applied to ego's MB and ZH, and ego's WMZC (and their reciprocals, among other kintypes), suggesting the possible gloss 'exchange partners'. This interpretation is supported by comments made by Ferry (1974:626-627) and

by Ferry and Guignard (1984:53). MB and ZS are also, in a sense, 'exchange partners', since a man may inherit or claim his MBW (his *alindawòn*, the same term used for W and BW) (Ferry and Guignard 1984:49, 53).

Furthermore, assuming a closed structure of alliance based on five matrilineages or clusters of matrilineages, the model of figure 5.6 reproduces the kintype specifications (see (3) above): W or BW is equated with ZHBD and with FZSD and MZHZSD (if these last two kintypes are not classed as cross-cousins). Again, this is in agreement with the available data: there are only seven major lineages, and in 11 villages studied by Gessain (1963:160) more than 90% of the inhabitants can be allocated to five or six intermarrying lineages. Five is thus a plausible constraint for a closed structure.

Finally, interpreted as an age-constrained structure, the model is compatible with large husband-wife age differences. In fact, assuming that $d_{FC} = d_{HW} + d_{MC}$, $d_{HW} = d_{MC}$ and ideally, d_{FC} (the mean age difference between father and child) should be twice as great as d_{MC} (the mean age difference between mother and child). This prediction conforms to an even more 'gerontocratic' model than is actually described for the Beliyen. As mentioned in Chapter 3, in this class of age-constrained models all age differences between siblings are ignored. The assumptions may be adapted if such a distinction is required.

It is not my intention in this chapter to present a fully articulated, dynamic theory of Beliyen kinship; the partial structure of figure 5.6 is of course only a programmatic sketch, indicating the important gains to be made by adopting a more comprehensive framework. The main thrust of my argument is that a class of 'more complex' kinship models, formulated on extended sibling groups, can be defined directly as algebraic structures. Indeed, I maintain that the further development of such a

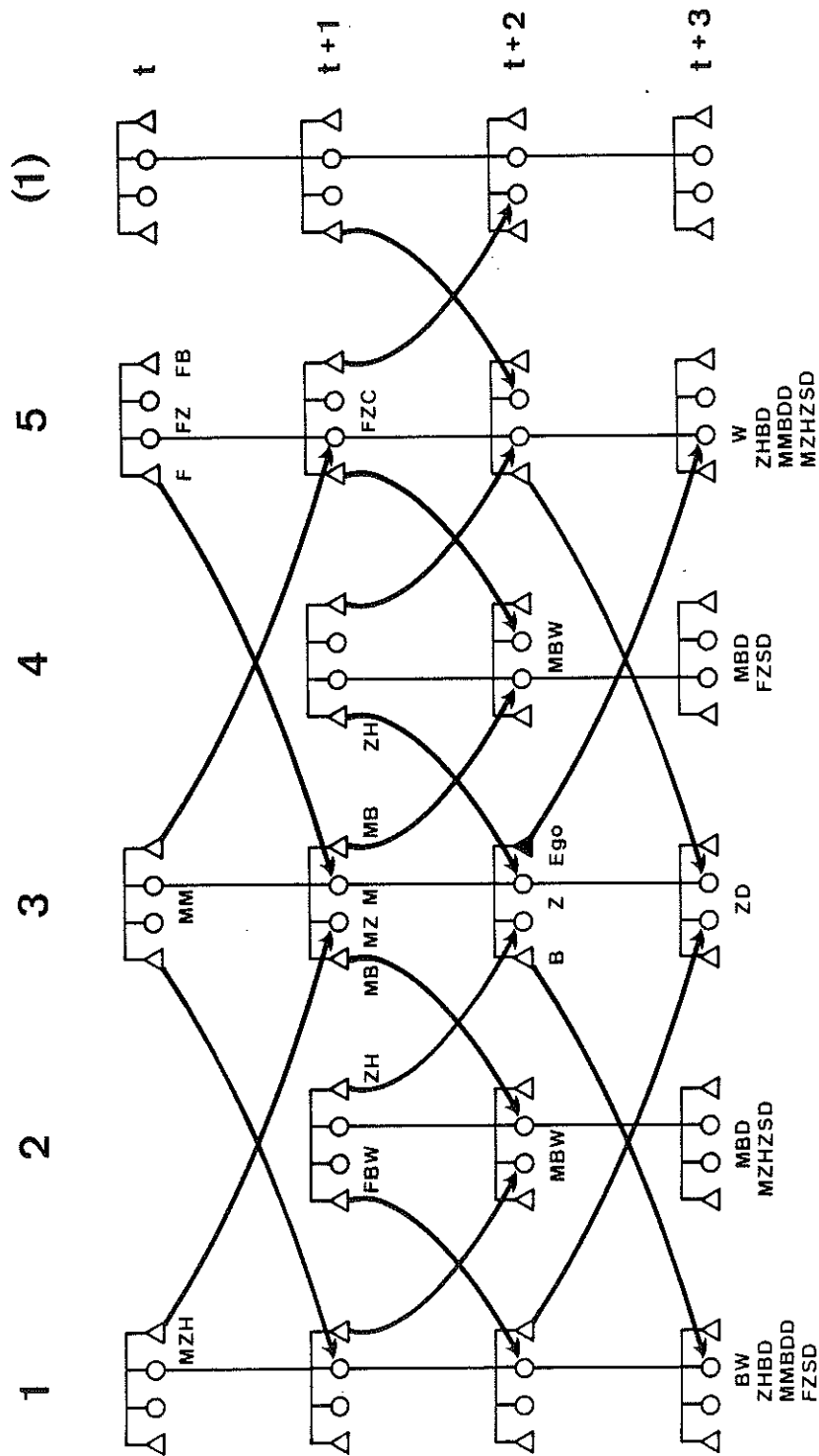


Fig. 5.6. Partial model representing the Beliyan kinship structure. Oblique marriage with the ZHBD and the exchange of sister's daughters generates an alternating generation structure on five clusters of matriline with d_{MC}/d_{FC} equal to .500.

formulation is mandatory if anything like an adequate theory rich enough to capture the full range of structural and dynamical phenomena exhibited by real kinship systems is to be realized.

Moreover, any general treatment of kinship structure should provide insights into the development of specific local variants of kinship systems. Under the research programme characterized by the work of the Leiden school (see Chapter 1 and Kuper 1987:110-133), specific local configurations (including variation in kin marriage formulae) are examined and explained as transformations of a shared cultural tradition. In terms of the non-statement conception of theories, local variation (both within and between the members of a series of historically related cultures) is represented by the class of partial structures, substructures of the class of proper models directly specified as a set-theoretical predicate.

As stressed by Kuper (1987:110-111, 131-133), a fundamental weakness of Lévi-Strauss's classic theory of elementary structures (and by implication, a weakness shared by Héritier's more recent extension to semi-complex structures) is the clear priority given to universally valid, culture-free formulations of exchange. Wives are exchanged for sisters, and all marriage formulae are largely independent of local cultural, historical and ecological constraints. On the argument of Kuper et al. the method of regional structural comparison produces more sophisticated results. The next step is to generalize the Belian analysis sketched above through a systematic examination of the regional configuration of variables and historical factors for the entire group of Tenda societies, in conjunction with a formal analysis of the structural transformations operating on the class of proper kinship models.

Finally, the notions of 'elementary', 'semi-complex' and 'complex' kinship structures central to the structuralist programme must be reconsidered. At the very

least, there is no necessary or contingent relationship with distinct classes of kinship structures or formal models, a point made repeatedly in this chapter. Kuper's historical critique, *The Invention of Primitive Society. Transformations of an Illusion* (1988) points to a more fundamental defect. On Kuper's argument, Lévi-Strauss's theory of marriage exchange (and indeed, most of kinship theory) is severely constrained by the idea of 'primitive society' which crystallized in the late nineteenth century. Although cycling through repeated transformations, all variants of the basic paradigm continued to be formulated reflexively, as a contrary vision of the own society or by negating and inverting the models of one's predecessors. Thus (Kuper 1988:240-241):

The most powerful images of primitive society were produced by very disparate political thinkers — Maine, Engels, Durkheim and Freud. Yet all were transformations of a single basic model. What each did, in effect, was to use it as a foil. They had particular ideas about modern society and constructed a directly contrary account of primitive society. Primitive society was the mirror image of modern society — or, rather, primitive society as they imagined it inverted the characteristics of modern society as they saw it. ... Once established, this kind of thinking was sustained by social inertia, like any other orthodoxy. At the same time it was never static. It lent itself to the most dazzling transformations. ... Boas could construct an alternative to Morgan, Radcliffe-Brown to Rivers, Leach to Lévi-Strauss, simply by realizing a new transformation of the basic model.

In the final analysis, Lévi-Strauss's category of elementary structures (closed, homogeneous systems with exogamous kin groups in which the Maussian principle of reciprocity generates a limited range of kin marriage formulae) represents yet another modality of the notion of primitive society, opposed to the category of complex structures and open systems in which modern society is situated (Kuper 1988:210-230).

The models and research strategies introduced in the present study represent a powerful new extension to the structure of orthodox kinship theory. If Kuper's analysis

is valid (and, admittedly, his arguments are most persuasive), further progress must involve an even more radical rethinking of our key assumptions.

NOTES

- 1 Berlinski (1986:241).
- 2 The anecdote is recounted in John Harte's splendid introduction to the modelling of environmental problems. See also Stewart (1989:215).
- 3 The first revolution is identified with the institution of scientific method by Galileo, Newton, and their successors. The second revolution introduced the theory of relativity, quantum physics and other fundamental paradigm shifts around the turn of the century. In terms of the Kuhnian framework, these fundamental transformations are not, of course, the only scientific revolutions that have occurred (cf. Kuhn 1970). The so-called 'third revolution' in the study of complexity and 'chaos' now sweeping through a number of disciplines was not of course treated in Kuhn's analysis. See the references in note 4 below.
- 4 Gleick (1987), Davies (1987), and Stewart (1989) present fascinating accounts of recent developments. Gleick's highly readable book has also been reviewed for an anthropological journal (see Friedrich 1988). For a more technical introduction to the literature on chaotic dynamics and nonlinear systems, see Barnsley (1988), Mandelbrot (1982) (on fractals), and Berry et al. (1987), Glass and Mackey (1988), Hao Bailin (1985), Holden (1987), Prigogine and Stengers (1984), and Schuster (1984). Other relevant publications are mentioned throughout this chapter.
- 5 Although Lévi-Strauss opposes 'mechanical' (or deterministic) models to 'statistical' models, I would argue that the term 'probabilistic' provides a better gloss for many of the 'statistical' examples he refers to (cf. Lévi-Strauss 1953, 1966).
- 6 I am indebted to professor P.E. de Josselin de Jong for pointing out the following passage in the final chapter of *Tristes Tropiques* (Lévi-Strauss 1976 [1955]: 543): 'Anthropology could with advantage be changed into "entropology" [my emphasis], as the name of the discipline concerned with the study of the highest manifestations of this process of disintegration'.
- 7 By a curious coincidence, Claude Lévi-Strauss and Claude Elwood Shannon once lived in the same apartment building in Greenwich Village during the 1940s

- (Lévi-Strauss 1983:347).
- 8 For a more comprehensive discussion, see Barnard and Good (1984:104-106), Lounsbury (1964), Buchler and Selby (1968). Strictly speaking, the classic 'Crow' equations are: $FZS = F$, $FZD = FZ$, $\delta MBC = \delta C$, $\varphi MBC = \varphi BC$, and the analogous 'Omaha' equations are: $MBS = MB$, $MBD = M$, $\delta FZC = \delta ZC$, $\varphi FZC = \varphi C$. See Lowie (1917) and Murdock (1949:166-167).
 - 9 See McKinley (1971a, 1971b) and Barnes (1976, 1984).
 - 10 Casti's relativistic approach to the study of complexity is foreshadowed in earlier work by W.R. Ashby, H. Simon, and others (Casti 1986:169).
 - 11 The term is Héritier's. See Héritier (1981).
 - 12 See in particular, De Heusch (1974), Barnes (1975, 1982, 1984), McKinley (1971a, 1971b), and Muller (1978, 1980, 1981, 1982).
 - 13 From information in Poundstone (1987). Ulam has told Jeremy Campbell (1984:108) that Noam Chomsky's early work on generative grammars was the point of departure for his own work on computer games.
 - 14 As defined by Romney and D'Andrade (1964), δ stands for 'male', φ for 'female', $+$ for 'parent of', $-$ for 'child of', and $*$ for 'sibling of'. For example, the string $\delta + \varphi * \delta - \varphi$ stands for a man's mother's brother's daughter.
 - 15 See the discussion of the lattice of quotient structures of $M_4 \times P_4$ and the definition of a 'cover' in Chapter 1 (especially figure 1.6) for an analogous procedure.
 - 16 Lounsbury (1964) actually defines the Omaha skewing rules as follows: Skewing rule I ($FZ \dots \rightarrow Z \dots$) with corollary ($\dots \varphi BS \rightarrow \dots \varphi B$) and ($\dots \varphi BD \rightarrow \dots \varphi Z$); Skewing rule II ($FZ \rightarrow Z$) with corollary ($\varphi BS \rightarrow \varphi B$) and ($\varphi BD \rightarrow \varphi Z$); Skewing rule III ($\delta Z \dots \rightarrow \delta D \dots$) with corollary ($\dots \varphi B \rightarrow \dots \varphi F$). Hence Boyd et al. only define the *corollaries* to Lounsbury's skewing rules. However, if necessary, the complete set of rules and their corollaries may be easily defined as a set of equations (Crow rules as well as Omaha rules). See also the discussion by Greechie and Ottenheimer (1974).
 - 17 Actually, to the corollary of Lounsbury's (1964) Crow skewing rule I. See note 16 above.
 - 18 For other important developments of the semigroup approach to kinship studies, see Lehman and Witz (1974) and other contributions in Ballonoff (1974a), and the paper by Sydney Gould. Levin's method of analysis (1974) is a parallel development, although not explicitly formulated in terms of semigroup theory. See also Liu (1986); Liu has collaborated with Gould on the formalization of kinship structures.
 - 19 See also the comments by Read (1986), Greechie and Ottenheimer (1974), Ottenheimer (1985), and Jorion (1980, 1981, 1986). Scheffler's assumptions are apparently not subject to discussion: any formal theory

- of systems of kin classification must be consistent with the formal properties of genealogical extension (Scheffler 1986:369). See also Scheffler (1982, 1984). For a more general critique of transformational analysis as applied to kinship semantics, see Fjellman (1978), Borland (1979), Shapiro (1982), Woolford (1984), Hirschfeld (1986), and Givón (1989:355-367).
- 20 In the social sciences, semigroup theory has also been applied to the modelling of interactive behaviour, social networks and role structures, and other applications falling under the term 'blockmodel algebras'. See the short bibliography in Lidl and Pilz (1984:530), and the paper by Lorrain and White (1971).
 - 21 However, the Nuer say '*tut thilke rual*' ('there is no incest among bulls'): wives of a man's paternal half-uncles, paternal half-brothers and paternal cousins are 'wives of our cattle', and in a general social sense the wives of the 'bulls', i.e., of the joint family and of the lineage. To have relations with the wives of these close agnates is not considered *rual*, although it shows lack of respect and compensation may be asked for (Evans-Pritchard 1949:92, 100).
 - 22 The French version was first published in 1979.
 - 23 The informal notion of 'specialization' applied here should not be confused with the technical concept of an 'idealized specialization relation' as defined by Balzer et al. (1987:170).
 - 24 Further specialization of the same-sex and opposite-sex sibling relations is also necessary if one is to cope with the problem of modelling parallel-cousin marriage and with the influence of birth-order constraints on marriage choices. See, for example, Gottlieb's discussion of the multiple alliance models co-existing within the Beng marriage system and their relationship with birth-order (Gottlieb 1986).
 - 25 It is suggested that such a perspective, modelling the interaction of individual choices (from among a limited number of alternatives) and the emergent properties of the global system of exchange, might encompass the wide range of exchange phenomena described by, say, Tapper (1981) for the Durrani Pashtuns of Afghan Turkestan (or similar systems) within a single, unified framework.
 - 26 There is a small but growing number of publications on the Tenda. See in particular the volume edited by Gessain and de Lestrangé (1980) which includes a bibliography.

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DUTCH SUMMARY

Dit boek is bedoeld als een pleidooi voor het formaliseren van theorieën binnen de culturele antropologie, in het bijzonder van theorieën die betrekking hebben op verwantschap en sociale organisatie. Theorie-verandering binnen de antropologie wordt vaak gekenschetst als een niet-cumulatief verschijnsel: paradigma's verdringen elkaar, zonder dat men kan spreken van een wezenlijke cumulatie aan nieuwe inzichten. De ontwikkeling van een adequaat raamwerk voor het weergeven en verklaren van verwantschapsverschijnselen vereist allereerst een systematische analyse van theorie-structuren; een structurele reconstructie van centrale delen van verwantschapstheorie is noodzakelijk, wil men komen tot theorie-vergelijking en tot een kritische evaluatie van mogelijke theorie-verandering. Dit zijn de centrale thesen bij dit onderzoek.

Het methodologische perspectief op de reconstructie van verwantschapstheorieën is gebaseerd op de 'structuralistische' vooronderstellingen van Joseph Sneed, Wolfgang Stegmüller, Wolfgang Balzer en anderen: theorieën worden gespecificeerd als klassen van modellen en hun substructuren, en niet (zoals het logisch-positivisme veronderstelt), als de geïnterpreteerde uitspraken en formules van een formele calculus. De 'semantiek' van een theorie wordt geïntroduceerd door de centrale klasse van modellen rechtstreeks te definiëren door middel van een verzamelings-theoretisch predicaat, en niet met behulp van correspondentie regels. De modellen van een bepaalde theorie zijn wiskundige structuren, en de relatie die tussen deze modellen en de onderzochte verschijnselen wordt verondersteld is een relatie van homomorfie: een 'empirische bewering' is een toetsbare uitspraak over structurele overeenkomsten tussen de systemen van relaties die binnen een domein van verschijnselen worden onderscheiden, en bepaalde substructuren, d.w.z., substructuren van de

modellen die voor een bepaalde theorie zijn gedefinieerd. Deze algemene uitgangspunten worden in dit boek toegepast op een aantal centrale theorieën die binnen de antropologie zijn ontwikkeld voor de bestudering van verwantschap.

In Hoofdstuk 1 introduceer ik een aantal van de vroege verwantschapsmodellen die door de 'Leidse richting' zijn ontwikkeld. Daarna bespreek ik de 'structuralistische' benadering van theorieën en pas deze toe op de klassieke modellen van dubbel-unilineale afstamming en circulerend connubium. Door deze modellen te formaliseren als groepentheoretische structuren is het mogelijk om de volledige klasse van 'latente' of 'gereduceerde' structuren die uit de aannames van de vroege Leidse antropologen volgen, af te leiden, en om deze structuren te vergelijken met verwantschapsstructuren die beschreven zijn door Claude Lévi-Strauss en anderen.

De belangrijkste methodologische aannames en wiskundige concepten werden in Hoofdstuk 1 geïntroduceerd; in Hoofdstuk 2 pas ik deze toe op de theorie van 'elementaire verwantschapsstructuren' die door Lévi-Strauss is ontwikkeld. Een meer volledige familie van verwantschapsstructuren met asymmetrische ruil wordt afgeleid door ruilrelaties recursief te definiëren, d.w.z., opeenvolgende cycli worden geformuleerd als automorfismen van een specifiek systeem van asymmetrische relaties. Vervolgens toon ik aan dat substructuren van deze uitgebreide klasse van modellen isomorf zijn aan verwantschapstructuren die afkomstig zijn uit etnografische beschrijvingen.

Deze procedure wordt ook toegepast in Hoofdstuk 4, waar de Lévi-Straussiaanse modellen van symmetrische structuren worden geformaliseerd en verder gegeneraliseerd. In Hoofdstuk 3 formuleer ik een uitbreiding voor de klassieke modellen met exclusief matrilateraal cross-cousin huwelijk. Een groepentheoretische structuur met 'helische' ruilcircuits (in plaats van de traditionele gesloten connubia) wordt gecombineerd met een metrische structuur; deze nieuwe klasse van modellen biedt mogelijkheden voor de analyse

van verwantschapssystemen waarvoor systematische relatieve leeftijdsverschillen tussen echtgenoten worden beschreven. Ook hier blijkt het mogelijk te zijn om isomorfe relaties vast te stellen tussen de modellen en voorbeelden afkomstig uit etnografische beschrijvingen. De analyse laat tevens zien dat de oorspronkelijke theorie van Lévi-Strauss te beperkt is: de nieuwe klasse van 'helische' modellen is compatibel met regels voor de 'ruil' van andere nauwe vrouwelijke verwanten dan alleen maar zusters (zoals door Lévi-Strauss wordt aangenomen).

In Hoofdstuk 5 tenslotte ga ik nader in op het onderscheid dat door Lévi-Strauss wordt gemaakt tussen 'elementaire' en 'complexe' verwantschapssystemen enerzijds, en 'mechanische' en 'statistische' modellen anderzijds. Na een samenvatting van recente ontwikkelingen met betrekking tot 'complexiteit' en 'chaos'-theorie introduceer ik voorbeelden van dynamische systemen uit de theorie van cellulaire automaten. Deze voorbeelden laten zien dat 'simpele' regels en modellen niet noodzakelijk beperkt zijn tot het genereren van 'simpele' configuraties. Deze bevindingen, tezamen met de resultaten uit voorafgaande hoofdstukken, maken duidelijk dat de Lévi-Straussiaanse opposities ('elementair' vs. 'complex' en 'mechanisch' vs. 'statistisch') moeten worden herzien. Het hoofdstuk wordt afgesloten met een aantal concrete voorstellen voor een verdere uitbreiding van de modellen en suggesties voor de noodzakelijke reconstructie van verwantschapstheorie.

CURRICULUM VITAE

Franklin Edmund Tjon Sie Fat werd geboren op 23 november 1947 te Willemstad, Curaçao. Na het behalen van het HBS-B diploma aan de Algemene Middelbare School te Paramaribo, Suriname, ging hij in 1964 Wiskunde studeren aan de Rijksuniversiteit te Leiden; deze studie werd niet afge- maakt. In 1970 begon hij aan de studie Culturele Antropologie aan de Rijksuniversiteit te Leiden. Het kandidaatsexamen werd in 1974 afgelegd, waarna in 1978 het doctoraalexamen in de Culturele Antropologie werd behaald. Van 1973 tot en met 1978 was hij werkzaam als student-assistent, eerst bij de Vakgroep Methoden en Technieken (Subfakulteit Culturele Antropologie/Sociologie der Niet-Westerse Volken), later bij de afdeling Data- theorie. Van 1978 tot en met 1980, en van 1983 tot en met 1986 was hij (met onderbrekingen) op verscheidene korte aanstellingen verbonden aan de Rijksuniversiteit te Leiden. In 1981 verrichtte hij onderzoek op St. Maarten (Neder- landse Antillen) naar aspecten van sociale organisatie. Sinds juli 1986 is hij als part-time universitair docent verbonden aan het Instituut voor Cultureel Antropologie van de Rijksuniversiteit te Leiden.